CogSci 260: Introduction to Diffusion Tensor Imaging
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Week 5 Problem Set

Problem 1 The multivariate Gaussian distribution for 2 variables is:

$$p(x,y) = \frac{1}{\sqrt{(2\pi)^2|\mathbf{\Sigma}|}} \exp\left[-\frac{1}{2}(\mathbf{x}-\mu)^t \mathbf{\Sigma}^{-1}(\mathbf{x}-\mu)\right] \tag{1}$$

where

$$\mathbf{\Sigma} = \begin{pmatrix} a & c \\ c & b \end{pmatrix} \tag{2}$$

The inverse covariance matrix is

$$\mathbf{\Sigma}^{-1} = \frac{1}{ab-c^2}\begin{pmatrix} b & -c \\ -c & a \end{pmatrix} \tag{3}$$

and the *determinant* of the matrix $\mathbf{\Sigma}$ is

$$|\mathbf{\Sigma}| = ab - c^2 \tag{4}$$

Problem 1a Now assume that this distribution was estimated from a diffusion experiment. Write Σ^1 in terms of the diffusion coefficients $\{D_{xx}, D_{xy}, D_{yx}, D_{yy}\}$.

Problem 1b Write out the expression for $p(x,y)$ for $D_{xy} = 0$. What simplified form does $p(x,y)$ take?

Problem 1c What can we do to Σ to make this simplification true even if $D_{xy} \neq 0$?

Problem 2 Use the matlab program fit1D to estimate the diffusion coefficient from the signal

$$s(b) = s(0)e^{-bD} + \eta(b) \tag{5}$$

You can hit 'return' to begin with to use default values One plot shows an exponential fit, the other the linear approximation to the log fit.

Problem 2a What are these different terms in eq. (5)?

Problem 2b What happens when you use more samples for the same range of experimental parameters?

Problem 2c Now extend the range of sampling to higher b-values. What happens to the different fit methods, and why?

Problem 3 See fit2D.m - problems are embedded in the program - just search for the problem number.

Problem 3a (See fit2D.m)
Problem 3b (See fit2D.m)
Problem 3c (See fit2D.m)
Problem 3d (See fit2D.m)
Problem 3e (See fit2D.m)
Problem 3f (See fit2D.m)
Problem 3g (See fit2D.m)

Problem 3h (See fit2D.m)
Problem 3i (See fit2D.m)
Problem 3j
What is being plotted in the different figures?