CogSci 260: Introduction to Diffusion Tensor Imaging
Instructor: L. Frank (lfrank@ucsd.edu)

Lecture 3 Problem Set

1. Vectors

Problem 1a

Consider two vectors u and v:

$$u = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad , \quad v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

1. $u^t v = ax + by + cz$
2. $u \cdot v == ax + by + cz$
3. $\sum_{i=1}^{3} u_i v_i == ax + by + cz$
4. The length of vector u is $\sqrt{u \cdot u} = \sqrt{a^2 + b^2 + c^2}$

Problem 1b

1. $\|u_{\|}\| = \|\left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}\| = \|\vec{u}\|\cos(\theta)$
2. $\vec{u} \times \vec{v} = |u||v|\sin(\theta)\hat{n}$ where $\hat{n}$ is perpendicular to the plane containing both $\vec{u}$ and $\vec{v}$.

2. Matrices

Problem 2a

1.

$$m^t = \begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix} \tag{2}$$

2.

$$m + m = 2\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 2c & 2d \\ 2e & 2f \end{pmatrix} \tag{3}$$

3.

$$m^t m = \begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = \begin{pmatrix} a^2 + c^2 + e^2 & ab + cd + ef \\ ab + cd + ef & b^2 + d^2 + f^2 \end{pmatrix} \tag{4}$$

Problem 2b

$$\begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = \begin{pmatrix} a & b & c \\ \alpha & \beta & \gamma \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{5}$$

3. Complex numbers

Problem 3a

1. $z_1 + z_2 = (a + c) + i(b + d)$
2. $z_1 * z_2 = (a + ib) * (c + id) = (ac - bd) + i(bc + ad)$
3. $z_1/z_2 = \left(\frac{a+ib}{c+id}\right) = \left(\frac{ac+bd}{c^2+d^2}\right) + i\left(\frac{bc-ad}{c^2+d^2}\right)$
4. $z_1 = A_1 \exp(i\theta_1)$ where $A_1 = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}(b/a)$
   $z_2 = A_2 \exp(i\theta_2)$ where $A_2 = \sqrt{c^2 + d^2}$ and $\theta = \tan^{-1}(d/c)$
5. Amplitude: $A = \sqrt{(a + c)^2 + (b + d)^2}$
   Phase: $\phi = \tan^{-1}\left(\frac{b+d}{a+c}\right)$
6. Amplitude: $A = \sqrt{(ac - bd)^2 + (bc + ad)^2}$
   Phase: $\phi = \tan^{-1}\left(\frac{bc+ad}{ac-bd}\right)$
7. Amplitude: $A = \sqrt{\frac{a^2+b^2}{c^2+d^2}}$
   Phase: $\phi = \tan^{-1}\left(\frac{bc-ad}{ac+bd}\right)$