CogSci 260: Introduction to Diffusion Tensor Imaging
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Lecture 3 Problem Set

1. Vectors

Problem 1a

Consider two vectors u and v:

$$u = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad , \quad v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

1. Compute $u^t v$
2. Compute $u \cdot v$
3. Compute $\sum_{i=1}^{3} u_i v_i$
4. What is the length of vector u?

Problem 1b

1. Consider to arbitrary vectors u and v that both start at the origin $(0, 0)$ but point in different directions. The angle between the vectors is θ. What is the length of the projection of u onto v?
2. What is the cross product of u and v in terms of the angle θ, and what direction is it pointing?

2. Matrices

Problem 2a Consider the matrix

$$m = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \tag{2}$$

1. What is m^t ?
2. What is $(m + m)$?
3. What is $m^t m$?

Problem 2b Consider the equations

$$\begin{aligned} g_1 &= ax + by + cz \\ g_2 &= \alpha x + \beta y + \gamma z \end{aligned}$$

Write these two equations as a product of a matrix and a column vector

3. Complex numbers

Problem 3a

Consider the two complex numbers

$$z_1 = a + ib \tag{3a}$$
$$z_2 = c + id \tag{3b}$$

where a, b, c, d are real numbers.

1. What is $z_1 + z_2$?
2. What is $z_1 * z_2$?
3. What is z_1/z_2?
4. Write z_1 and z_2 in terms of complex exponentials using Euler's relation.
5. What is the amplitude and phase of $z_1 + z_2$?
6. What is the amplitude and phase of $z_1 * z_2$?
7. What is the amplitude and phase of z_1/z_2?