CogSci 260: Introduction to Diffusion Tensor Imaging
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Week 3 Problem Set

Problem 1

Problem 1a

1. $\omega_0 = \gamma B_0 = 42.57Mhz/T \times 3T = 127.74Mhz$
2. $m_{\|}(t = 0) = M_0 \hat{z}$
3. $m_{\perp}(t = 0) = 0$

Problem 1b

1. $m_{\perp}(t = \tau) = M_0 e^{-\tau/T_2^*}$
2. $m_{\perp}(t = \tau_E) = M_0 e^{-\tau_E/T_2}$
3. T_2^* is due to local field inhomogeneities and is reversible (by the 180) whereas T_2 is due to random microscopic interactions and is not reversible.

Problem 2

Consider a normal human brain in the a 3T human MRI scanner.

Problem 2a The MRI signal is

$$s(\mathbf{k}) = \int m_{\perp}(\mathbf{x}, t) e^{-i\mathbf{k}\cdot\mathbf{x}} d\mathbf{x} \tag{1}$$

1. $i = \sqrt{-1}$
2. $\mathbf{k} = \gamma G_x t$
3. $\mathbf{x}$ is the spatial coordinate
4. The "·" $\mathbf{k} \cdot \mathbf{x}$ is the vector dot product and expresses the fact that the gradient produces a phase along a particular direction.

Problem 2b

1. $$\vec{G} = \begin{pmatrix} G_x \\ G_y \\ G_z \end{pmatrix} \tag{2}$$
2. $$\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{3}$$
3. $\mathbf{k} \cdot \mathbf{x} = k_x x + k_y y + k_z z = \gamma G_x xt + \gamma G_y ty + \gamma G_z tz$

Problem 2c

The applied gradients are $G_x = 1G/cm$, $G_y = 1G/cm$ and $G_z = 0G/cm$.

1. $\phi = \tan^{-1}(1) = 45°$
2. $\phi = \tan^{-1}(1/2) \approx 27°$
3. $\phi = \tan^{-1}(1/2) \approx 27°$