Lecture 17
Tissue Models and Q-Space Imaging

Physiological basis

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Review Article

The basis of anisotropic water diffusion in the nervous system – a technical review

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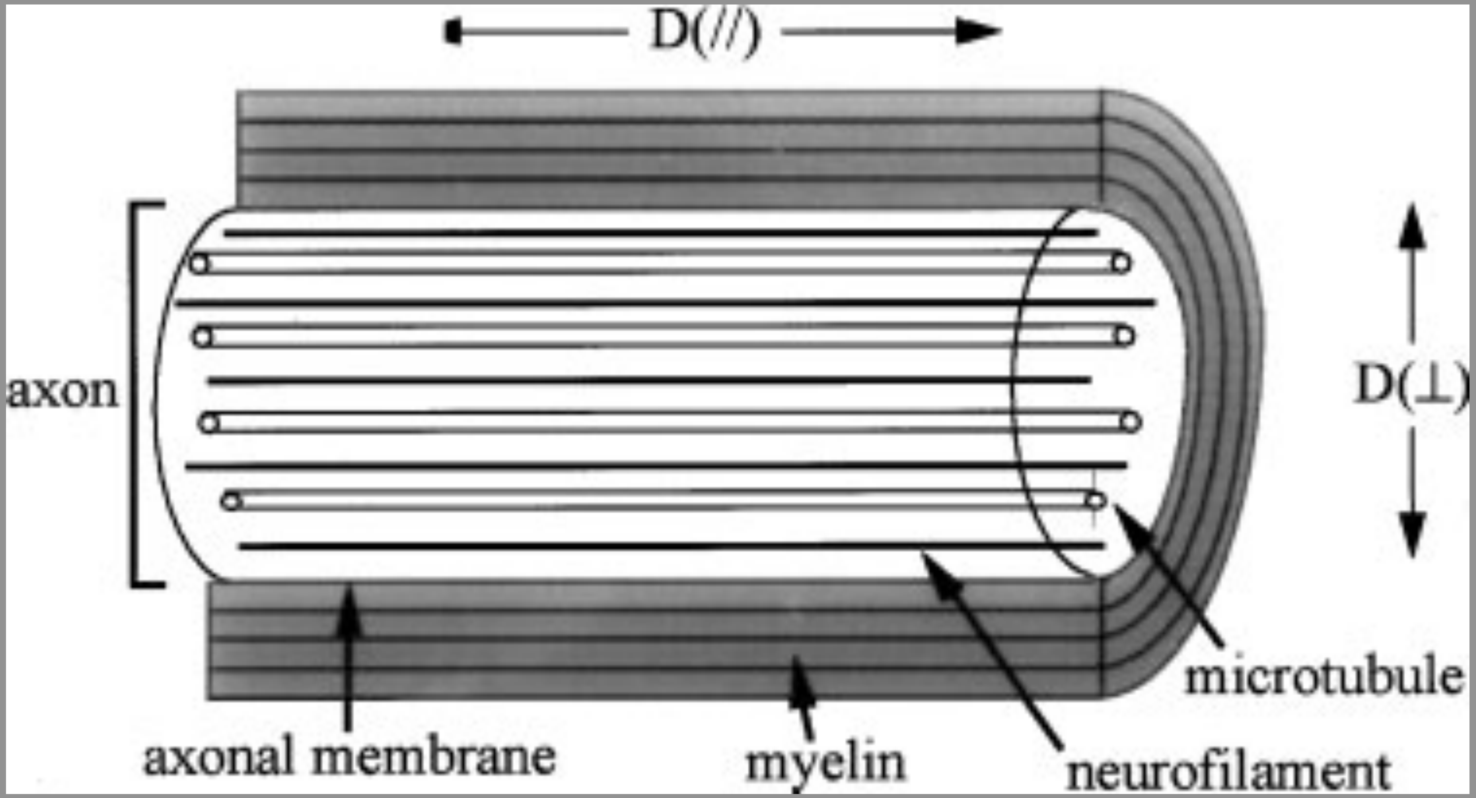
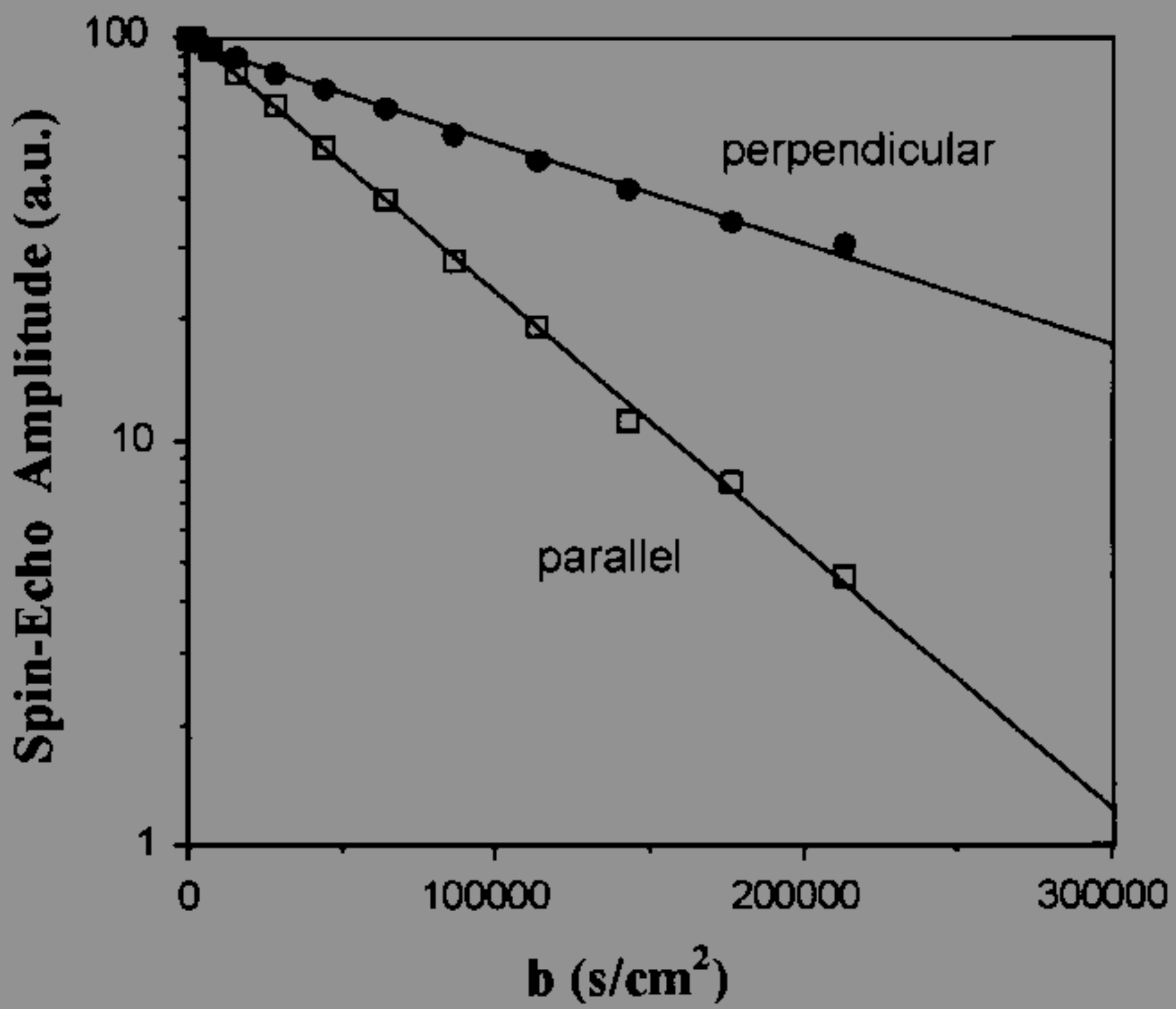
Diffusion in Fibers



Myelin schematic showing longitudinally oriented structures that could hinder water diffusion perpendicular to the length of the axon and cause

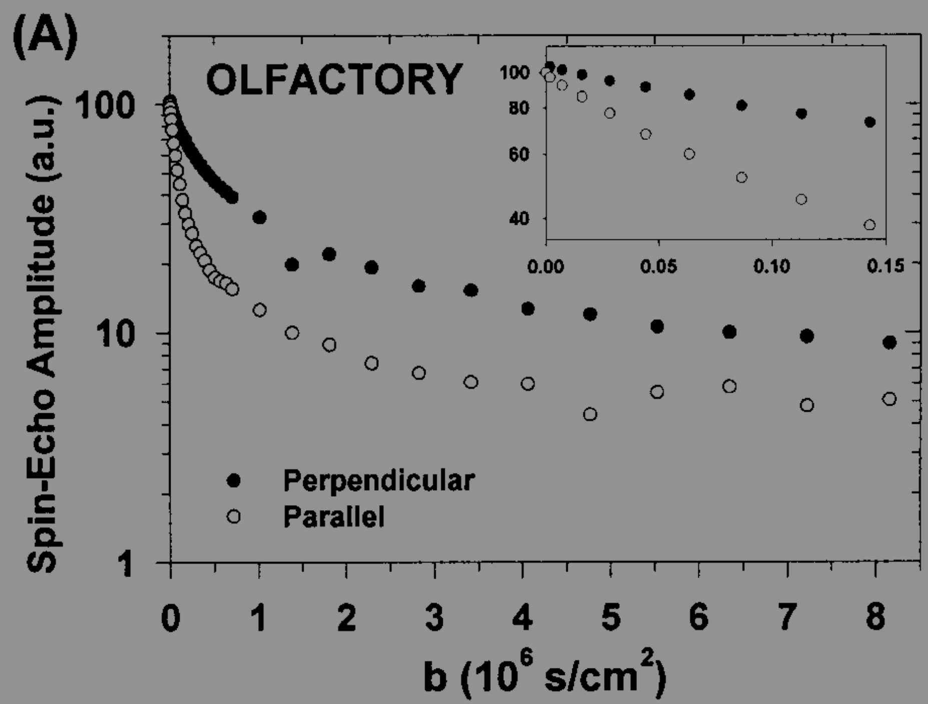
$$D_\perp < D_\parallel$$

Diffusion in Fibers

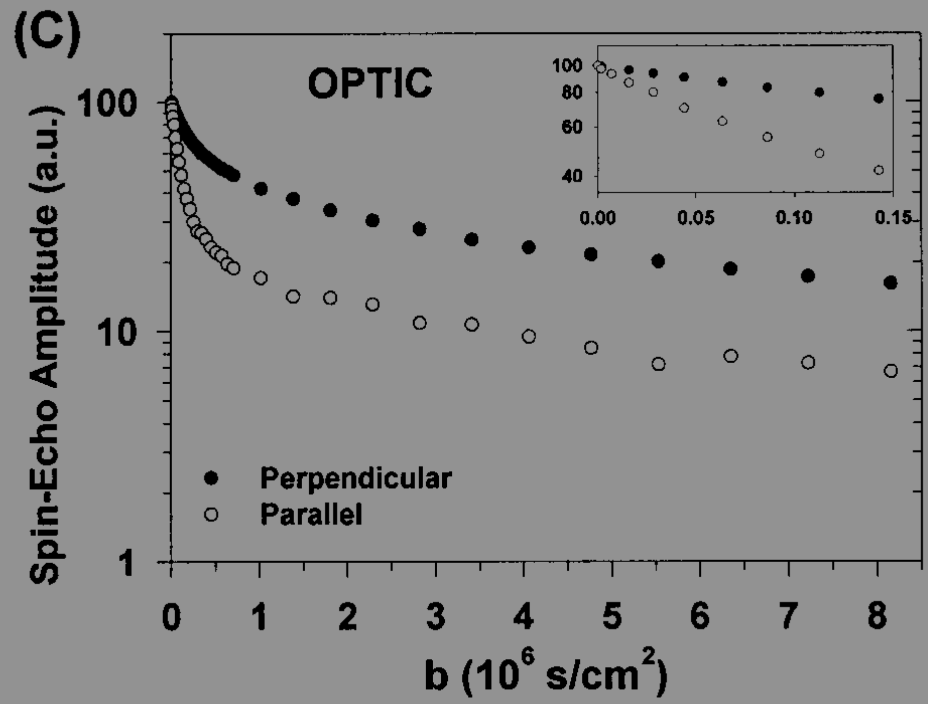


Diffusion curves of water in vascular bundles of celery measured parallel and perpendicular to their long axis

Diffusion in Gar Fish Neural Fibers



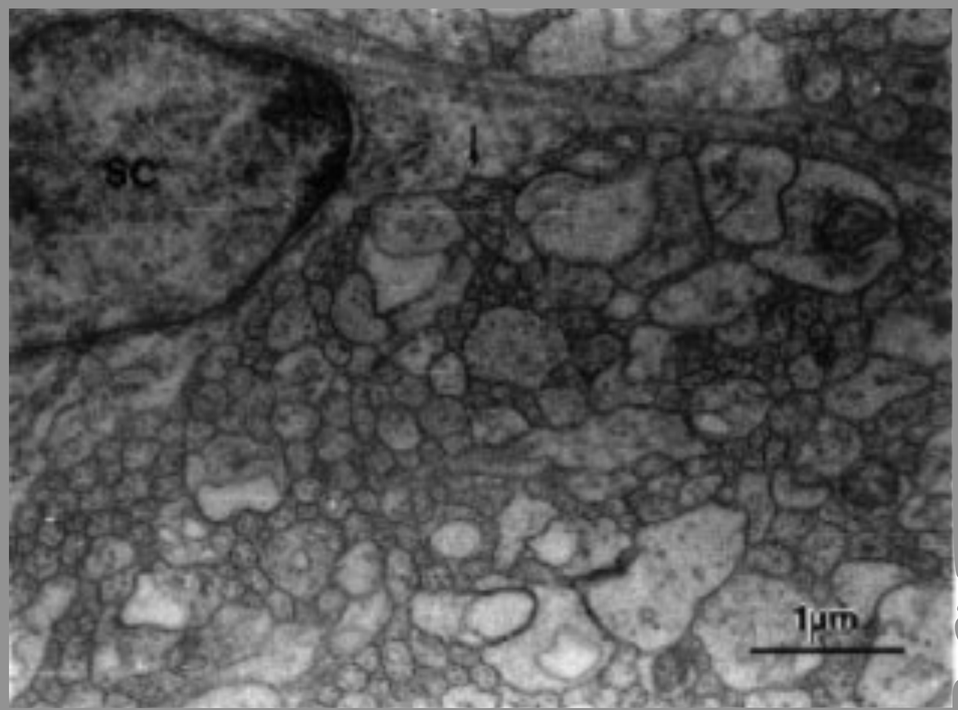
non-myelinated

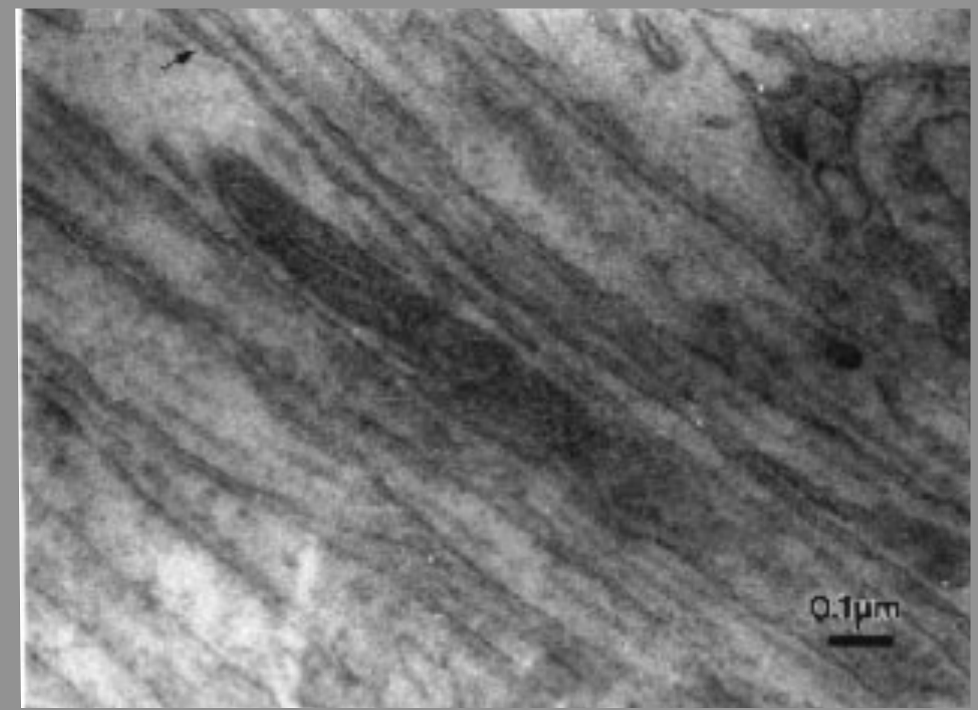


myelinated

Non-monoexponential!

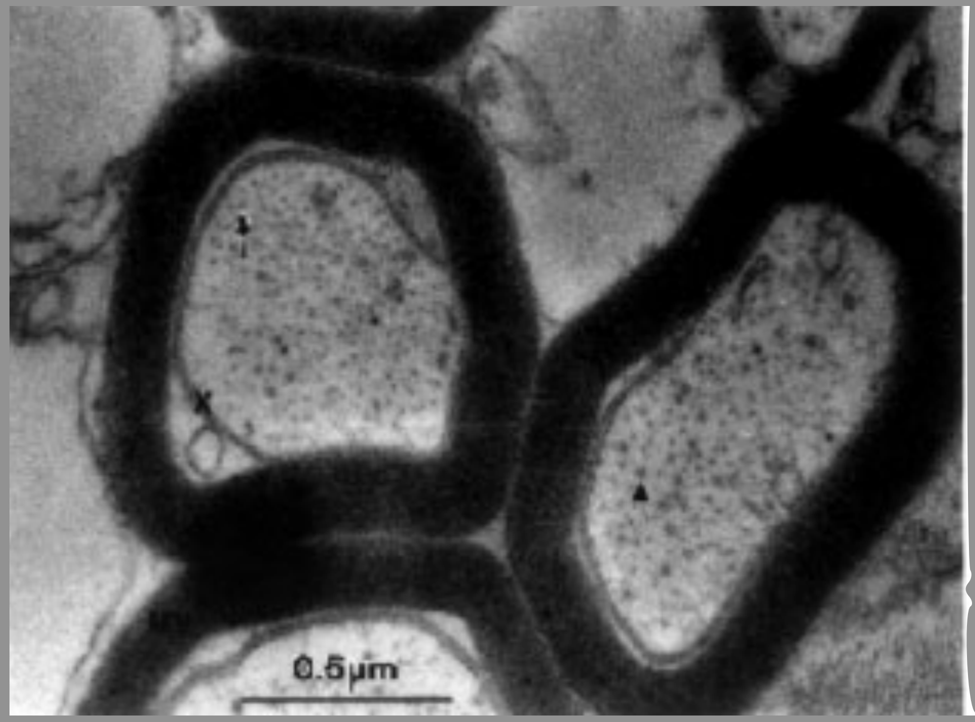
Diffusion in Gar Fish Neural Fibers





non-myelinated

diameter $\approx 0.25 \mu m$



myelinated

diameter $\approx 1 \mu m$

Diffusion in Gar Fish Neural Fibers

$$\text{Olfactory: } \frac{ADC_{\parallel}}{ADC_{\perp}} = 3.6$$

non-myelinated

$$\text{Optic nerve: } \frac{ADC_{\parallel}}{ADC_{\perp}} = 2.6$$

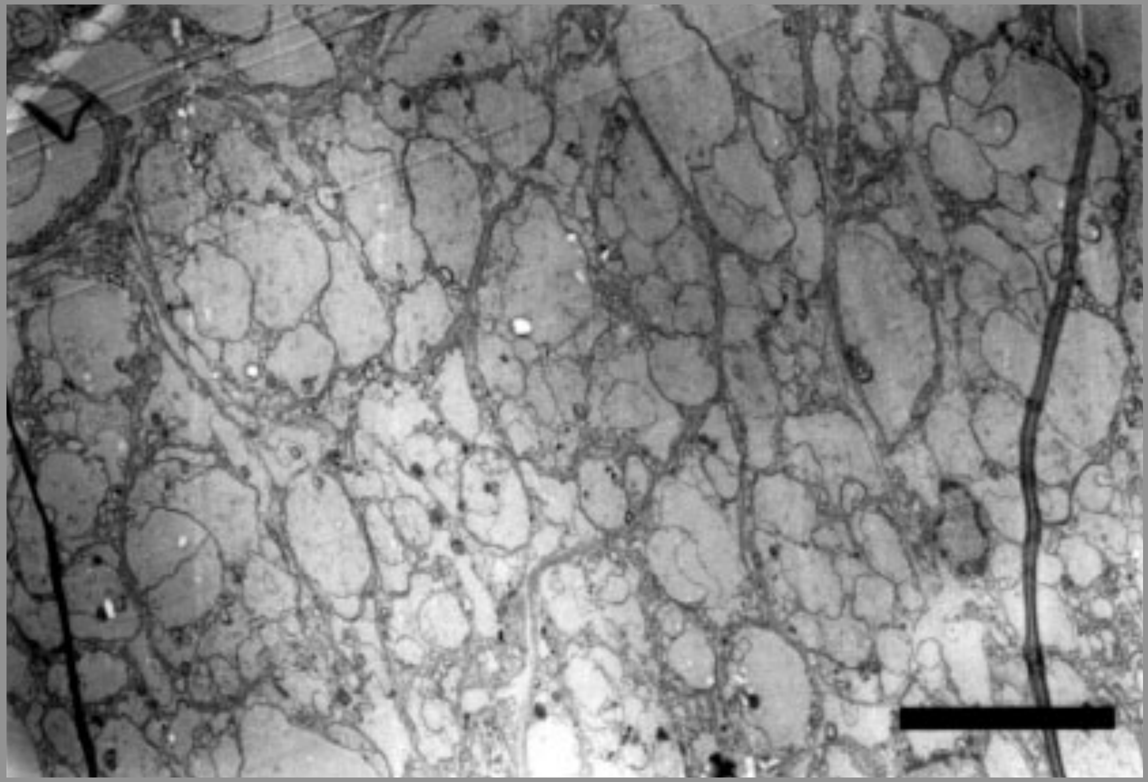
myelinated

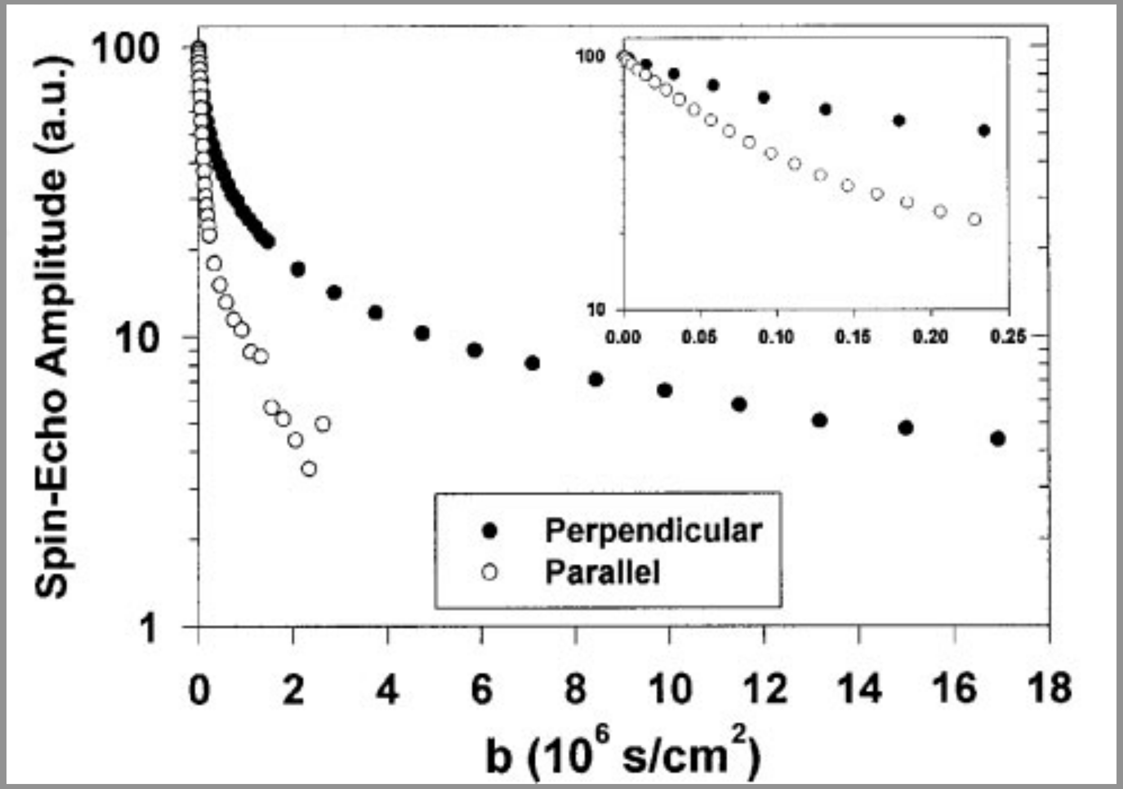
Diffusion in Gar Fish Neural Fibers

Myelination can modulate the degree of anisotropy

Increase anisotropy by a certain, albeit unknown, extent due to greater hindrance to intra-axonal diffusion and greater tortuosity for extra-axonal diffusion.

Diffusion in Lobster Leg muscle





non-myelinated

The Diffusion Signal

Signal and Distribution are
Fourier Transform pairs

$$\mathfrak{s}(q,\tau) = \int P(\bar{r},\tau)e^{-iq\cdot\bar{r}}d\bar{r}$$

$$\mathfrak{s}(q,\tau) \equiv \frac{s(q,\tau)}{s(0)}$$

$$P(\bar{r},\tau) = \int \mathfrak{s}(q,\tau)e^{iq\cdot\bar{r}}dq$$

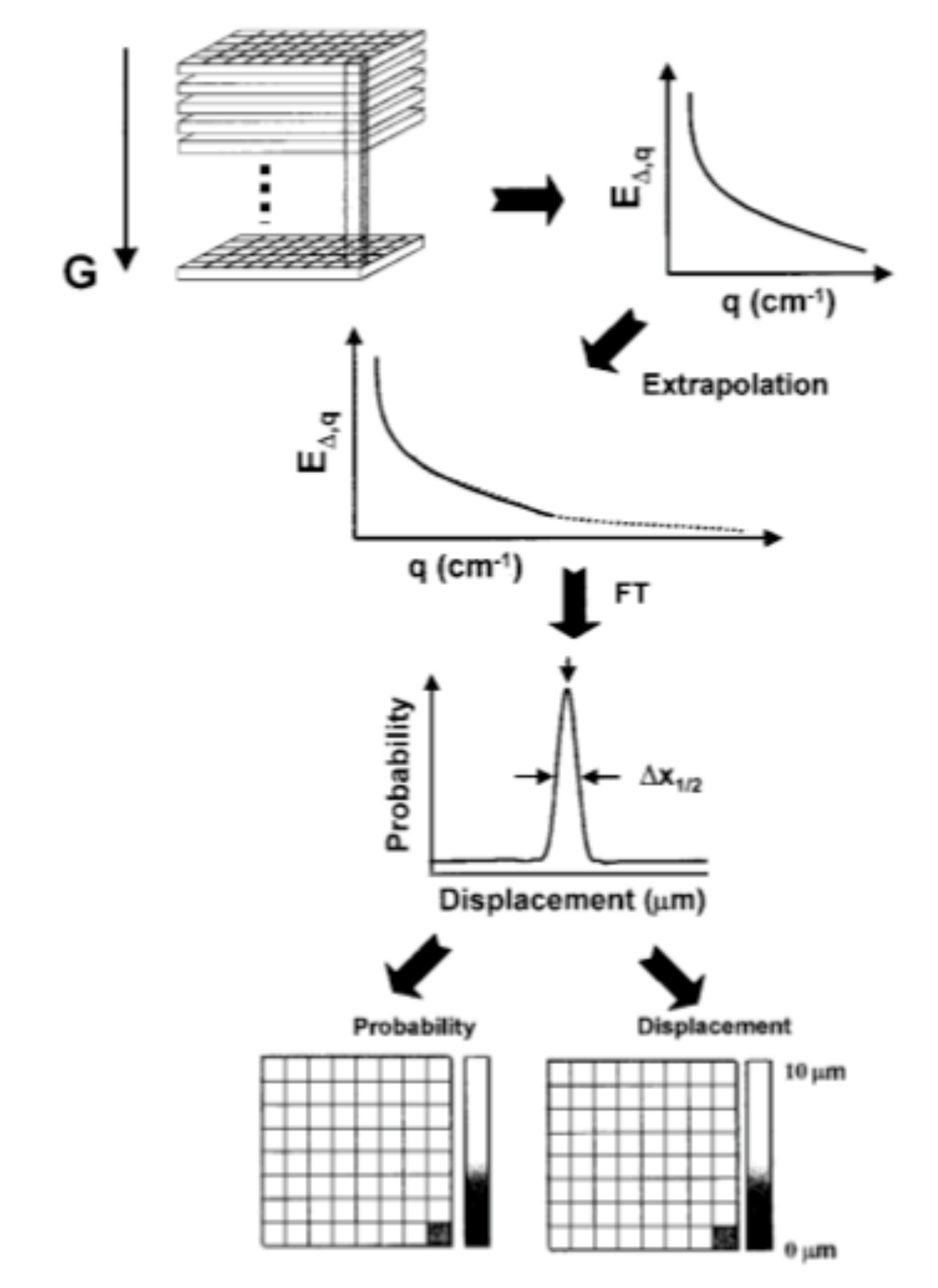
q-space imaging

Signal decay

$$E(\boldsymbol{q}, \Delta) = \int P(\boldsymbol{r}, \Delta) e^{i\boldsymbol{q}\cdot\boldsymbol{r}} d\boldsymbol{r}$$

q-space imaging

2D images with different q-values



Signal as a function of q

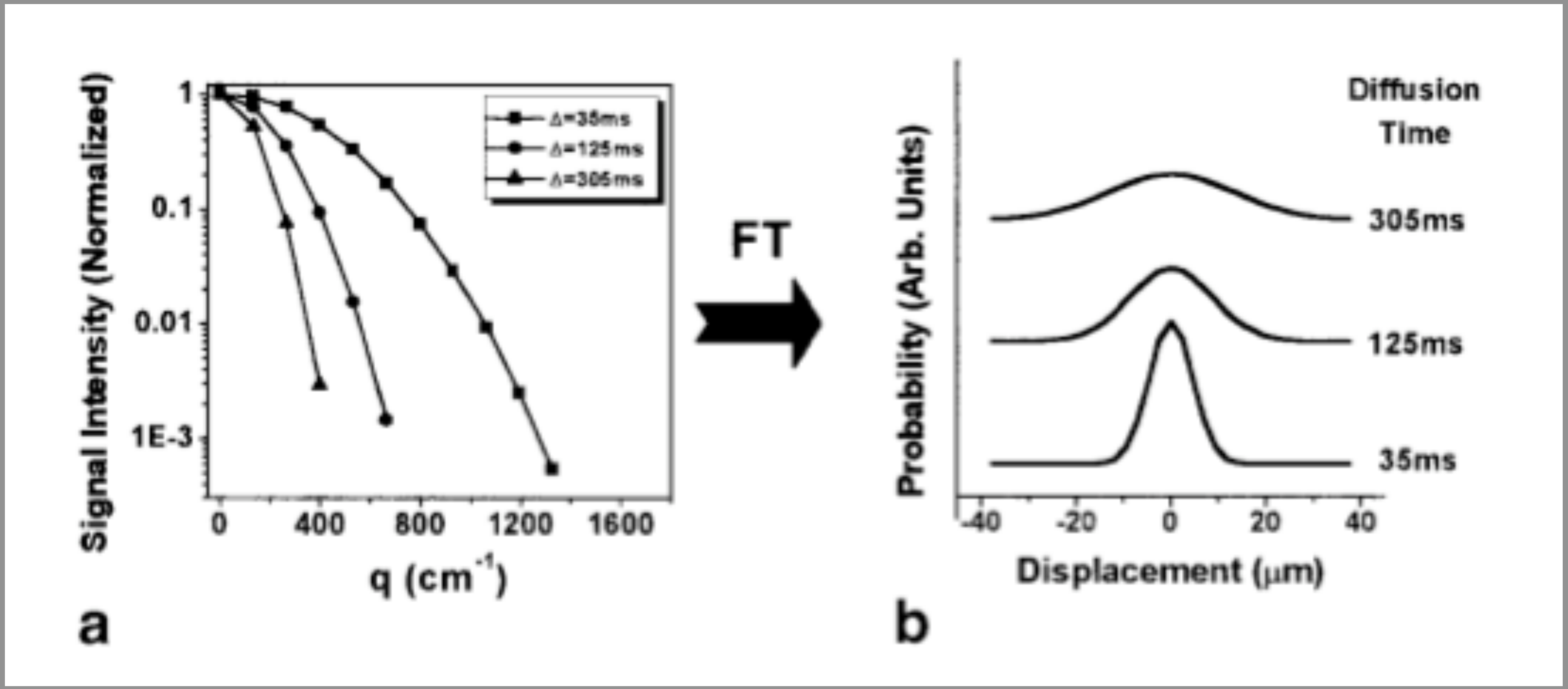
Displacement prob via Fourier Transform

Probability from peak

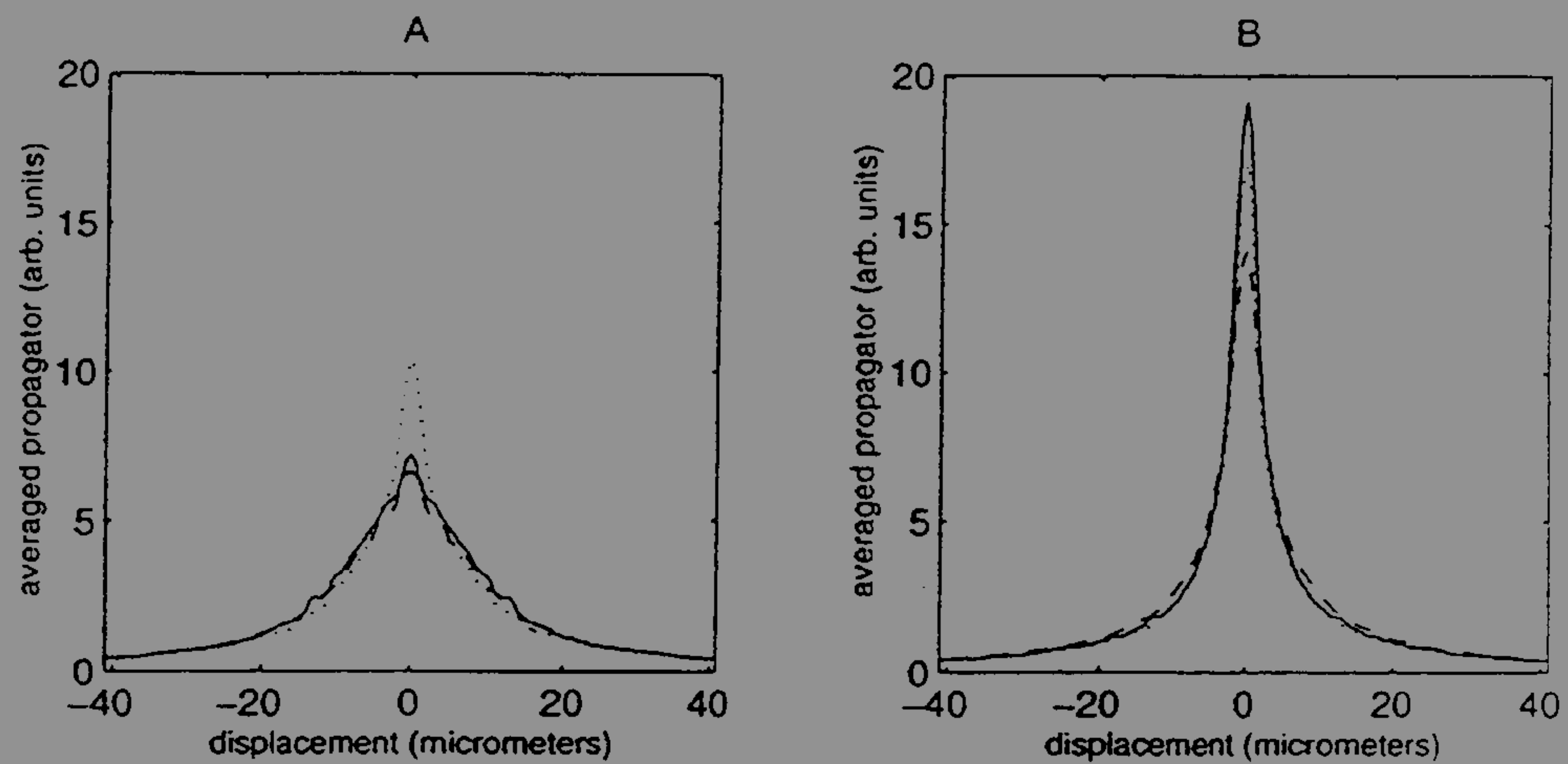
Displacement from width

Assaf, et. al. MRM 44:713 (2000)

q-space imaging



q-space imaging in Gar Fish Neural Fibers



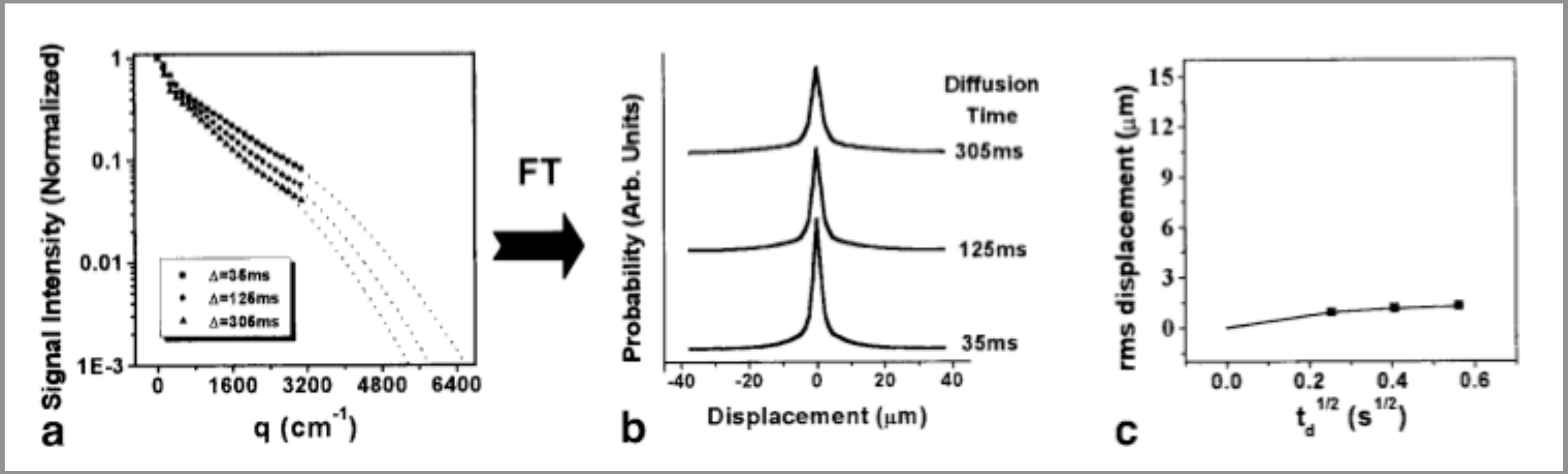
parallel

perpendicular

non-myelinated

Beaulieu, et al, 2001

q-space imaging in spinal cord

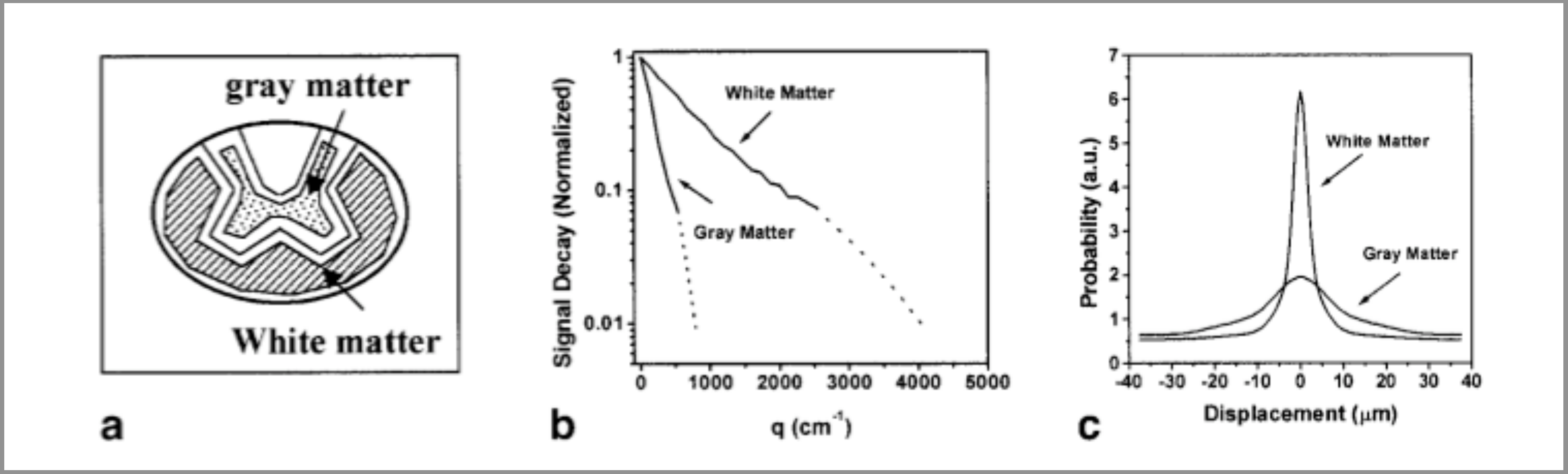


q-space analysis of
restricted water in a bovine
optic nerve.

D cannot be calculated from the slope of the graph in (c)

Assaf, et. al. MRM 44:713 (2000)

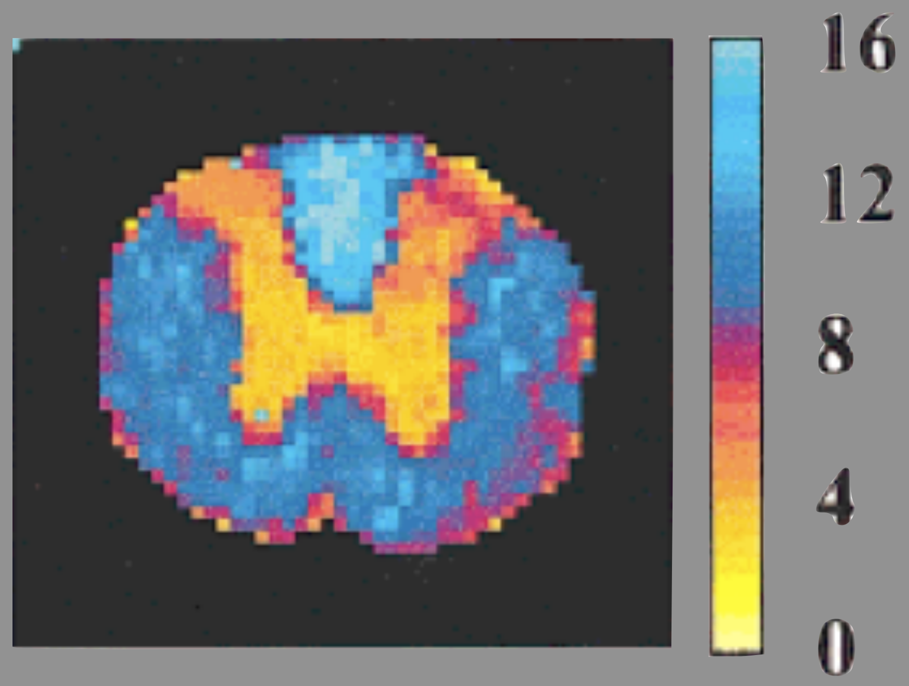
q-space imaging in spinal cord



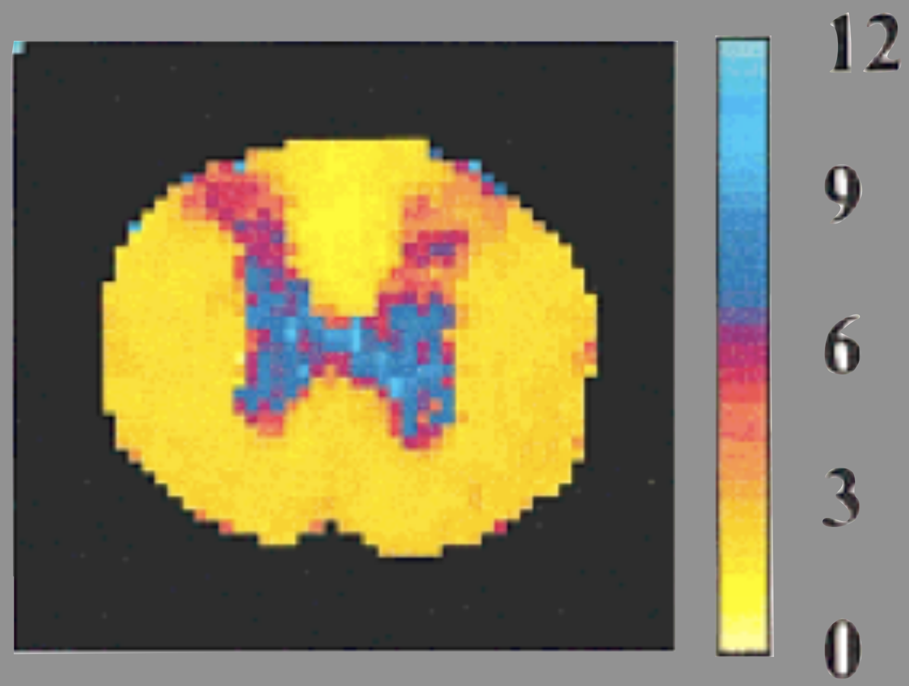
Hypothetical gray/white matter q-space experiment

Assaf, et. al. MRM 44:713 (2000)

q-space imaging in spinal cord



Probability of zero displacement (arbitrary units)

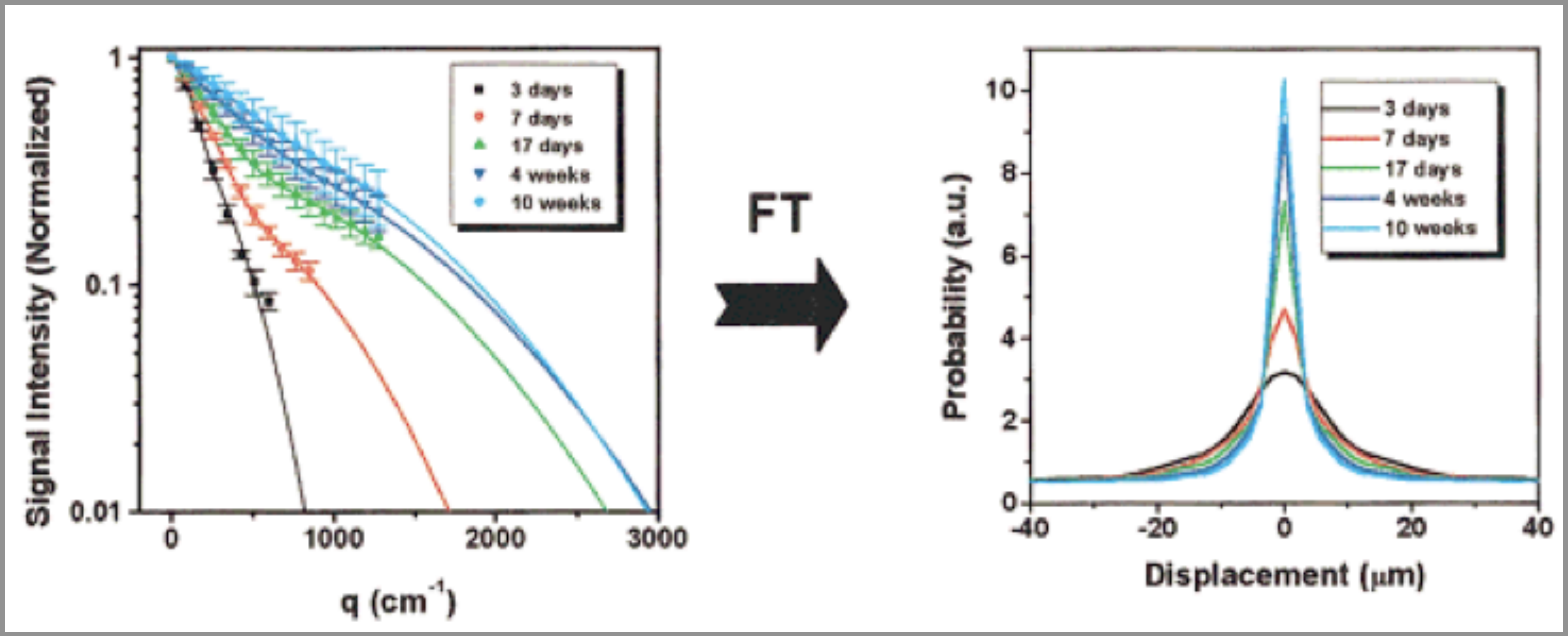


Displacement (microns)

Data acquired only in one direction
- perpendicular to cord direction

Assaf, et. al. MRM 44:713 (2000)

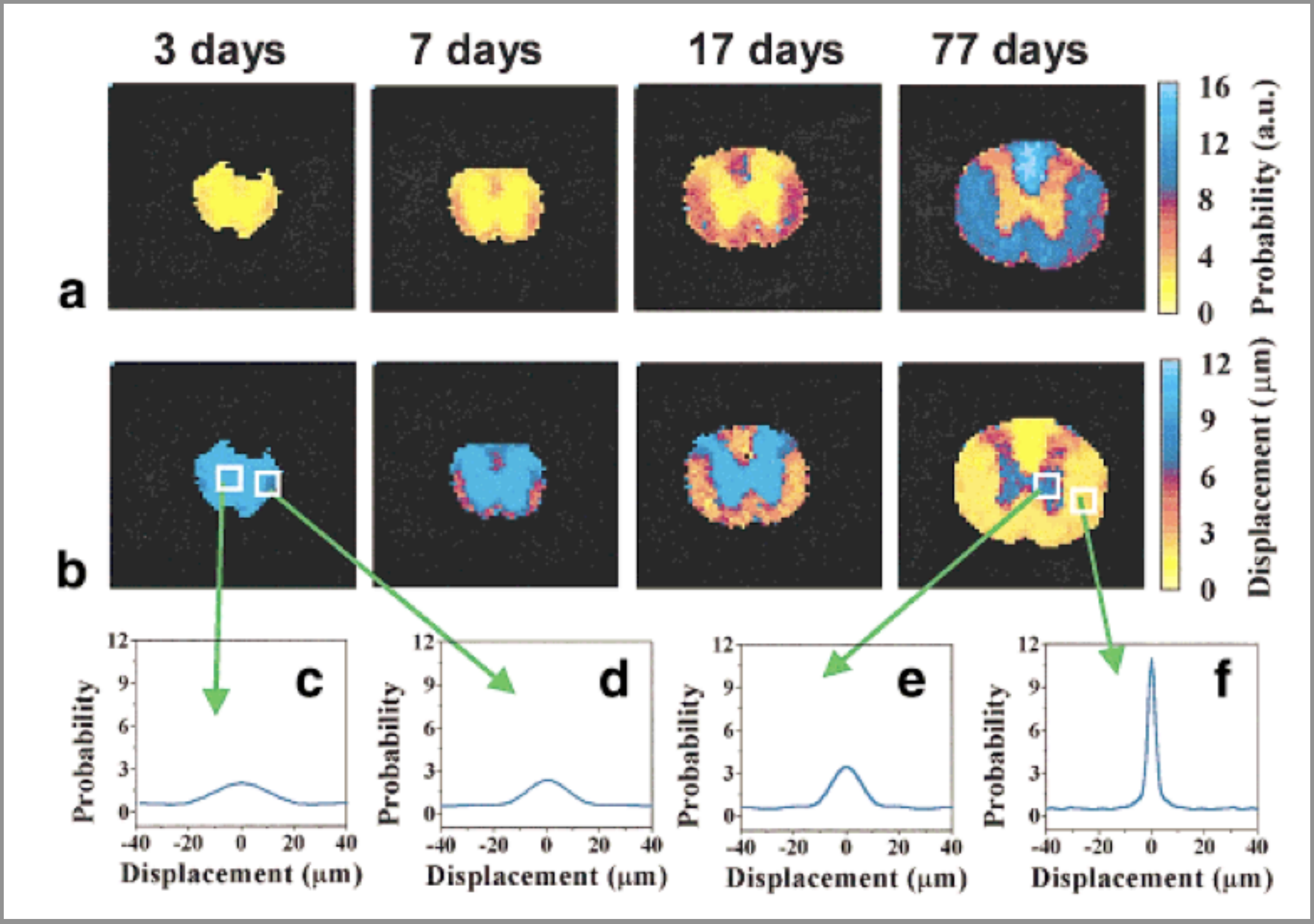
q-space imaging in spinal cord



Maturation of rat spinal cord

Assaf, et. al. MRM 44:713 (2000)

q-space imaging in spinal cord



Assaf, et. al. MRM 44:713 (2000)

Spherical Harmonic Decomposition

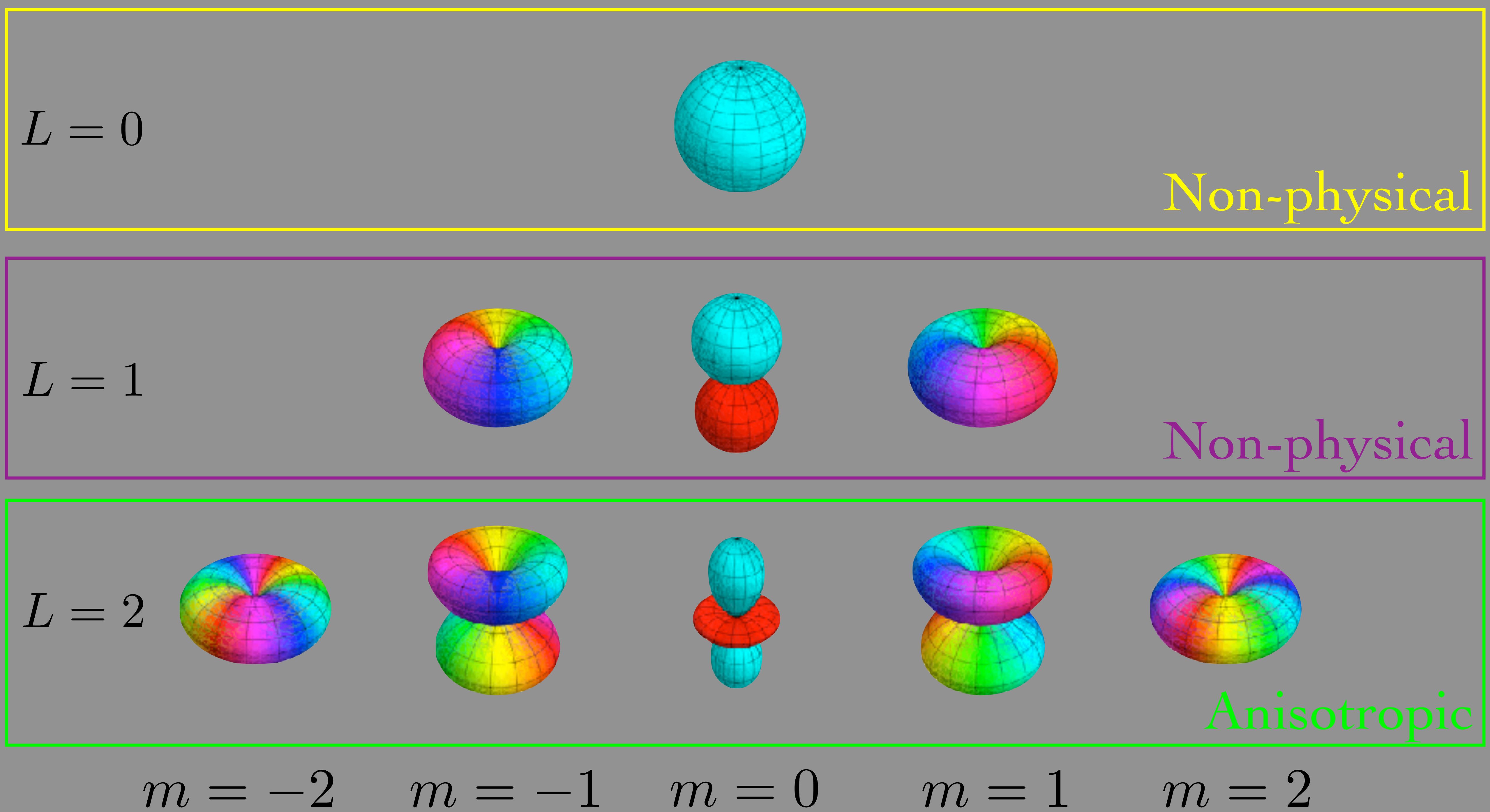
the signal

$$S(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} s_{lm} Y_{lm}(\theta, \phi)$$

the signal coefficients

$$s_{lm} = \int_0^{2\pi} \int_0^{\pi} Y_{lm}^*(\theta, \phi) S(\theta, \phi) \sin\theta \, d\theta \, d\phi$$

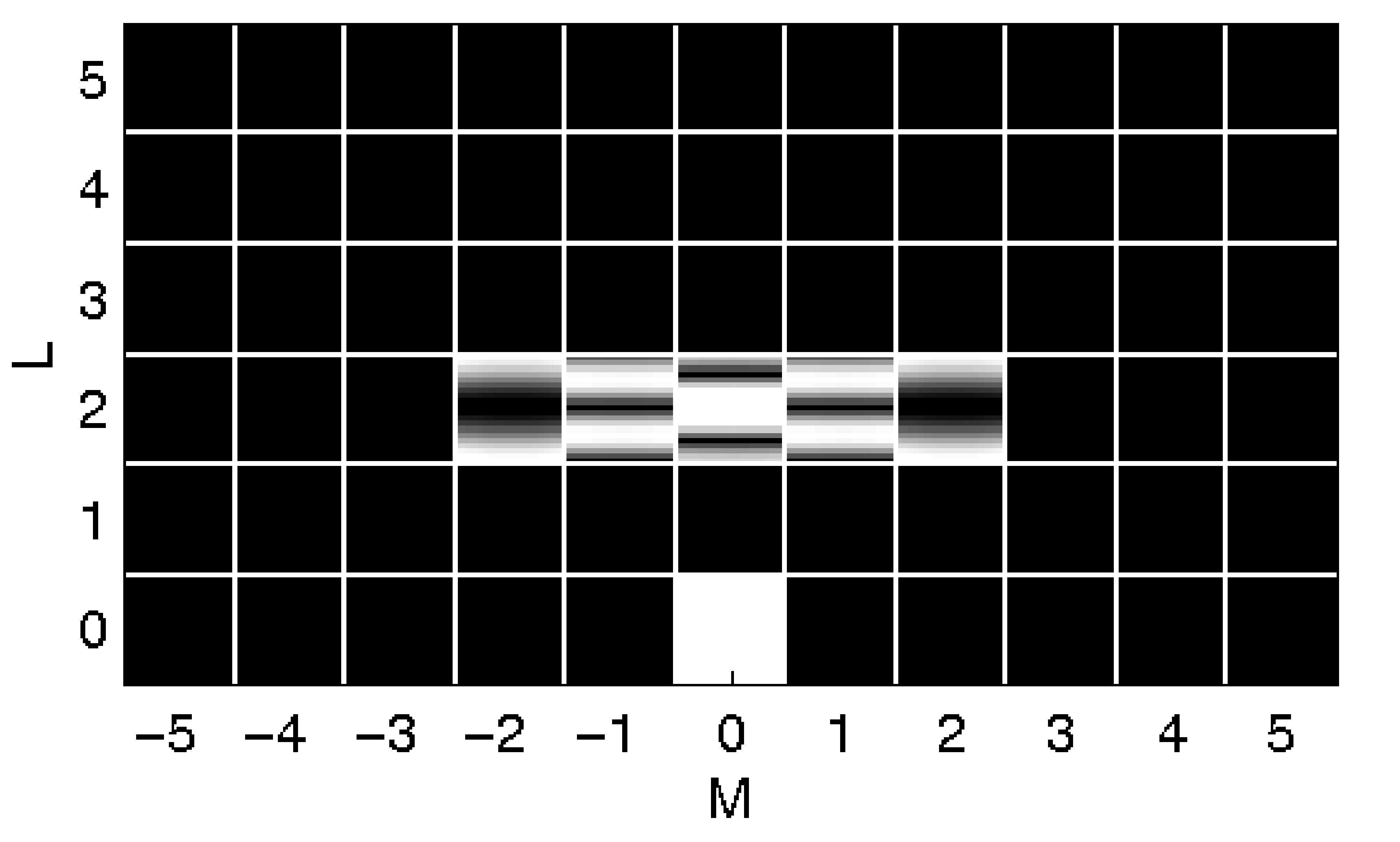
Spherical Harmonics



Frank, MRM *47*:1083 (2002)

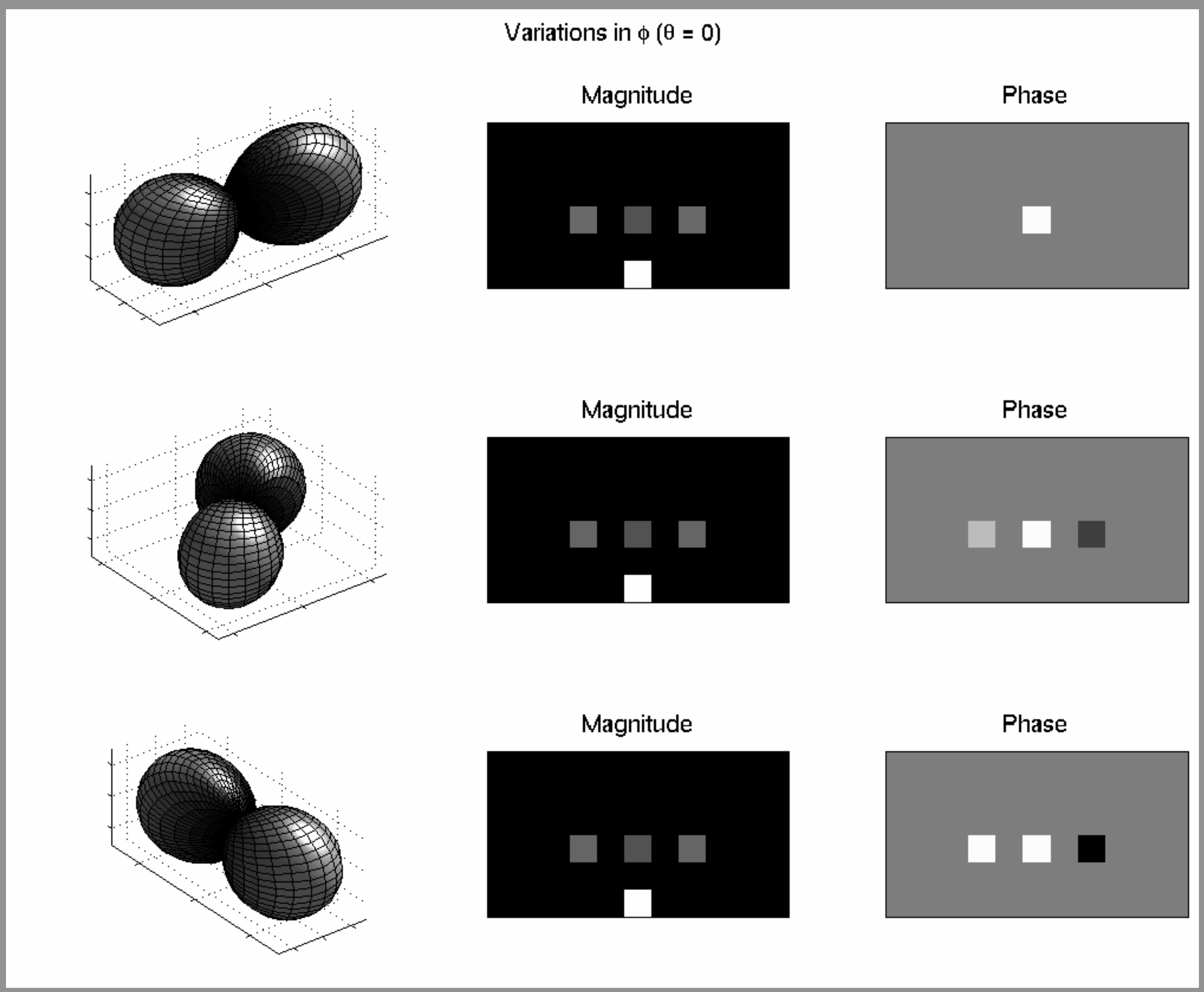
The Spherical Harmonic Decomposition

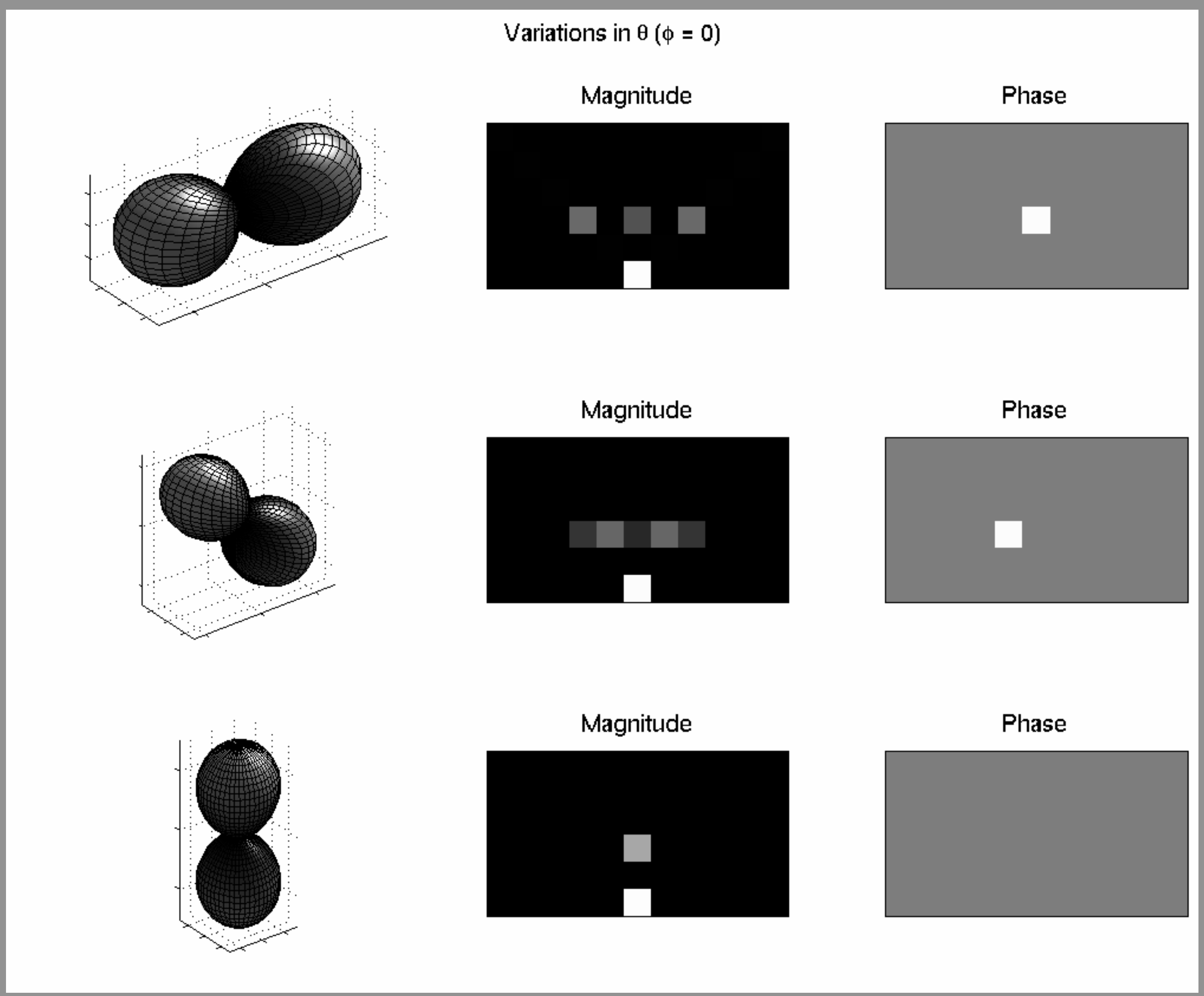
Single Fiber



fiber rotated through a full range of (θ, ϕ)

The Spherical Harmonic Decomposition

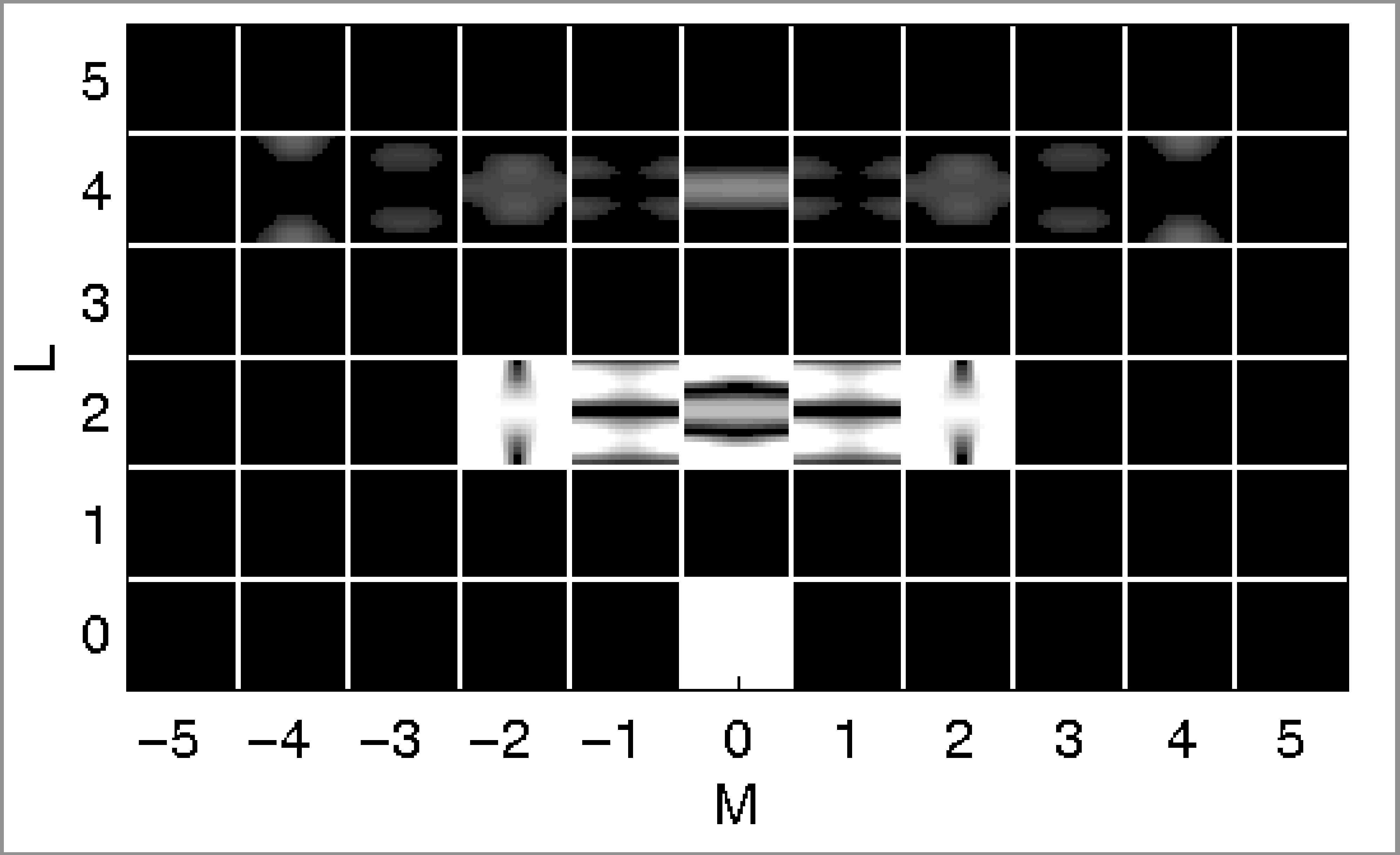




Frank, MRM *47*:1083 (2002)

The Spherical Harmonic Decomposition

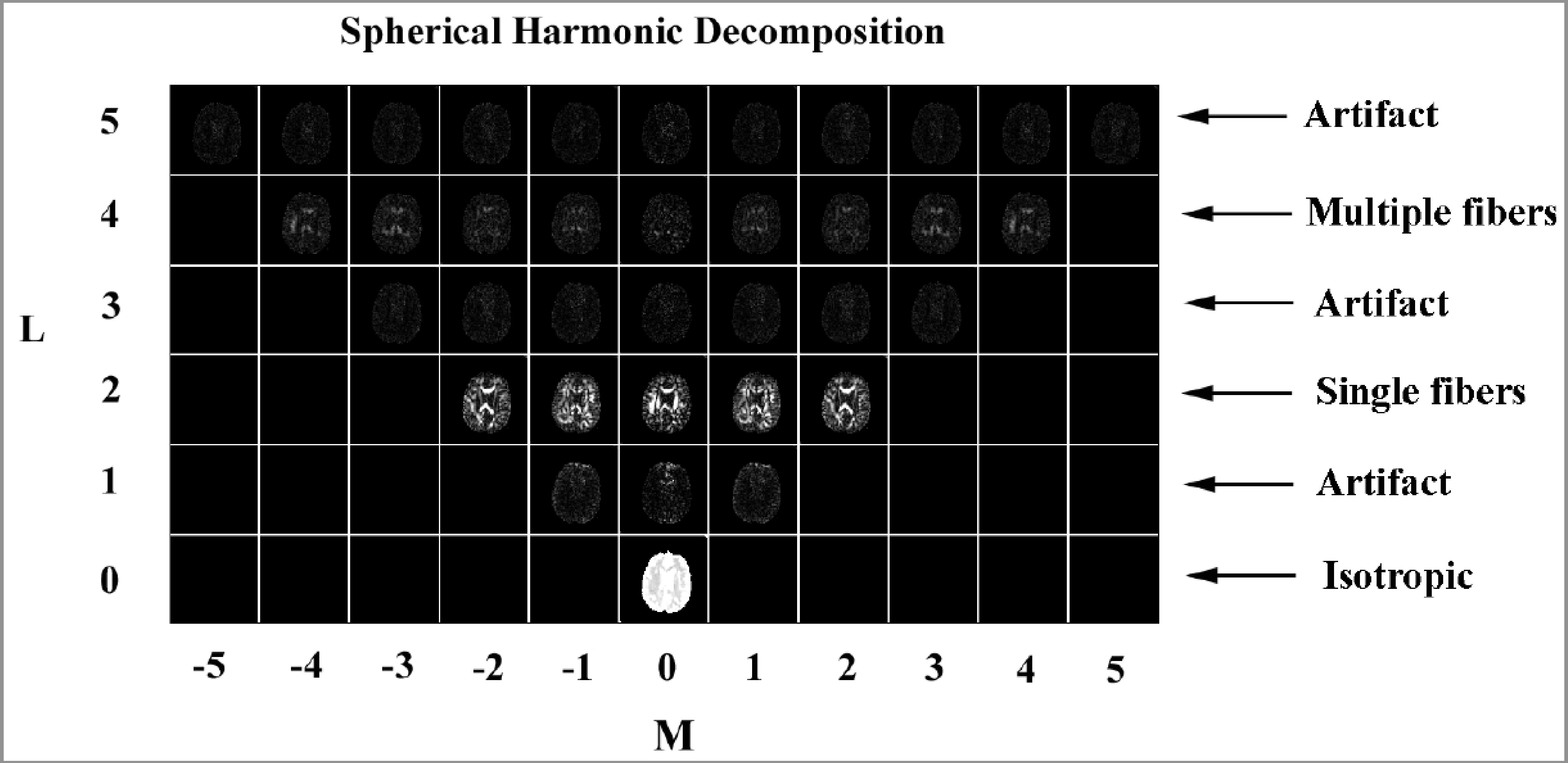
Two Fibers



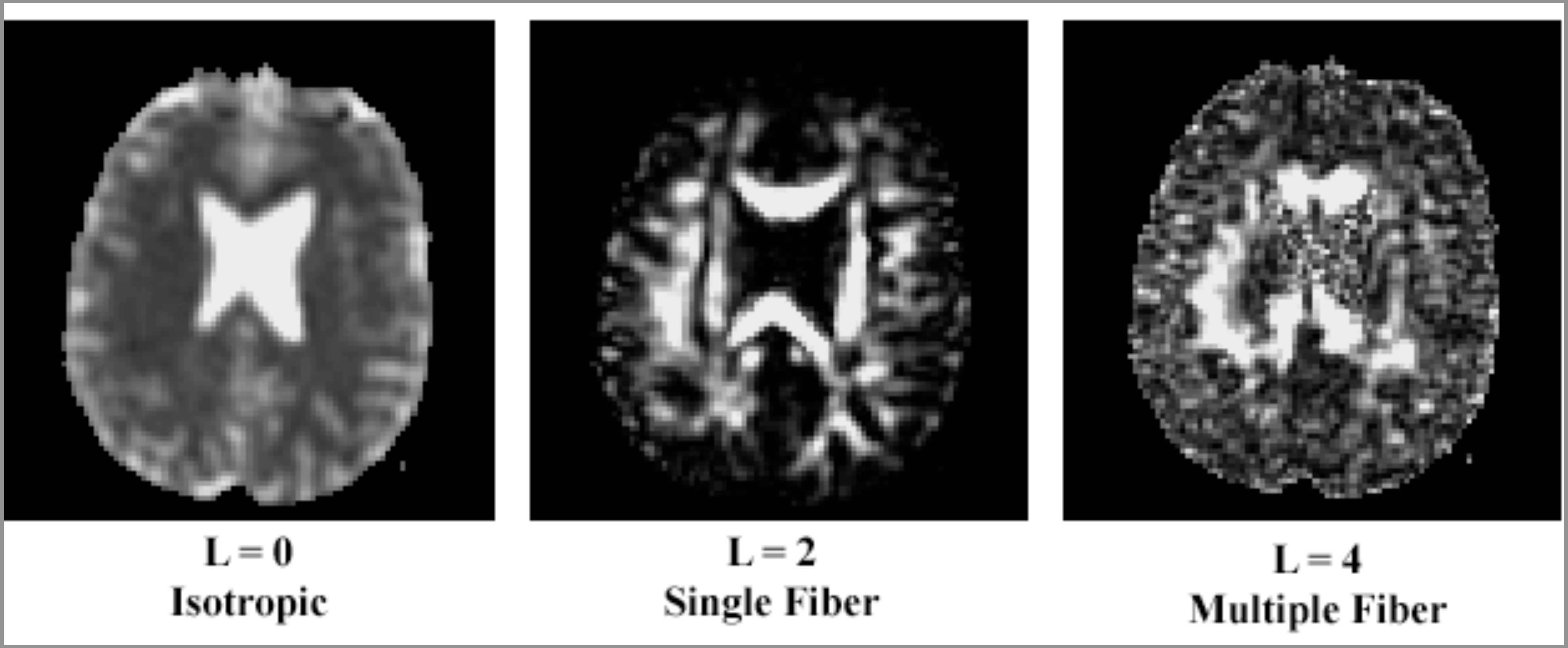
two fibers rotated relative to one another through a full range of $(\Delta\theta, \Delta\phi)$

Frank, MRM 47:1083 (2002)

The Spherical Harmonic Decomposition

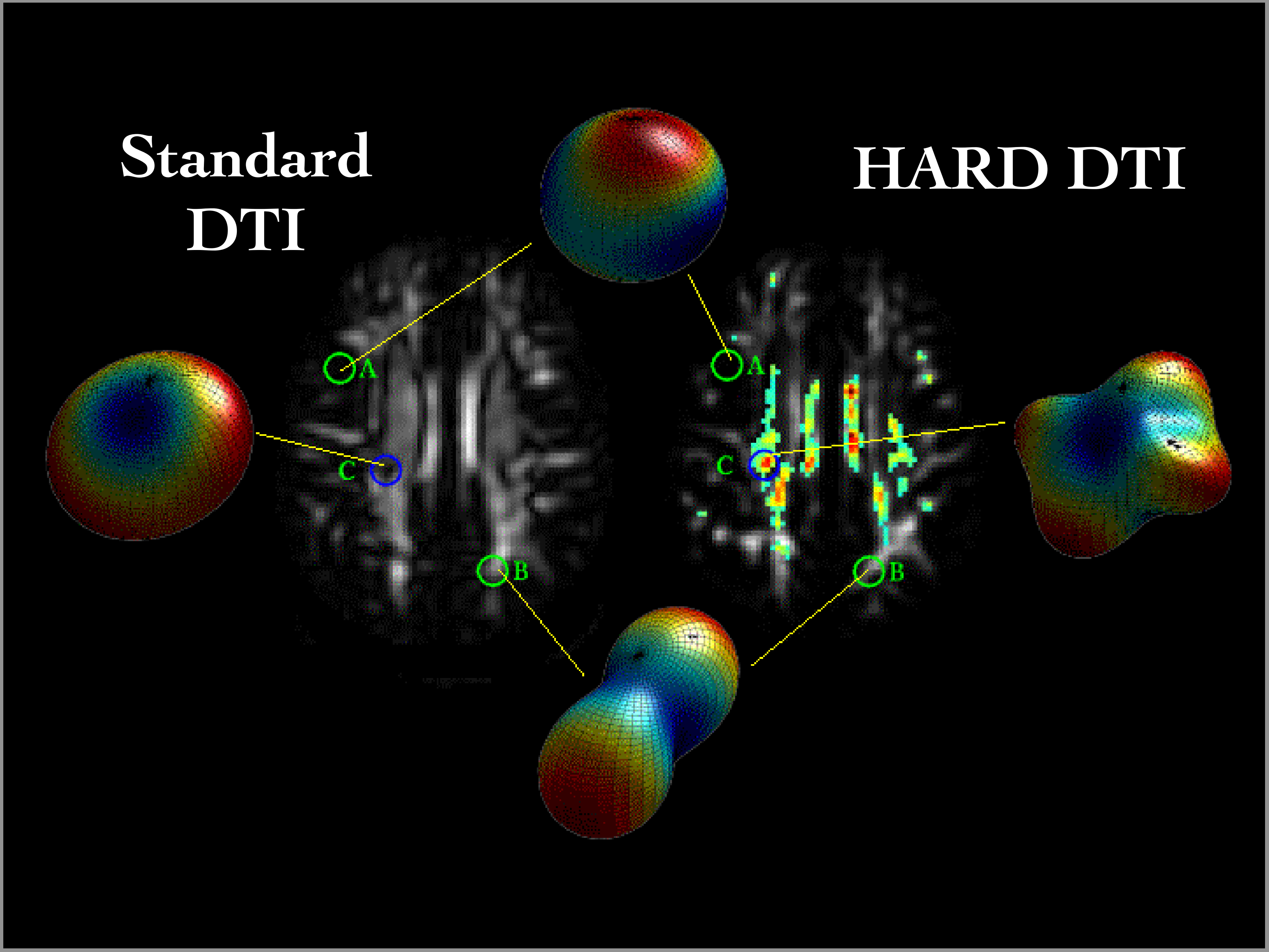


The Spherical Harmonic Decomposition



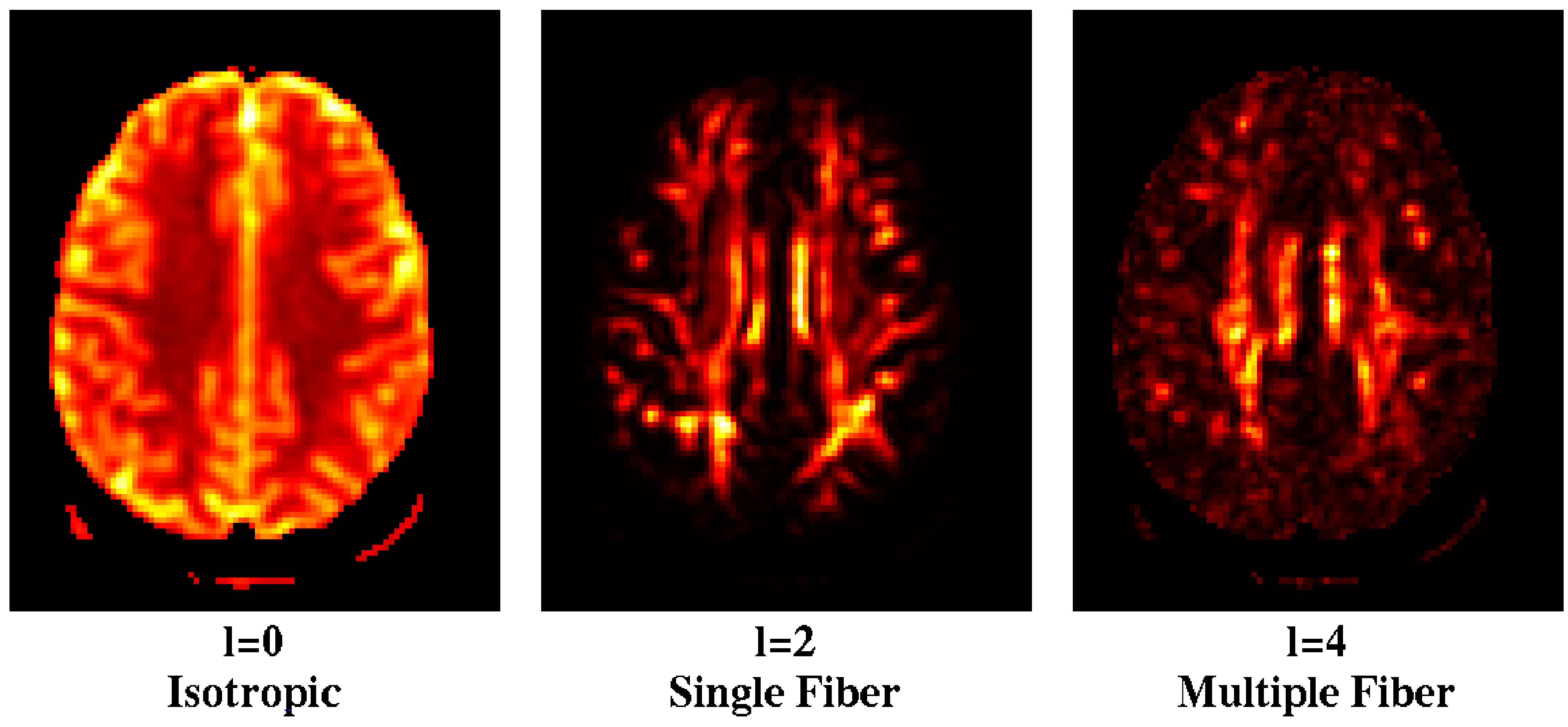
Frank, MRM 47:1083 (2002)

The Spherical Harmonic Decomposition



Frank, MRM *47*:1083 (2002)

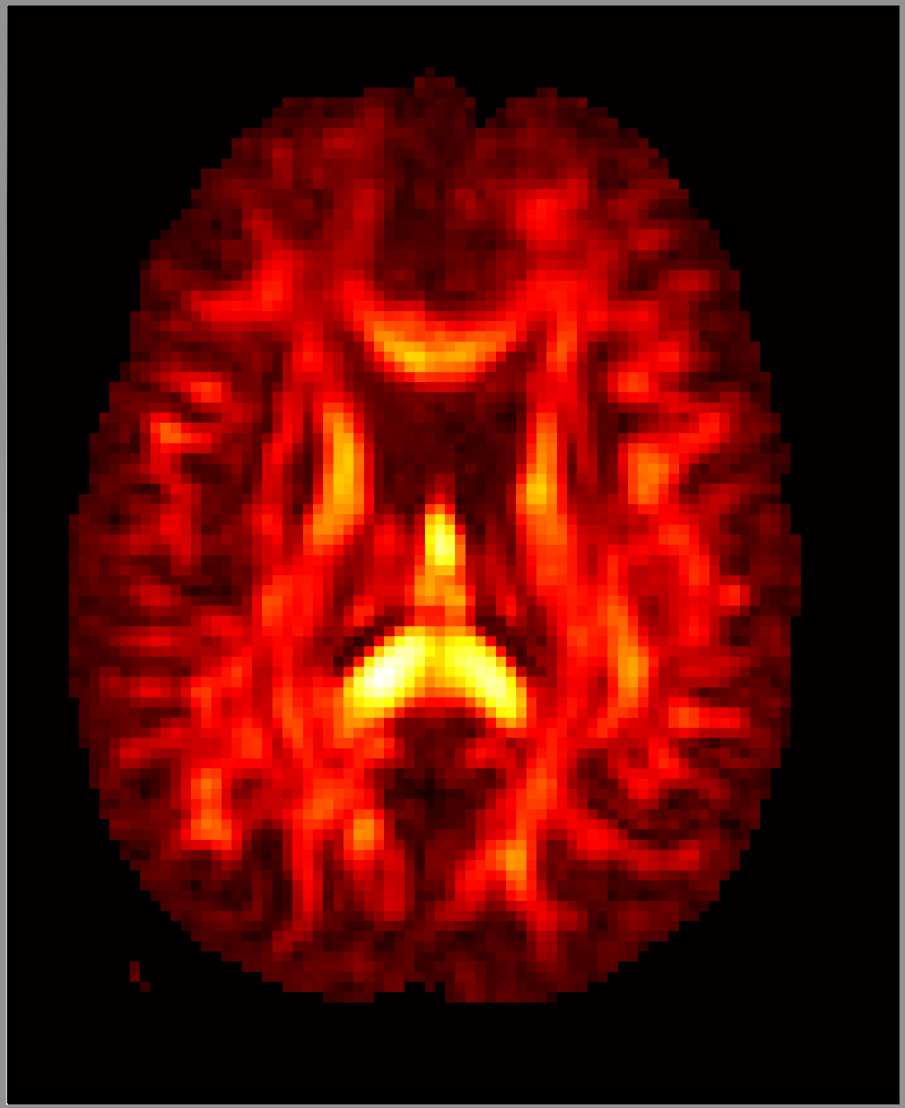
The Spherical Harmonic Decomposition



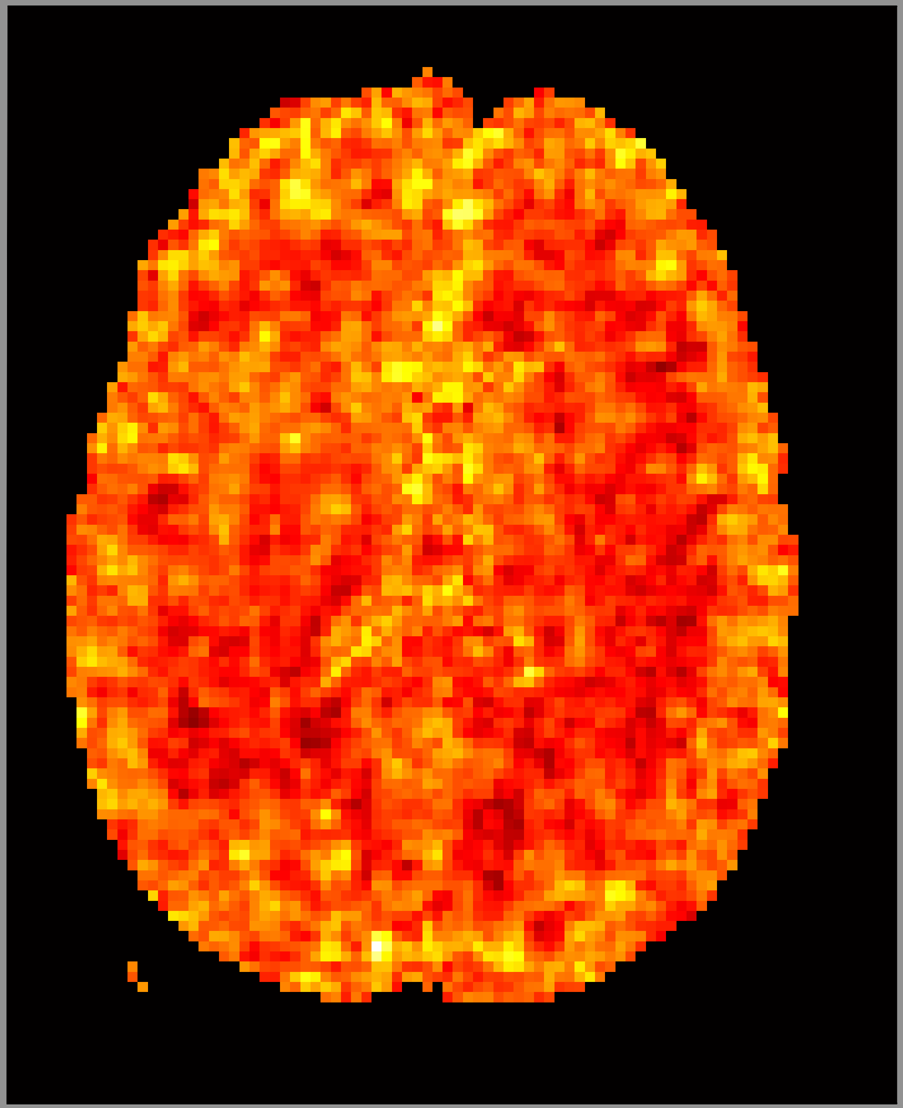
Sum amplitudes over all M for given L

Frank, MRM 47:1083 (2002)

The Spherical Harmonic Decomposition



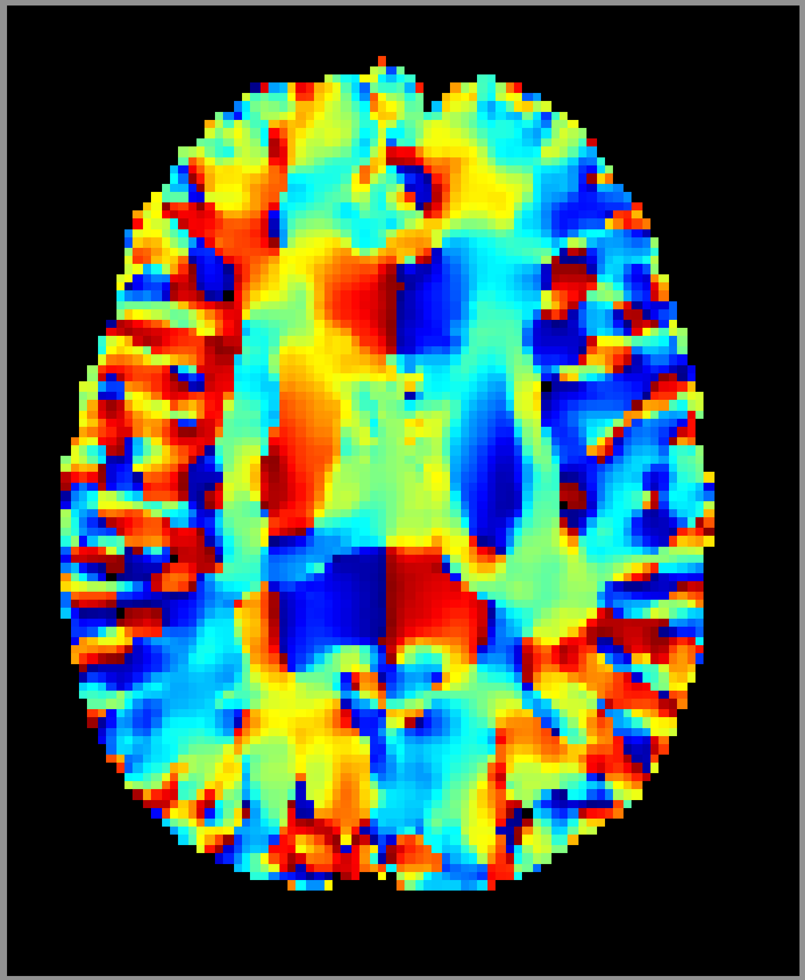
Even orders > 0



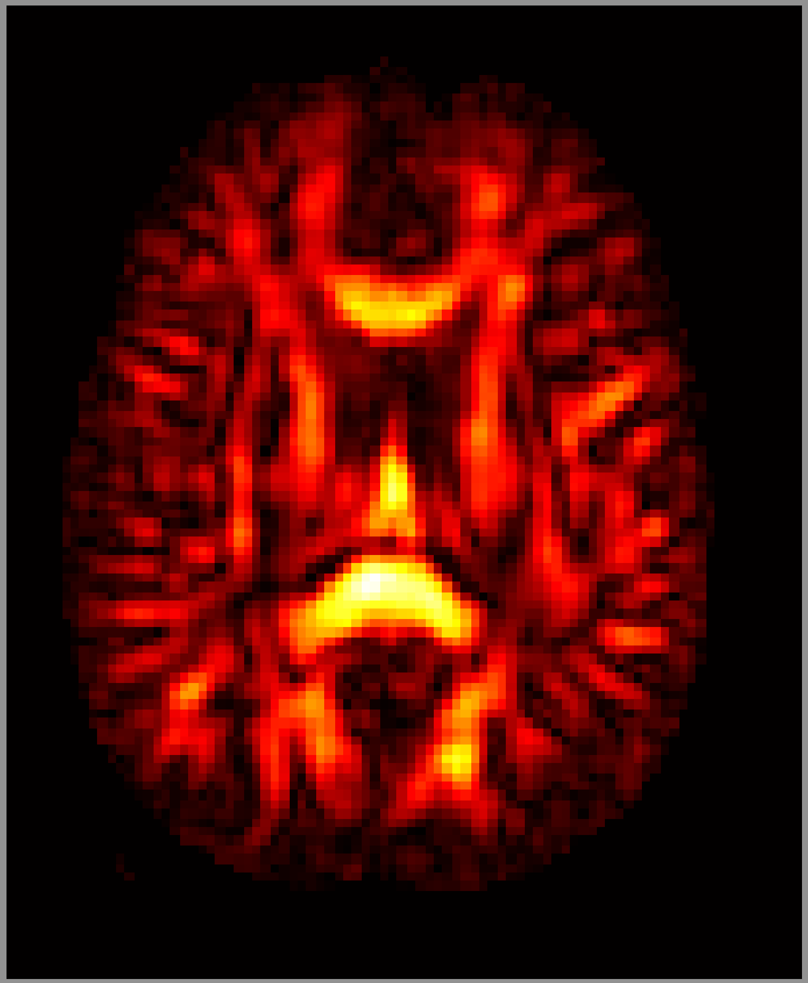
Odd orders

Frank, MRM *47*:1083 (2002)

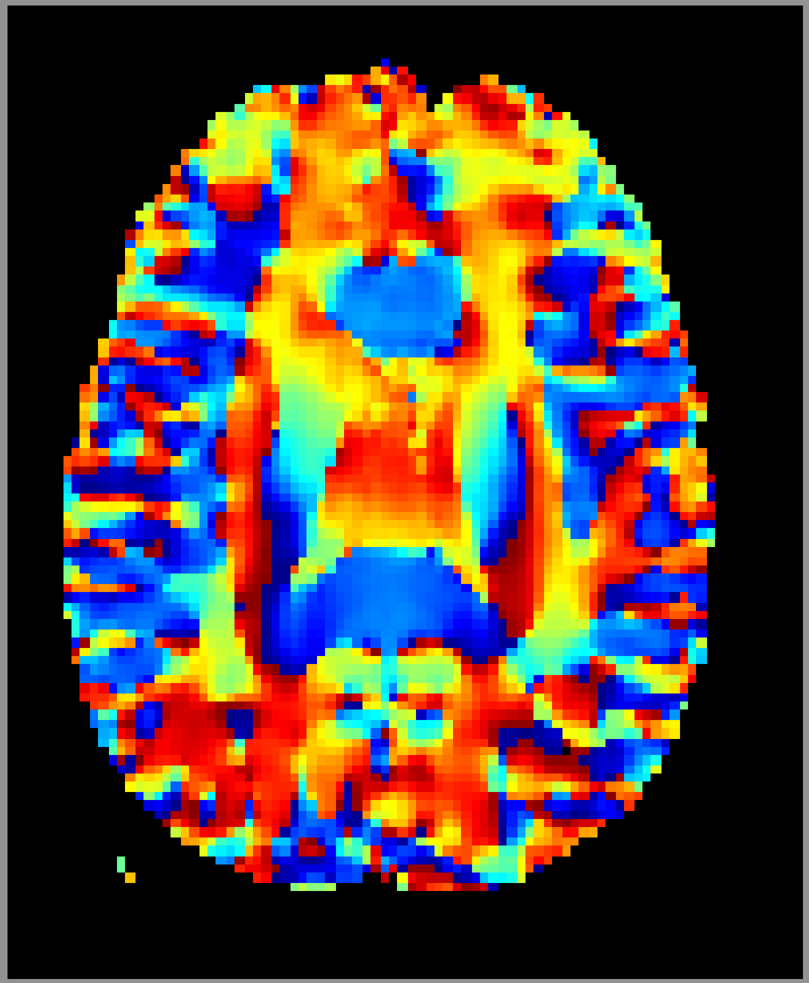
The Spherical Harmonic Decomposition



ϕ



Magnitude



θ

Frank, *MRM 47*:1083 (2002)

Anderson, MRM 54:1194 (2005)

The FORECAST Model

Fiber orientation by SHD of signal spherical harmonic representation of fiber orientation function but with the assumption of cylindrical symmetry

the signal

$$S(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} s_{lm} Y_{lm}(\theta, \phi)$$

the angular distribution of fibers

$$P(\vartheta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} p_{lm} Y_{lm}(\vartheta, \varphi)$$

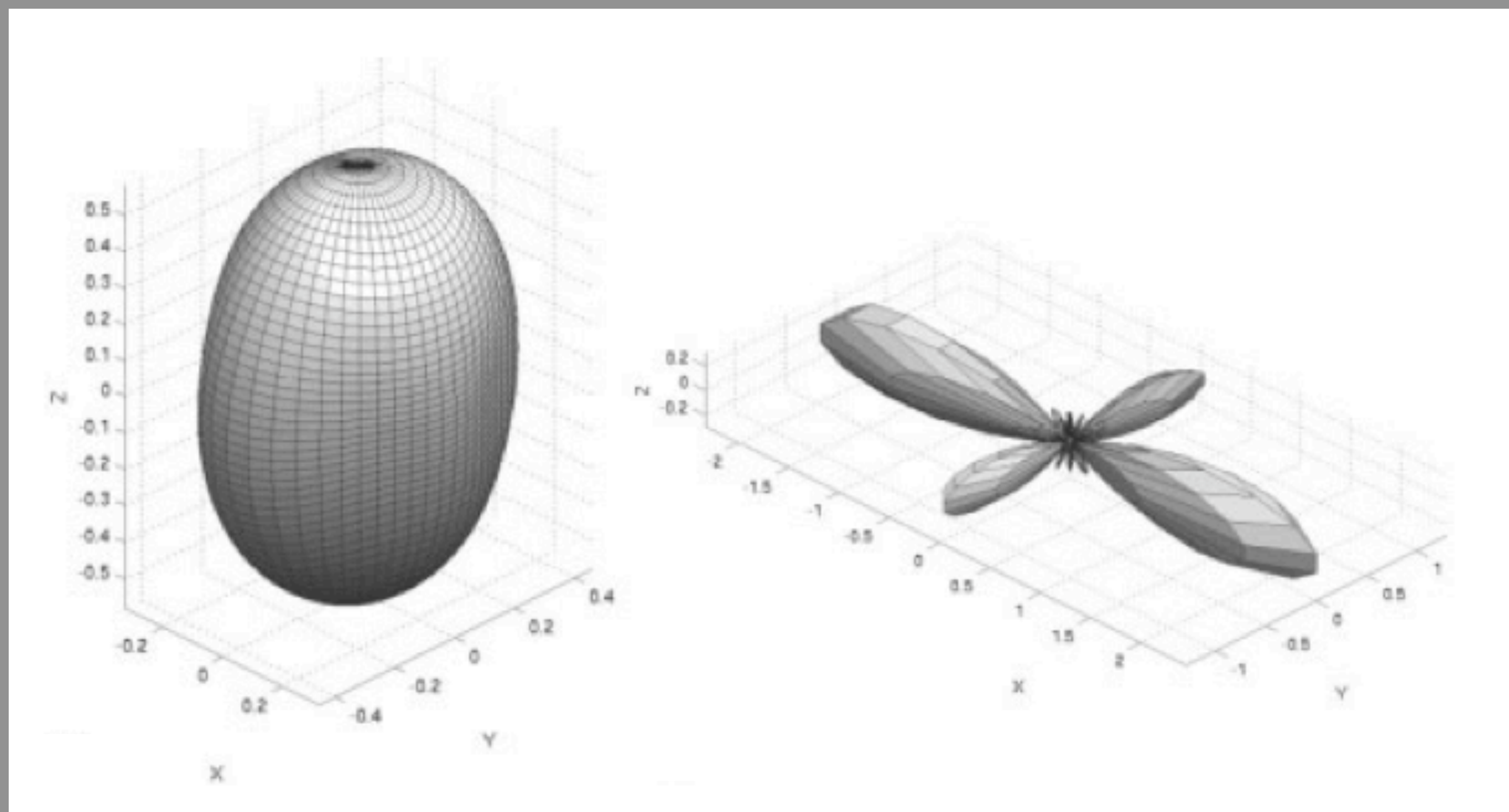
The FORECAST Model

Assumption of cylindrical symmetry

$$\boldsymbol{D} = \begin{pmatrix} \lambda_{\perp} & 0 & 0 \\ 0 & \lambda_{\perp} & 0 \\ 0 & 0 & \lambda_{\parallel} \end{pmatrix}$$

Anderson, MRM 54:1194 (2005)

The FORECAST Model

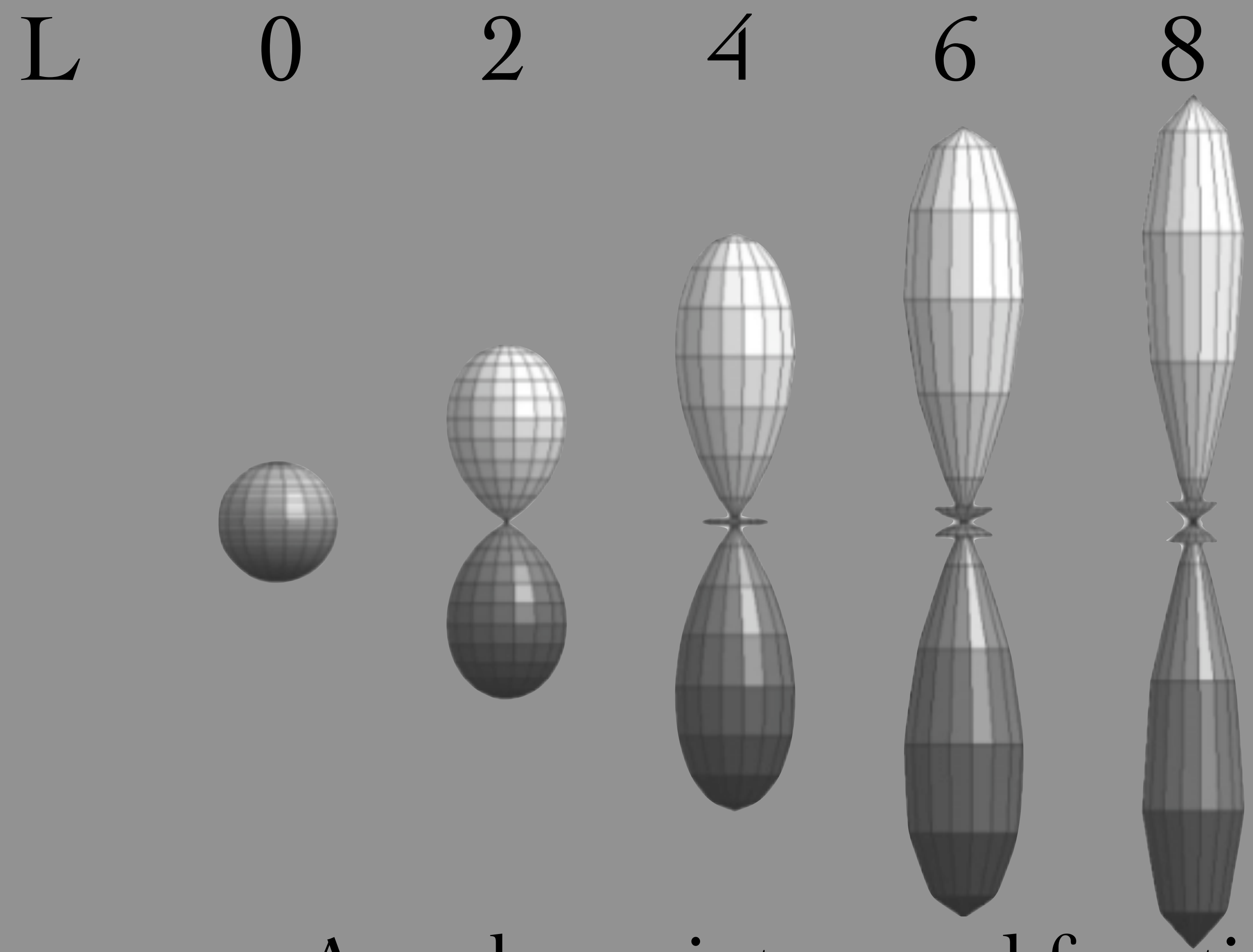


signal

orientation and
volume fraction (2/3 & 1/3)

Anderson, MRM 54:1194 (2005)

The FORECAST Model

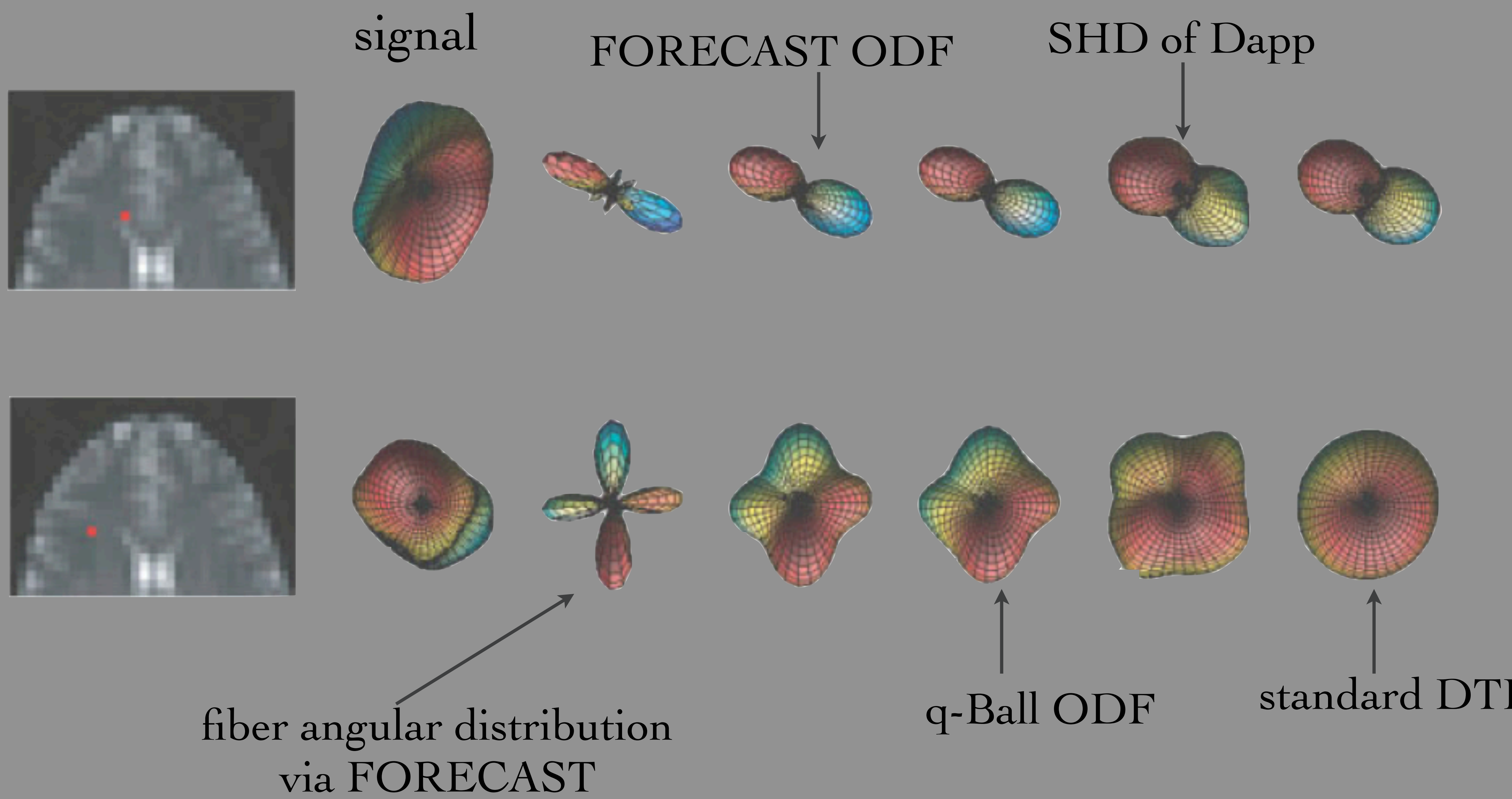


Angular point spread function

Higher order gives higher resolution but is more sensitive to noise as the coefficients are smaller

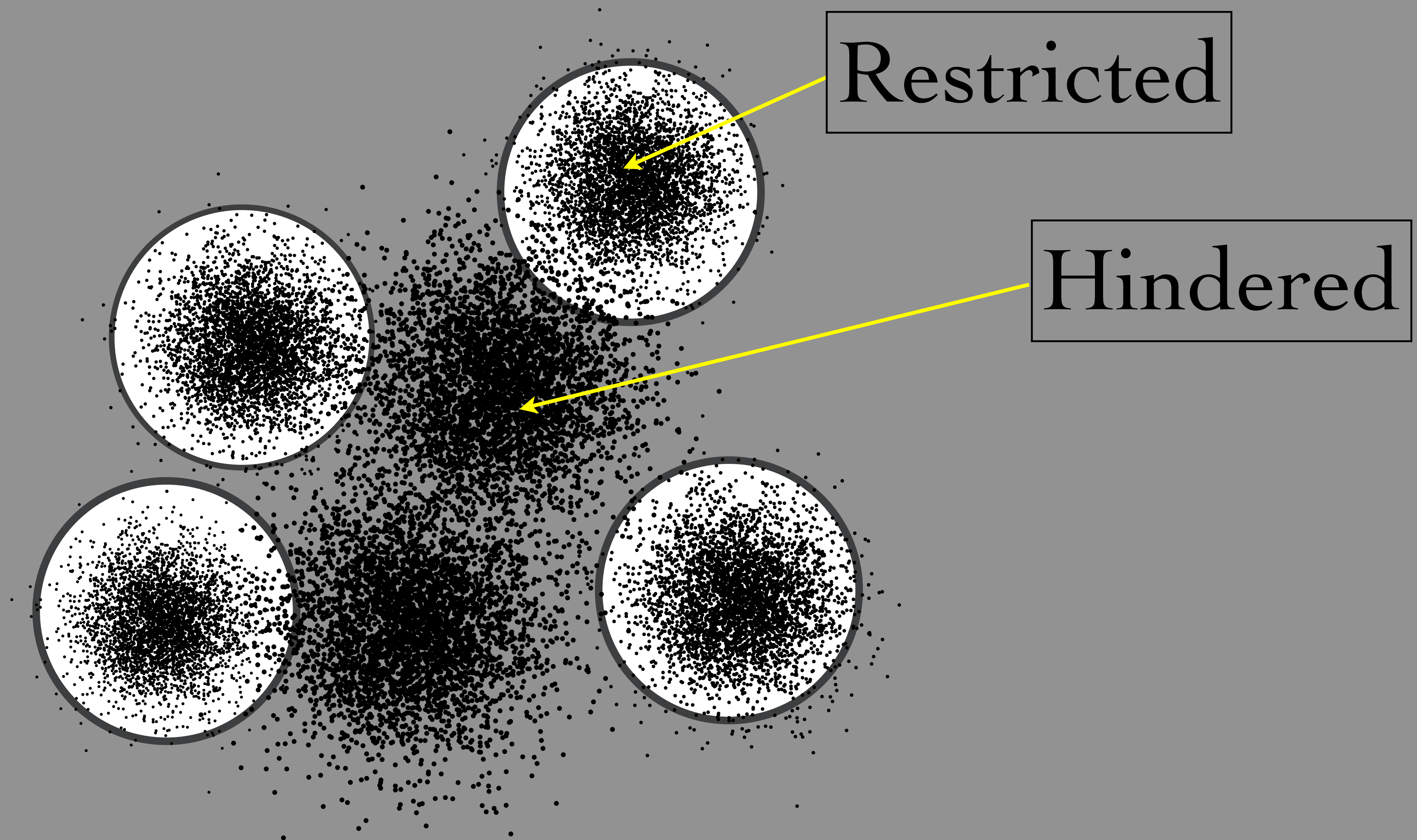
Anderson, MRM 54:1194 (2005)

The FORECAST Model



Anderson, MRM 54:1194 (2005)

The CHARMED Model



Assaf, et. al. MRM 52:965 (2004)

The CHARMED Model

$$E(q,\Delta) = f_h E_h(q,\Delta) + \sum_{j=1}^{n} f_r^j E_r^j(q,\Delta)$$

Signal decay

Hindered decay

Restricted decay

hindered volume fraction

restricted volume fraction

Assaf, et. al. MRM 52:965 (2004)

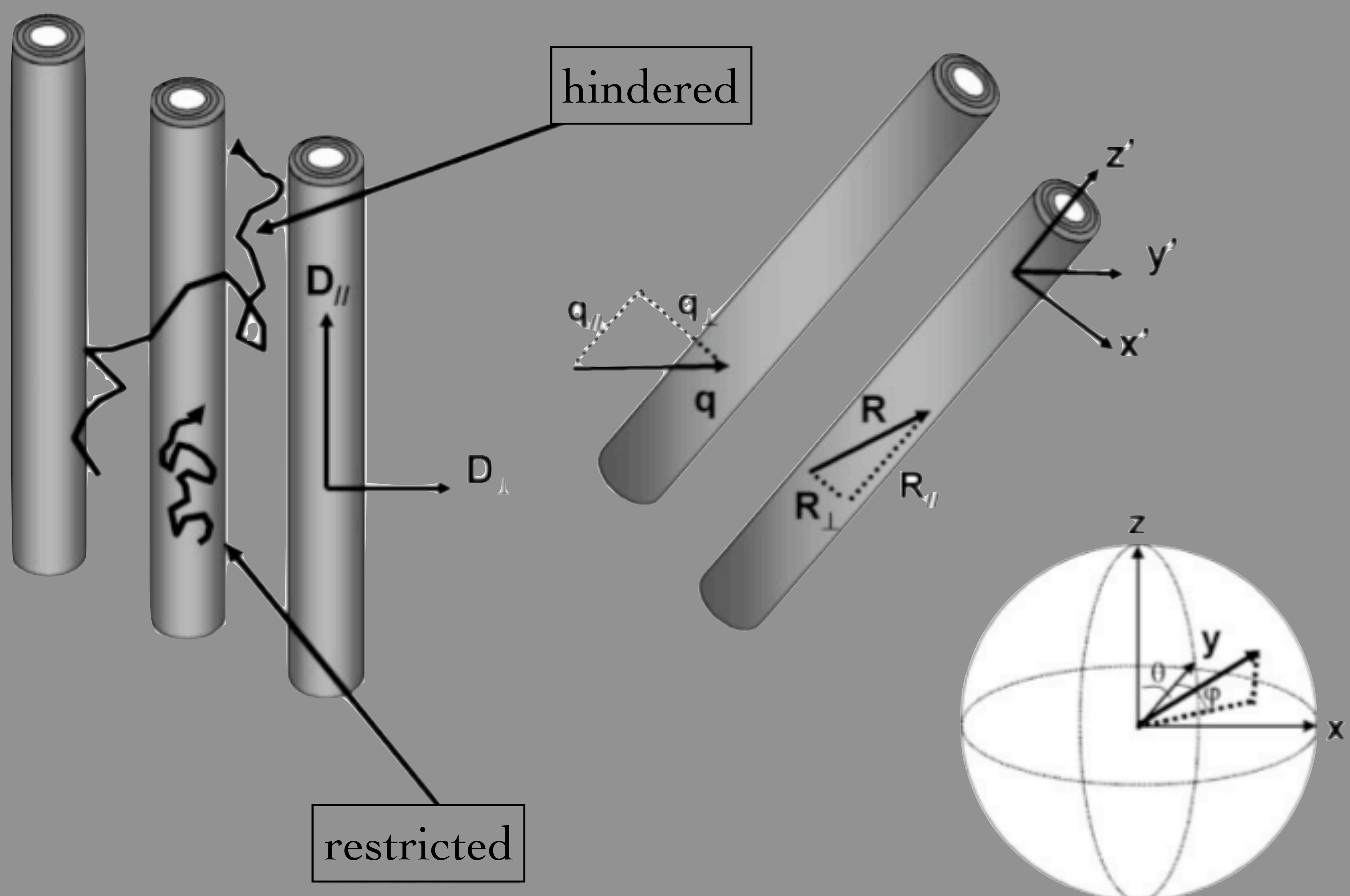
The CHARMED Model

Assume cylindrical symmetry

$$\boldsymbol{D} = \begin{pmatrix} \lambda_{\perp} & 0 & 0 \\ 0 & \lambda_{\perp} & 0 \\ 0 & 0 & \lambda_{\parallel} \end{pmatrix}$$

Assaf, et. al. MRM 52:965 (2004)

The CHARMED Model



Assume cylindrical symmetry

Assaf, et. al. MRM 52:965 (2004)

The CHARMED Model

Decoupling of $\boldsymbol{D}_{\|}$ and $\boldsymbol{D}_{\perp}$ in restricted compartment

$$E_R(\boldsymbol{q}, \Delta) = \int \overline{P}(\boldsymbol{r}, \Delta) e^{i\boldsymbol{q}\cdot\boldsymbol{r}} d\boldsymbol{r}$$

$$P_R(\boldsymbol{r}, \Delta) = P_{\perp}(\boldsymbol{r}_{\perp}, \Delta) P_{\|}(\boldsymbol{r}_{\|}, \Delta)$$

$$E_R(\boldsymbol{q}, \Delta) = E_{\perp}(\boldsymbol{q}_{\perp}, \Delta) E_{\|}(\boldsymbol{q}_{\|}, \Delta)$$

Assaf, et. al. MRM 52:965 (2004)

The CHARMED Model

Form of $E_\parallel$ and $E_\perp$ in restricted compartment

$$E_\parallel(\boldsymbol{q}_\parallel, \Delta) = e^{-4\pi^2|\boldsymbol{q}_\parallel|^2\tau D_\parallel}$$

$$\tau = \Delta - \delta/3$$

messy!

$$E_\perp(\boldsymbol{q}_\perp, \Delta) = e^{f(D_\perp)} = \text{restricted diffusion in a cylinder (Neuman)}$$

Assaf, et. al. MRM 52:965 (2004)

The CHARMED Model

Form of E_h in hindered compartment

$$E_h(\boldsymbol{q}, \Delta) = e^{-4\pi^2 \tau \boldsymbol{q}^t \boldsymbol{D} \boldsymbol{q}}$$

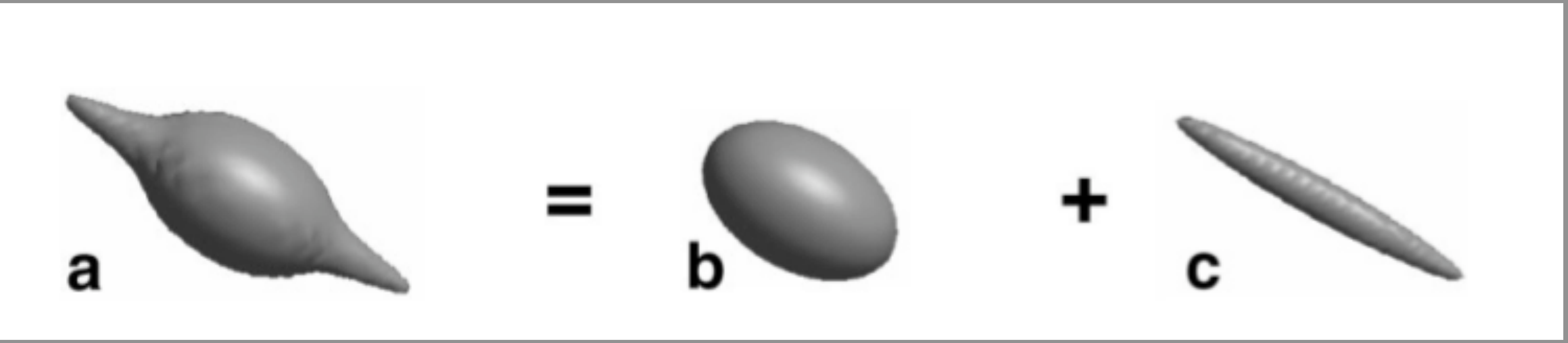
$$\boldsymbol{q} = \boldsymbol{q}_{\|} + \boldsymbol{q}_{\perp}$$

$$E_h(\boldsymbol{q}, \Delta) = e^{-4\pi^2 \tau (|\boldsymbol{q}_{\|}|^2 \lambda_{\|} + |\boldsymbol{q}_{\perp}|^2 \lambda_{\perp})}$$

Assaf, et. al. MRM 52:965 (2004)

The CHARMED Model

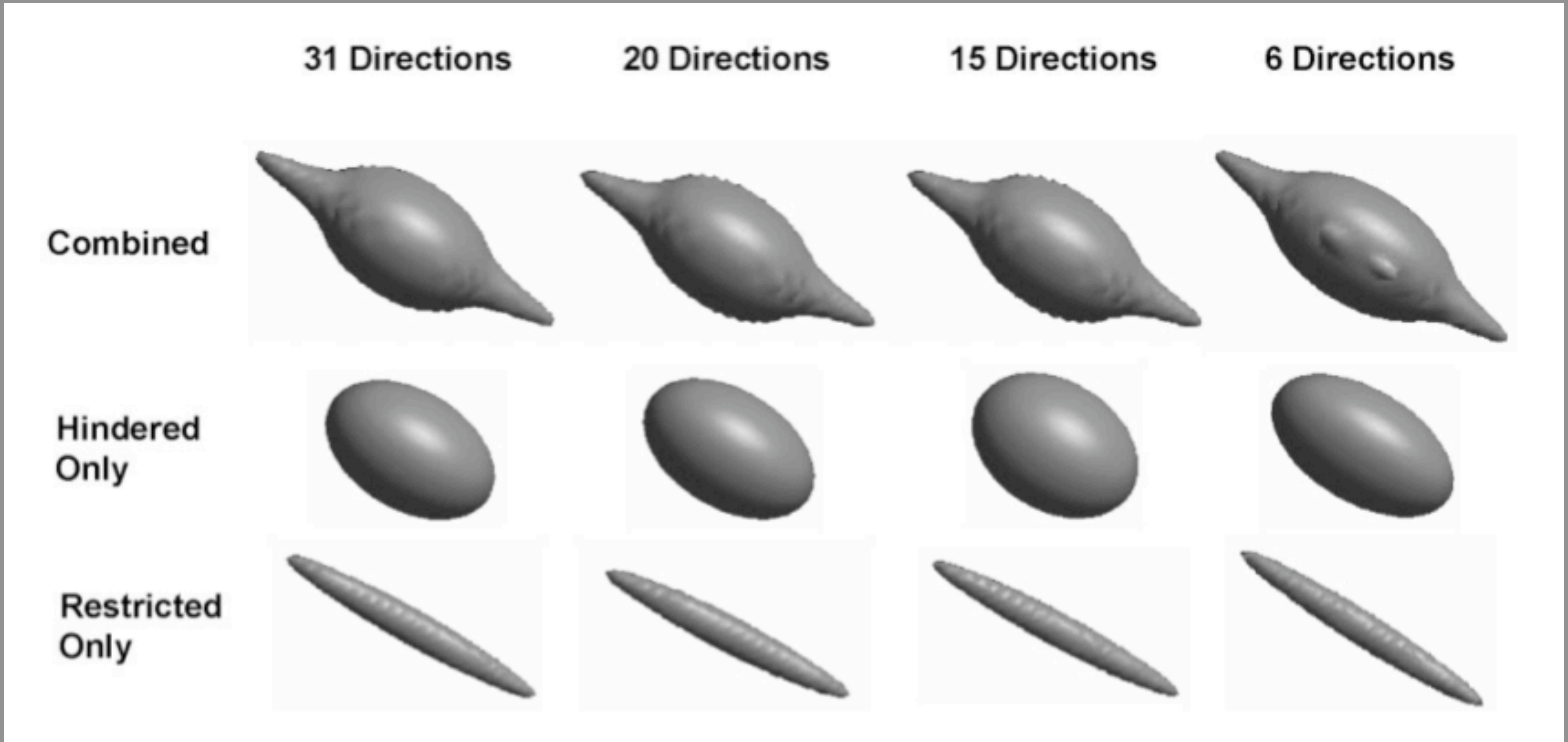
3D-FFT of simulated signal



Assaf, et. al. MRM 52:965 (2004)

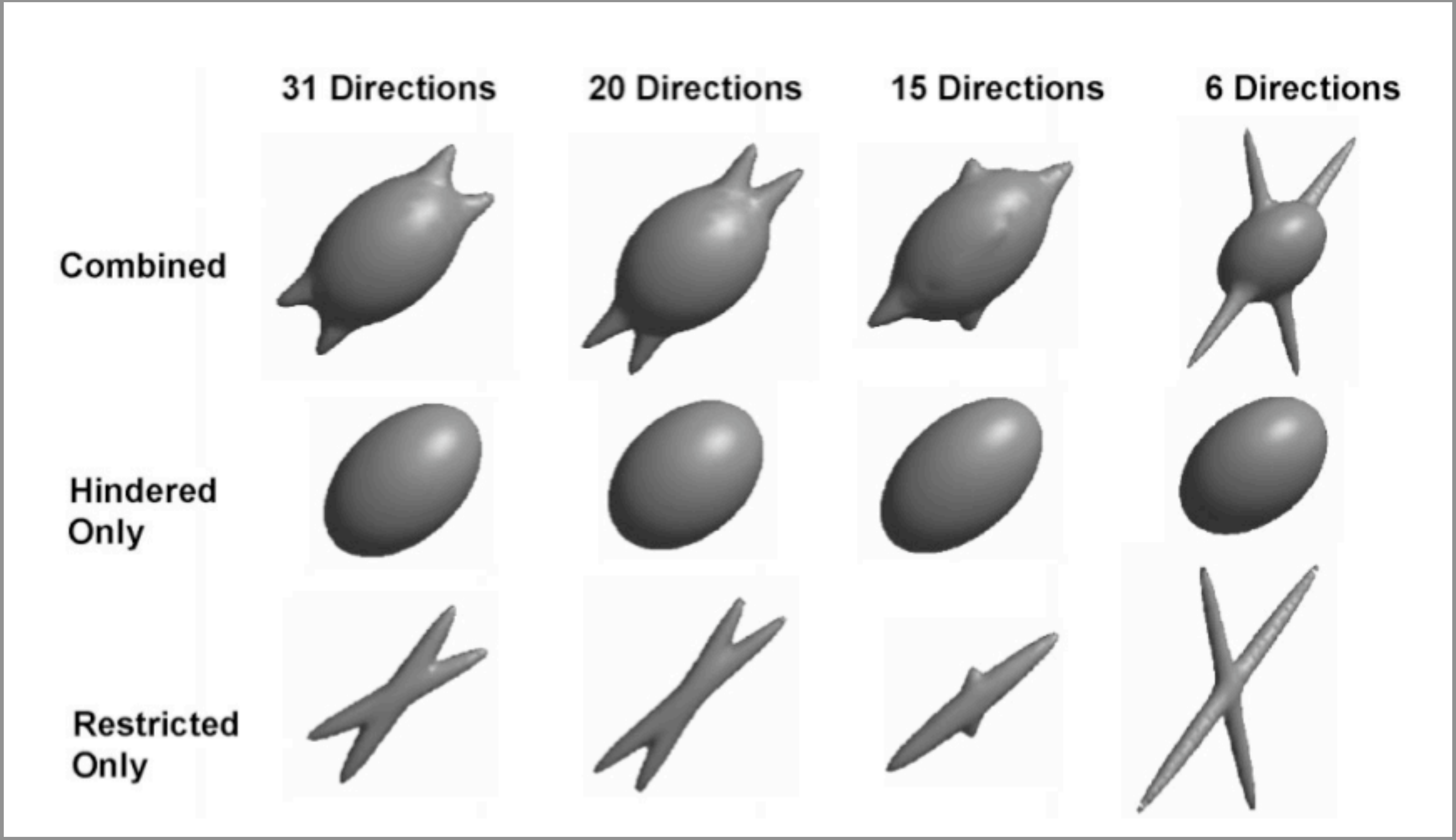
The CHARMED Model

3D-FFT of simulated signal



Assaf, et. al. MRM 52:965 (2004)

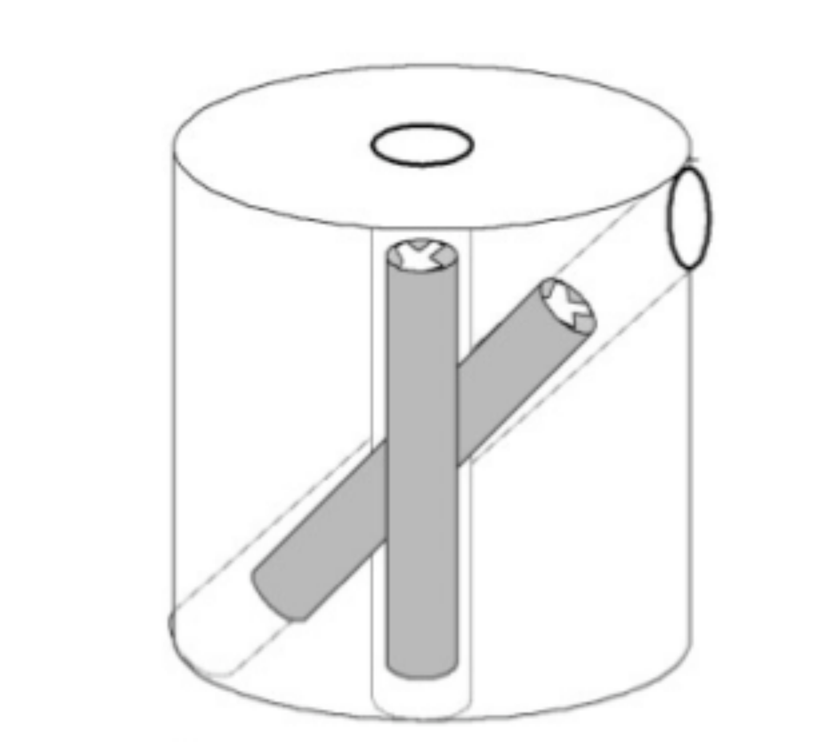
The CHARMED Model



Assaf, et. al. MRM 52:965 (2004)

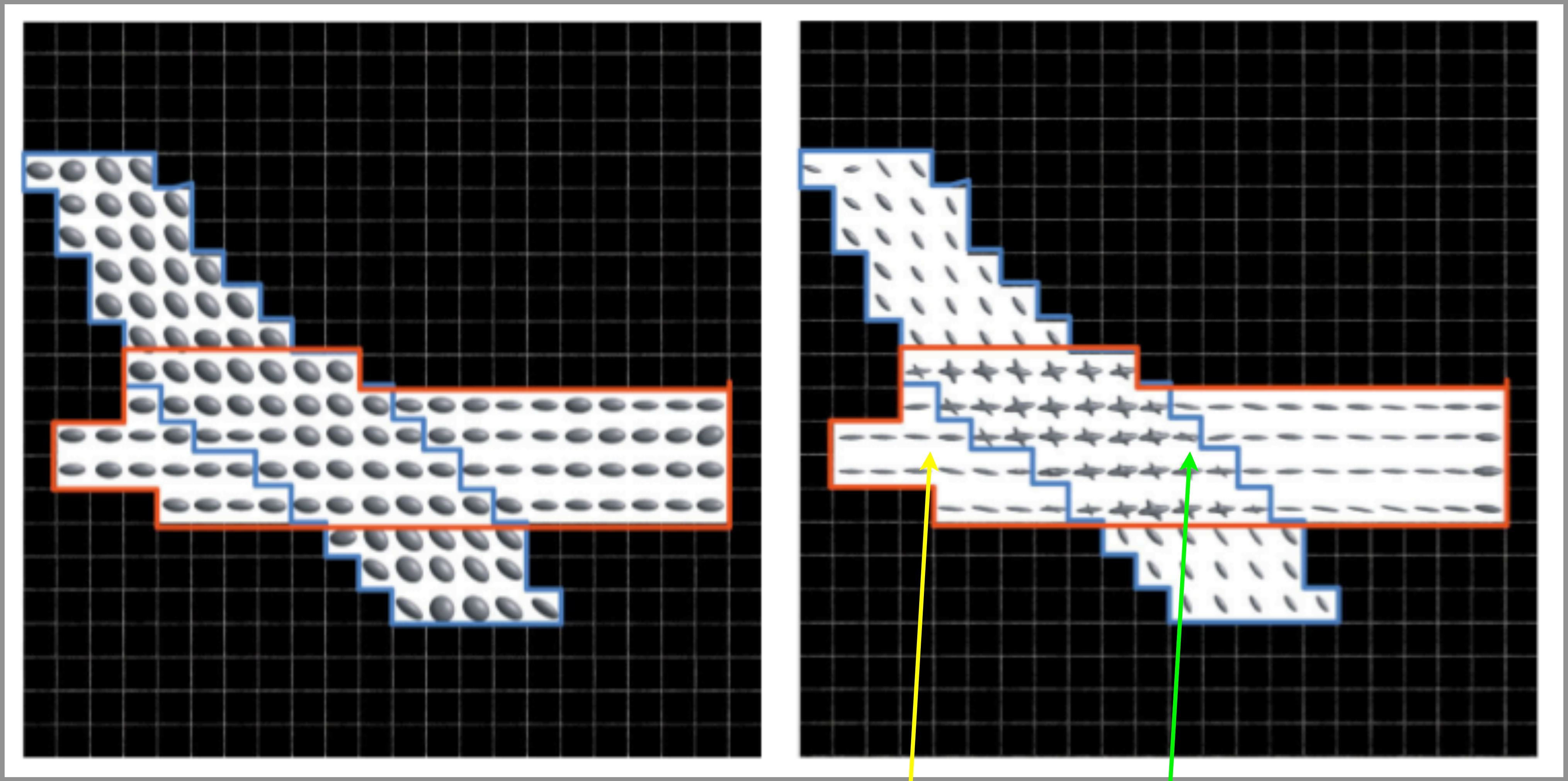
The CHARMED Model

pig spinal cord phantom



Assaf, et. al. MRM 52:965 (2004)

The CHARMED Model



one hindered
(i.e., standard DTI)

one hindered
one and two restricted

Assaf, et. al. MRM 52:965 (2004)

The CHARMED Model

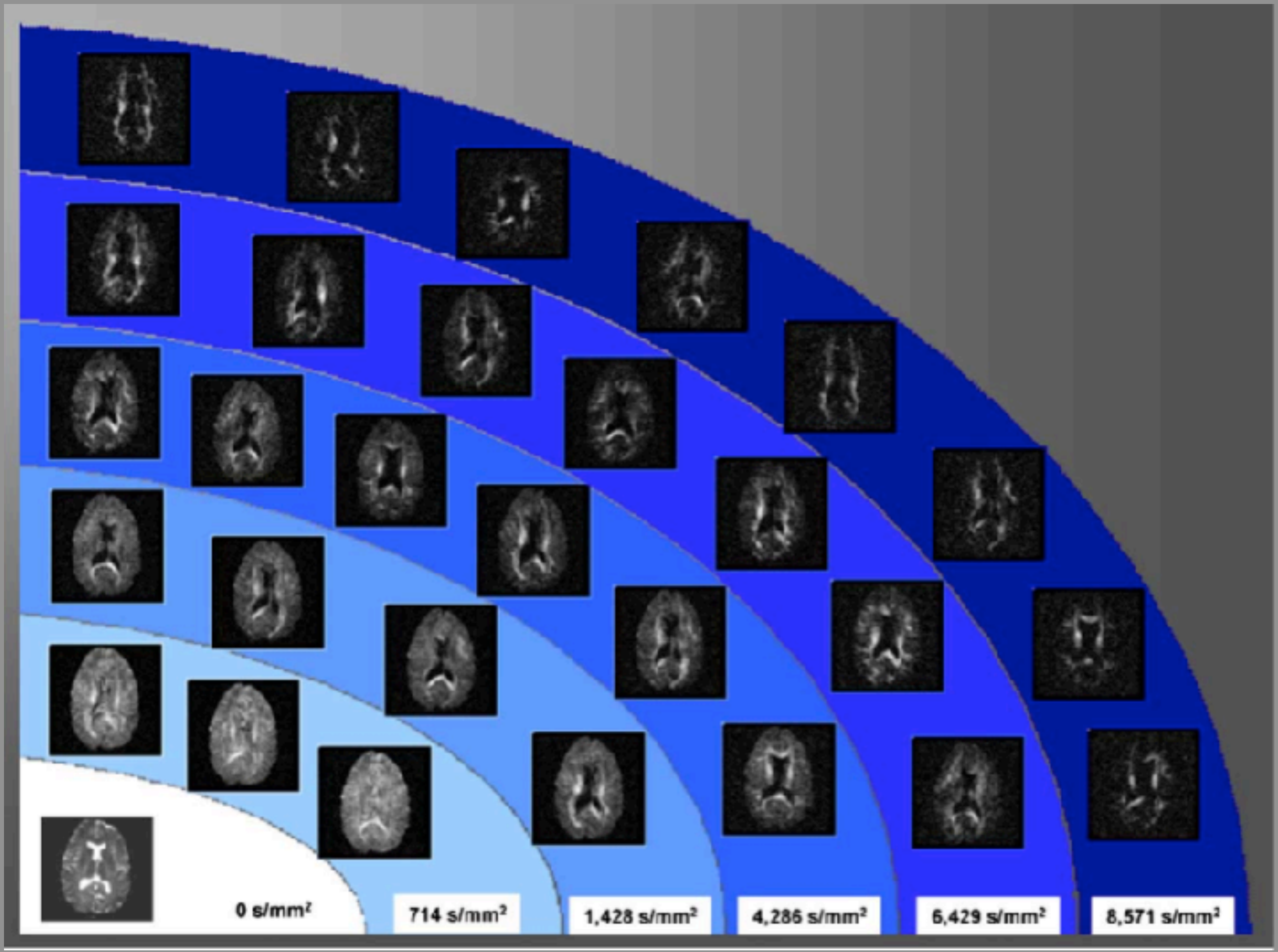
Three configurations

1. One hindered and no restricted (n=0)
2. One hindered and one restricted (n=1)
3. One hindered and two restricted (n=2)

Assaf and Basser, Neuroimage 27:48 (2005)

Assaf and Basser, Neuroimage 27:48 (2005)

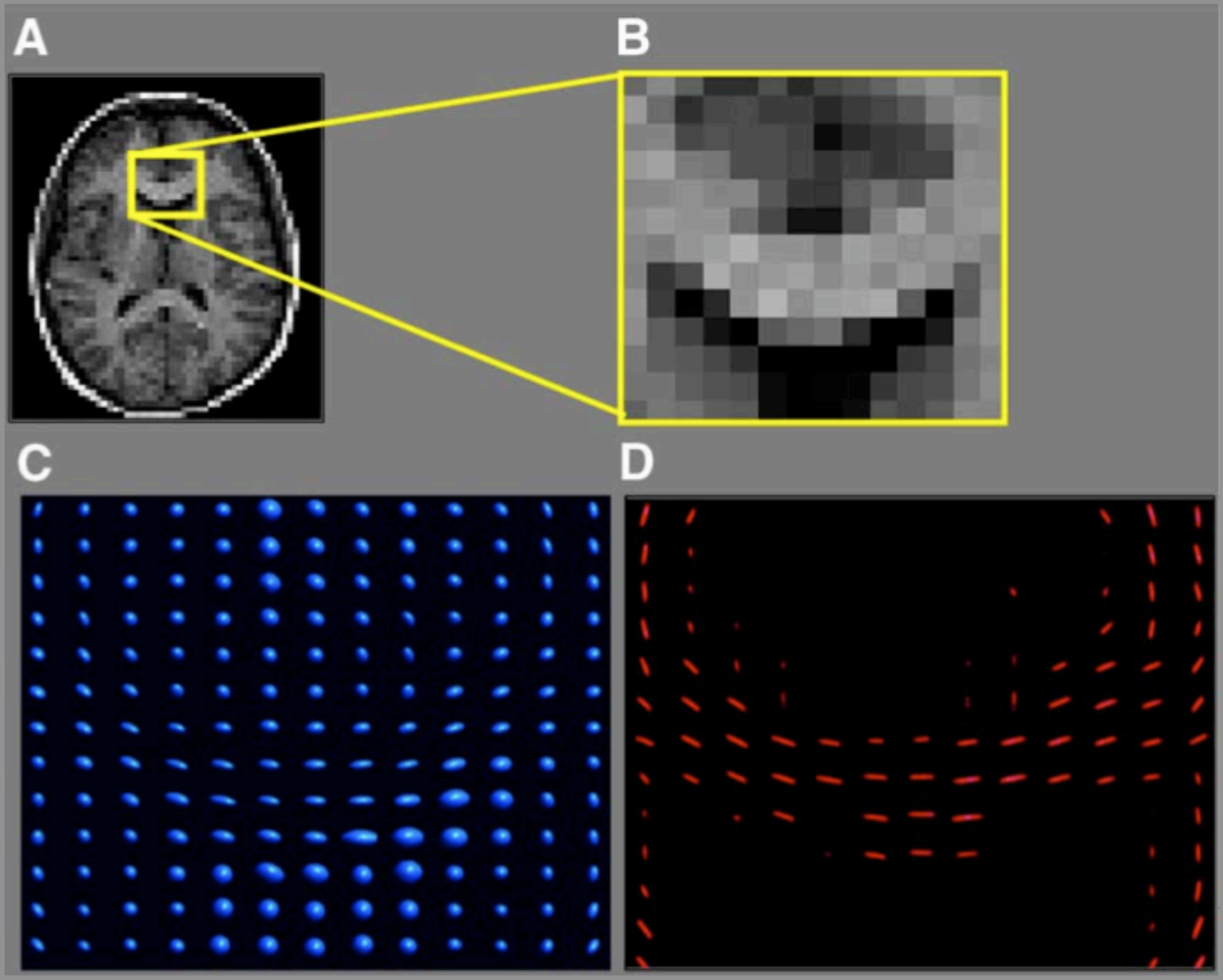
The CHARMED Model



10 shells of b-values from 0-10,000 s/mm^2,
from 6 directions (inner shell) to 30 directions (outer shell)

Assaf and Basser, Neuroimage 27:48 (2005)

The CHARMED Model



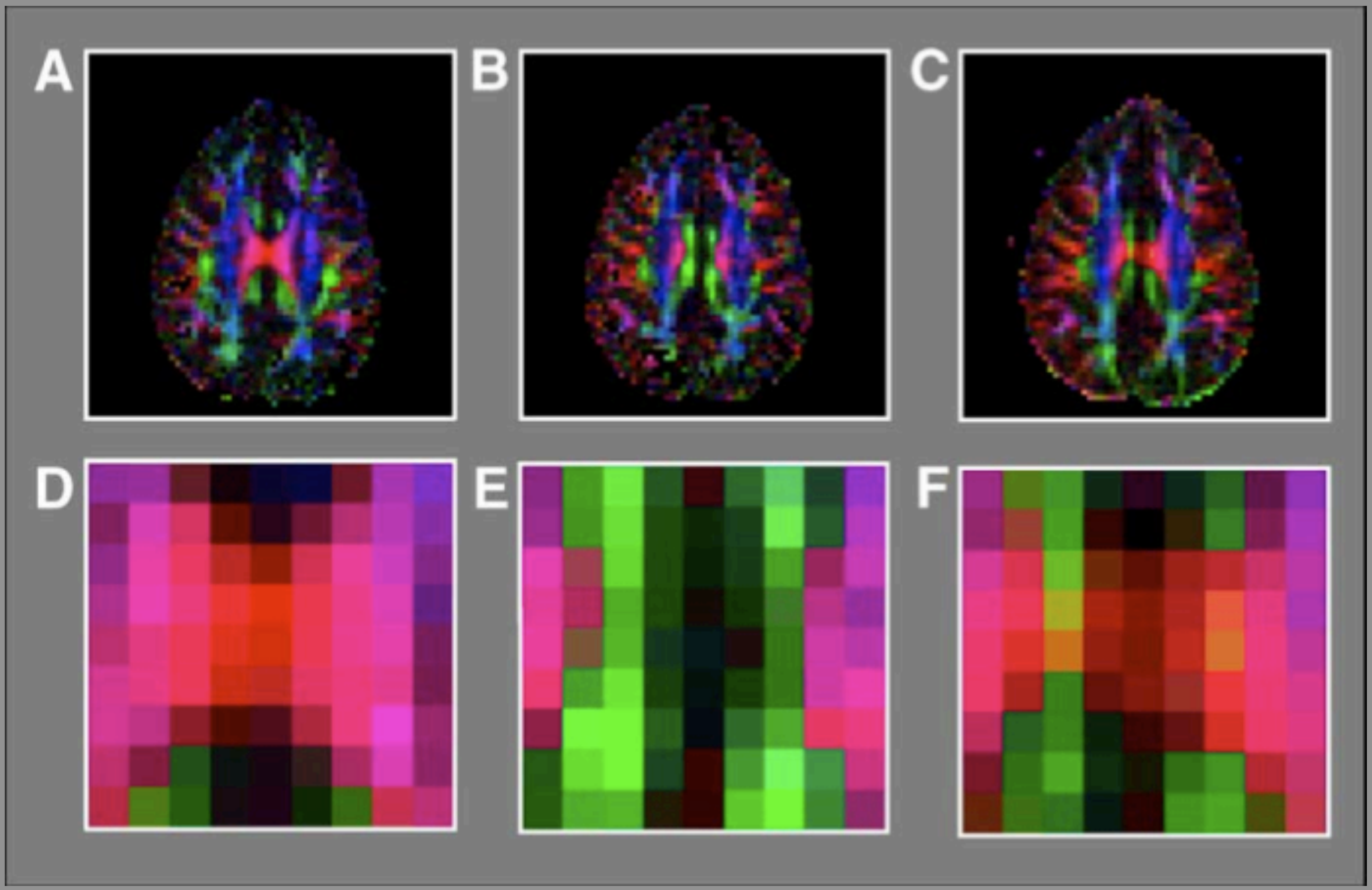
Hindered component

Restricted component

Assaf and Basser, Neuroimage 27:48 (2005)

The CHARMED Model

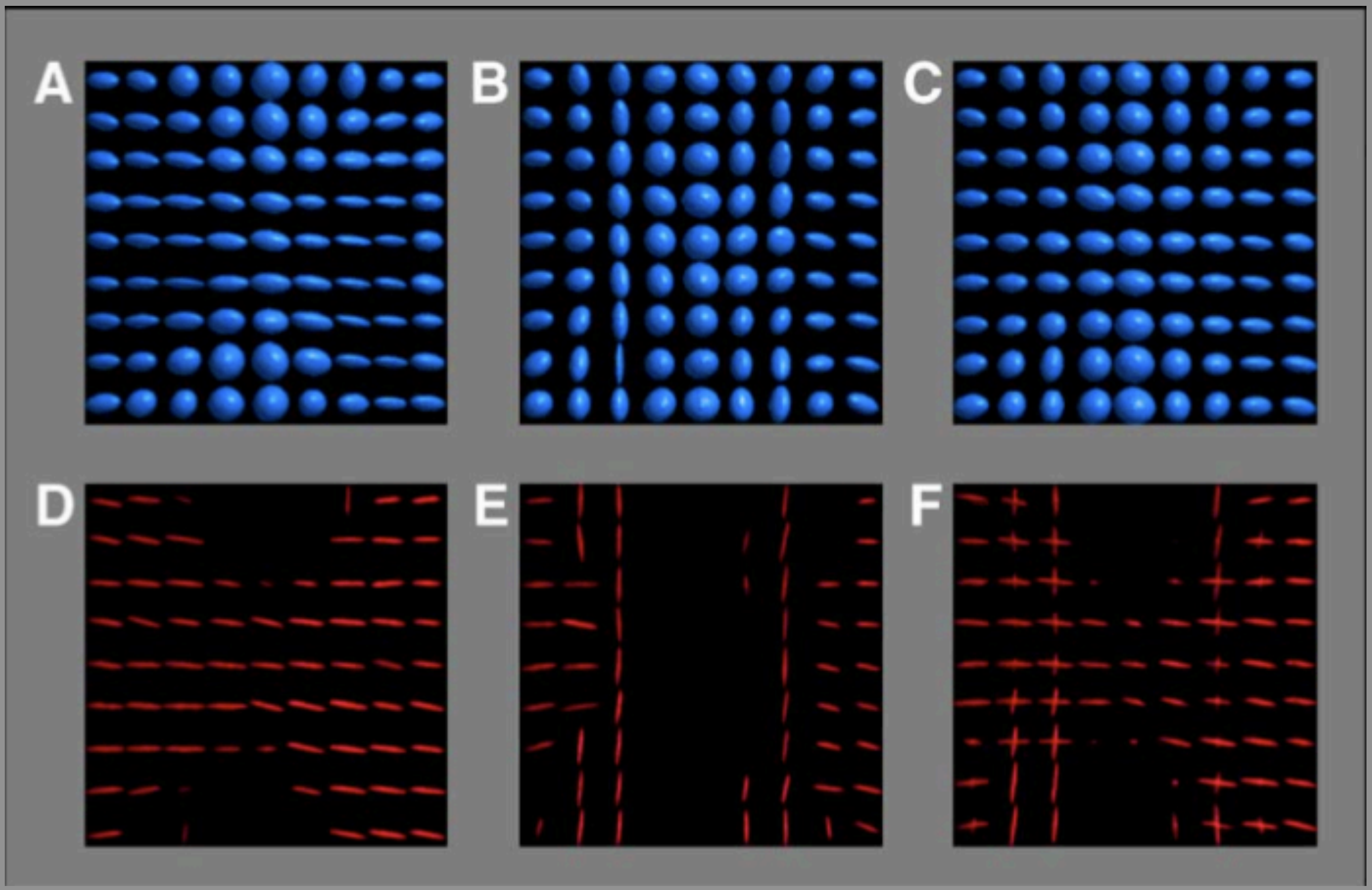
Directionality Map



Corpus callosum Cingulum A+B

Assaf and Basser, Neuroimage 27:48 (2005)

The CHARMED Model



Hindered

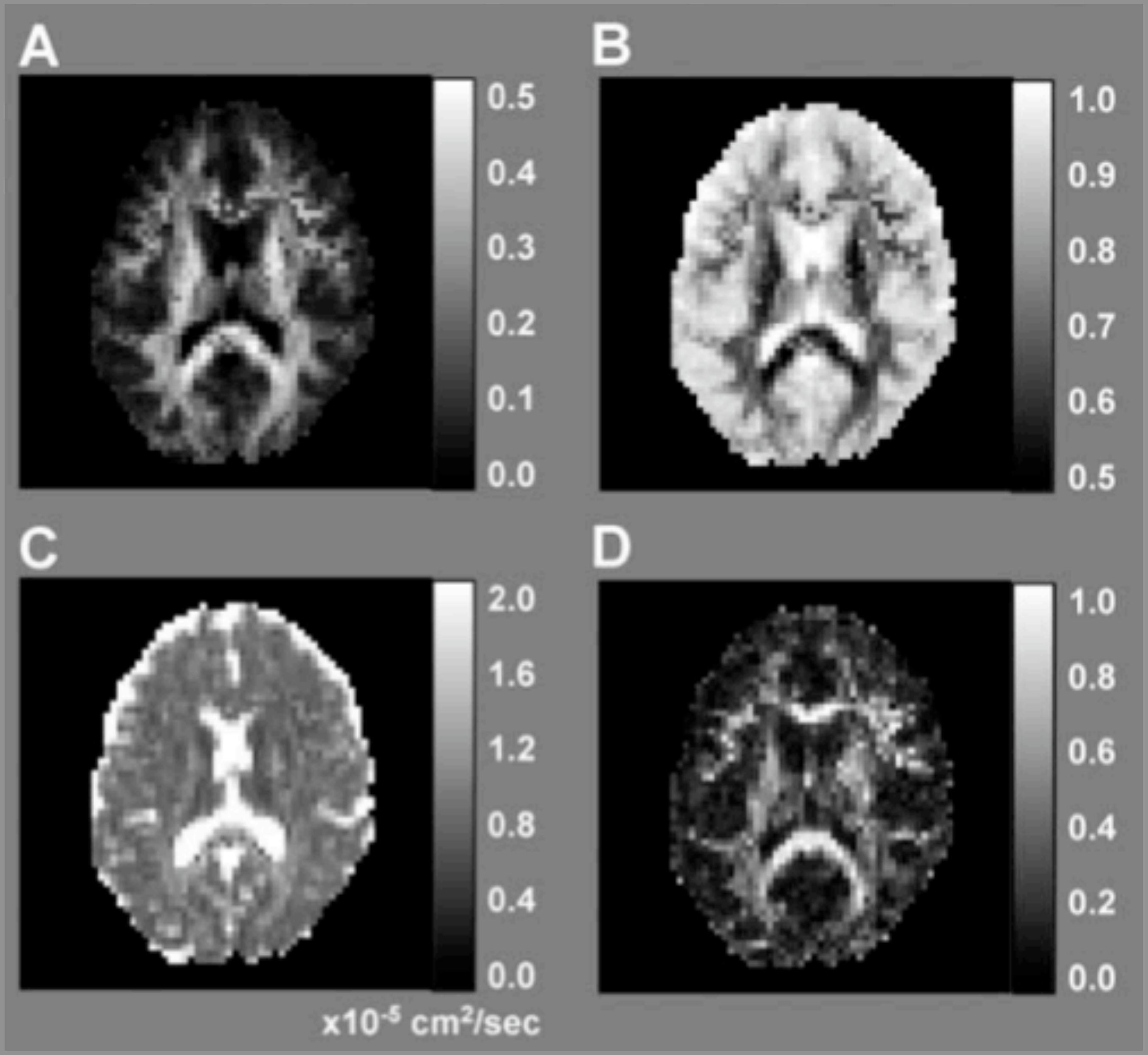
Restricted

Corpus callosum | Cingulum | A+B

The CHARMED Model

Restricted

Hindered



Mean diffusivity

FA

Assaf and Basser, Neuroimage 27:48 (2005)