Lecture 10
Basic DTI Acquisition

For next Lecture

Please install AFNI
http://afni.nimh.nih.gov/afni/

Download sample DTI data
on "Software" page of course website

For help, contact Tom Lesperance at:
tlesperance@ucsd.edu

Review

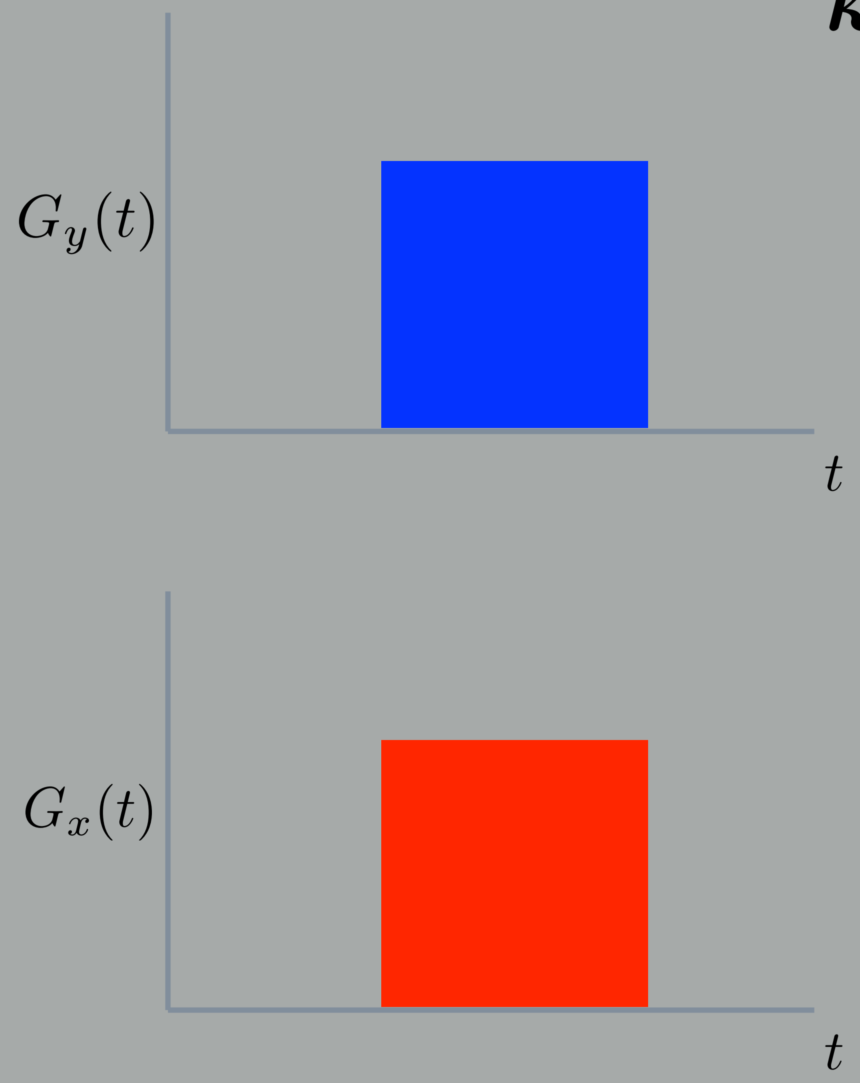
1. The effect of a bipolar gradient on stationary spins
   **** Imaging ****

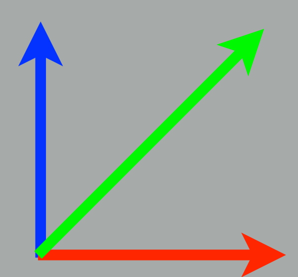
2. The effect of a bipolar gradient on diffusing spins
   **** Diffusion weighting ****

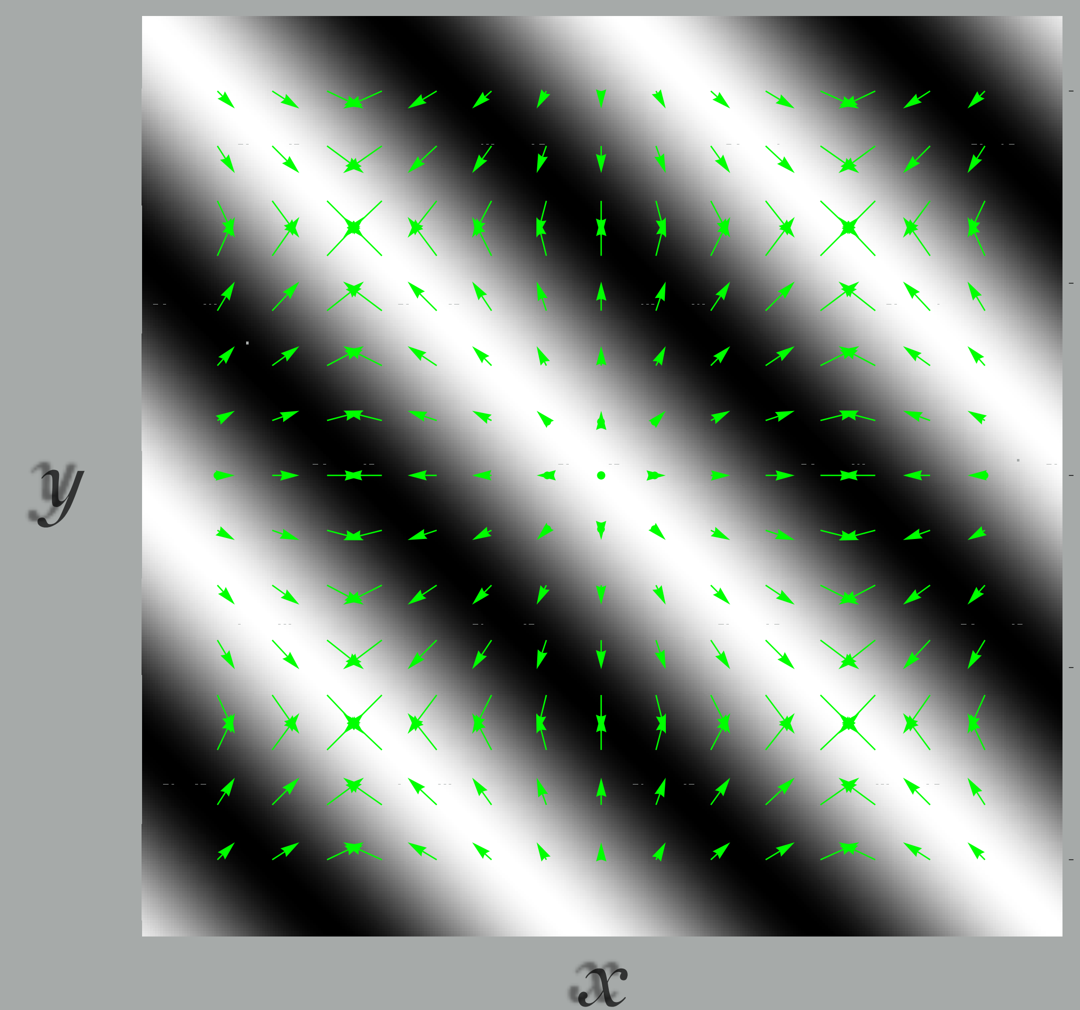
Then we'll put these together to get DTI

What gradients are doing

$$\boldsymbol{k} \cdot \boldsymbol{x} = k_x x + k_y y = \gamma G_x t x + \gamma G_y t y$$

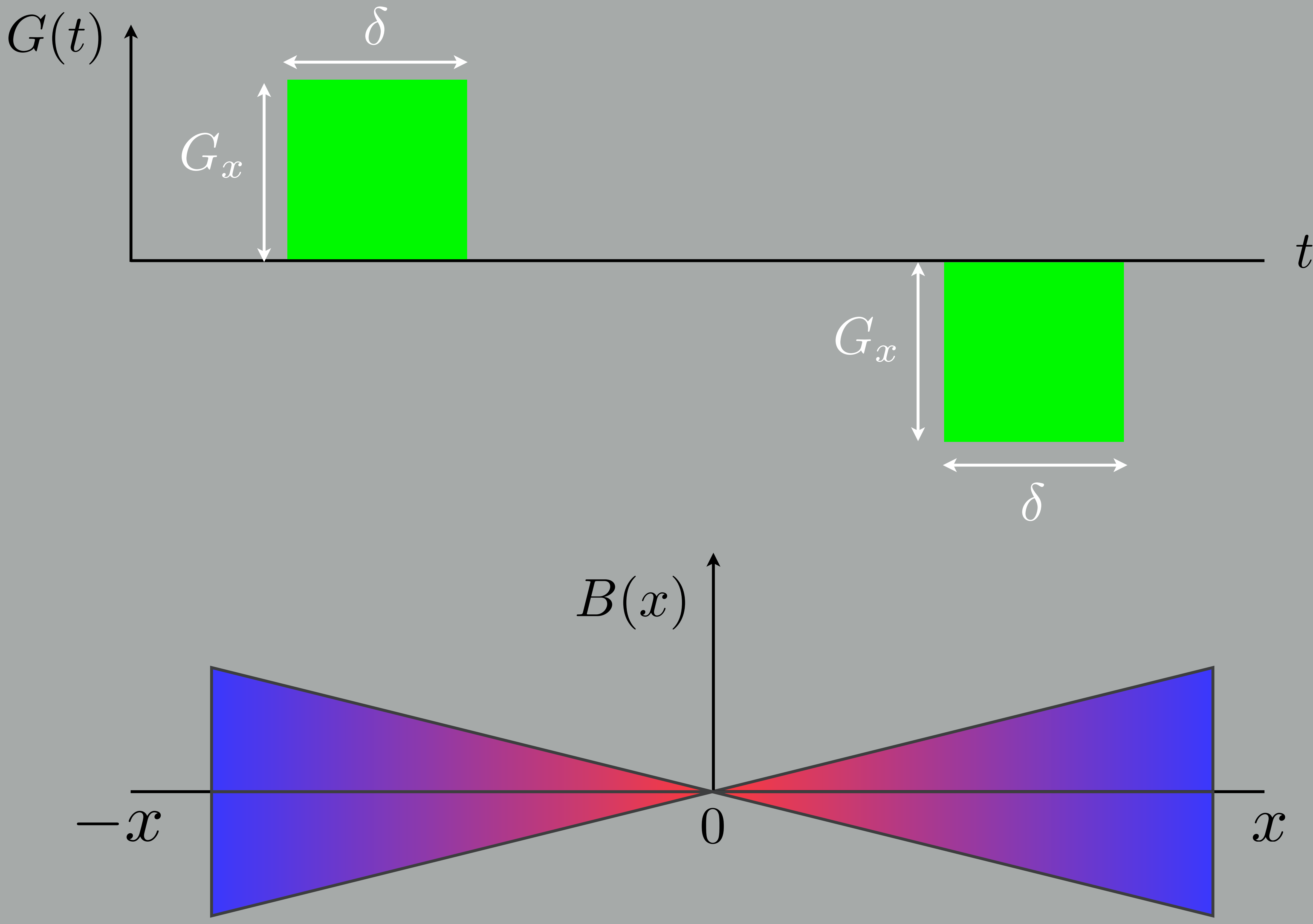




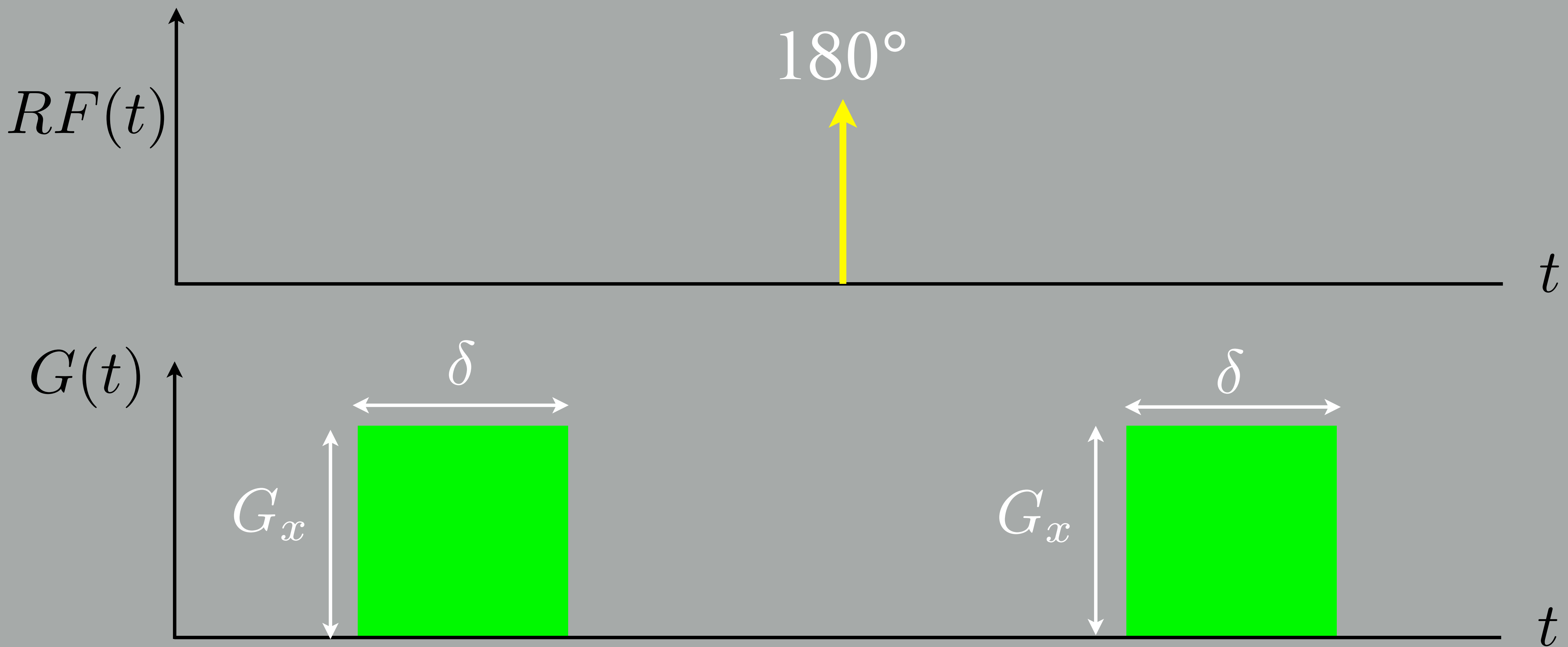


spatial modulation of the phase

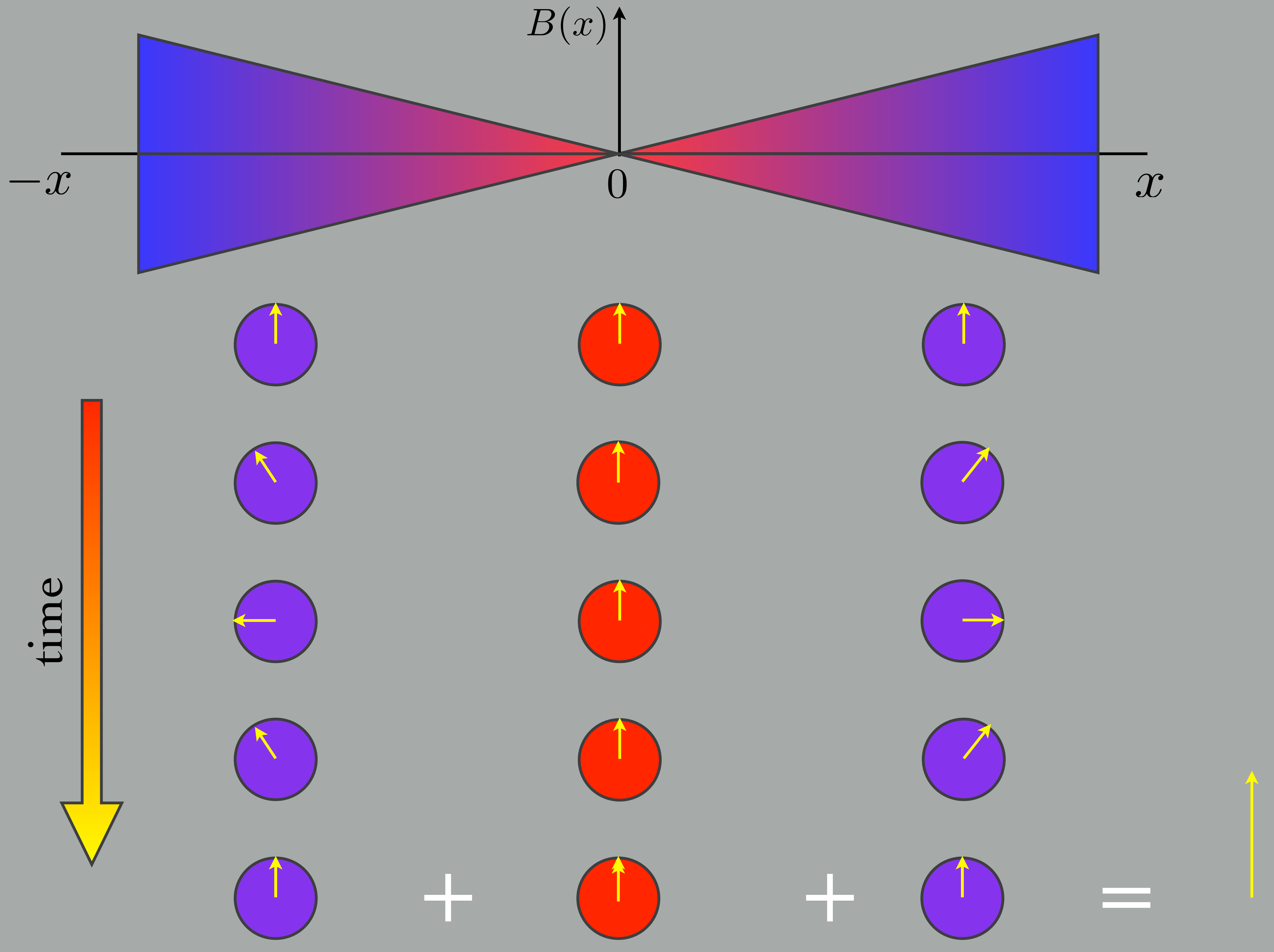
The Bipolar Gradient Pulse (gradient echo)



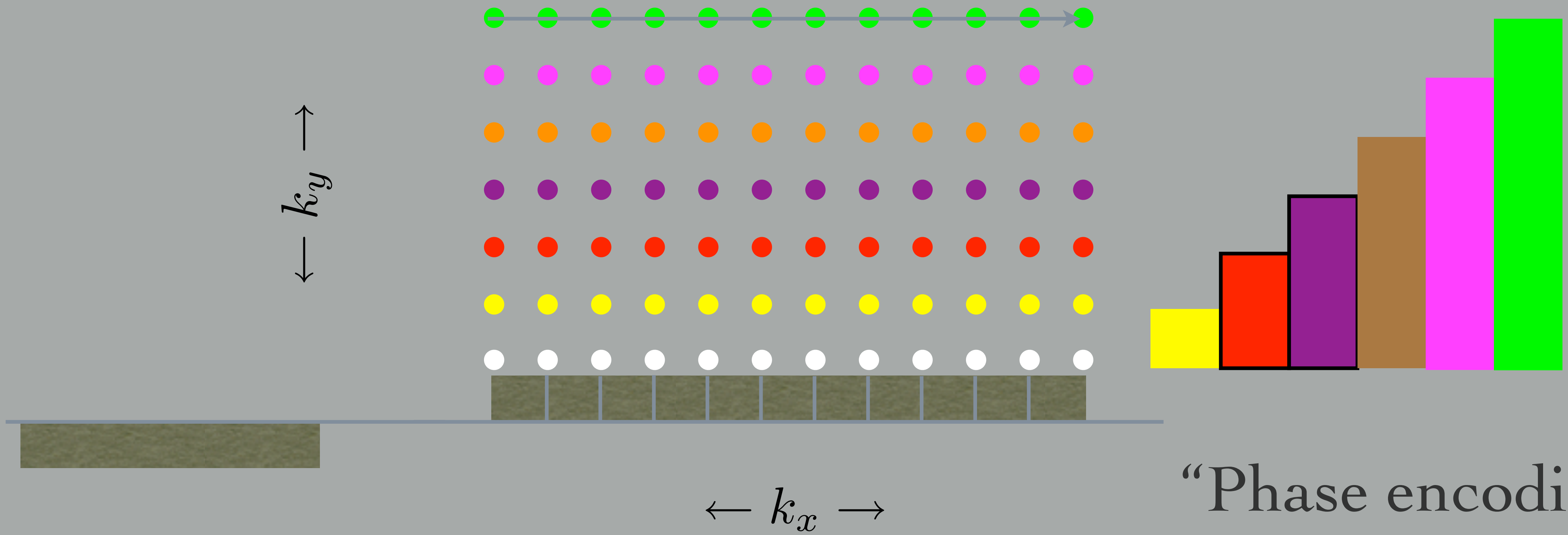
The Bipolar Gradient Pulse (spin echo)



Stationary Spins in Bipolar Pulse



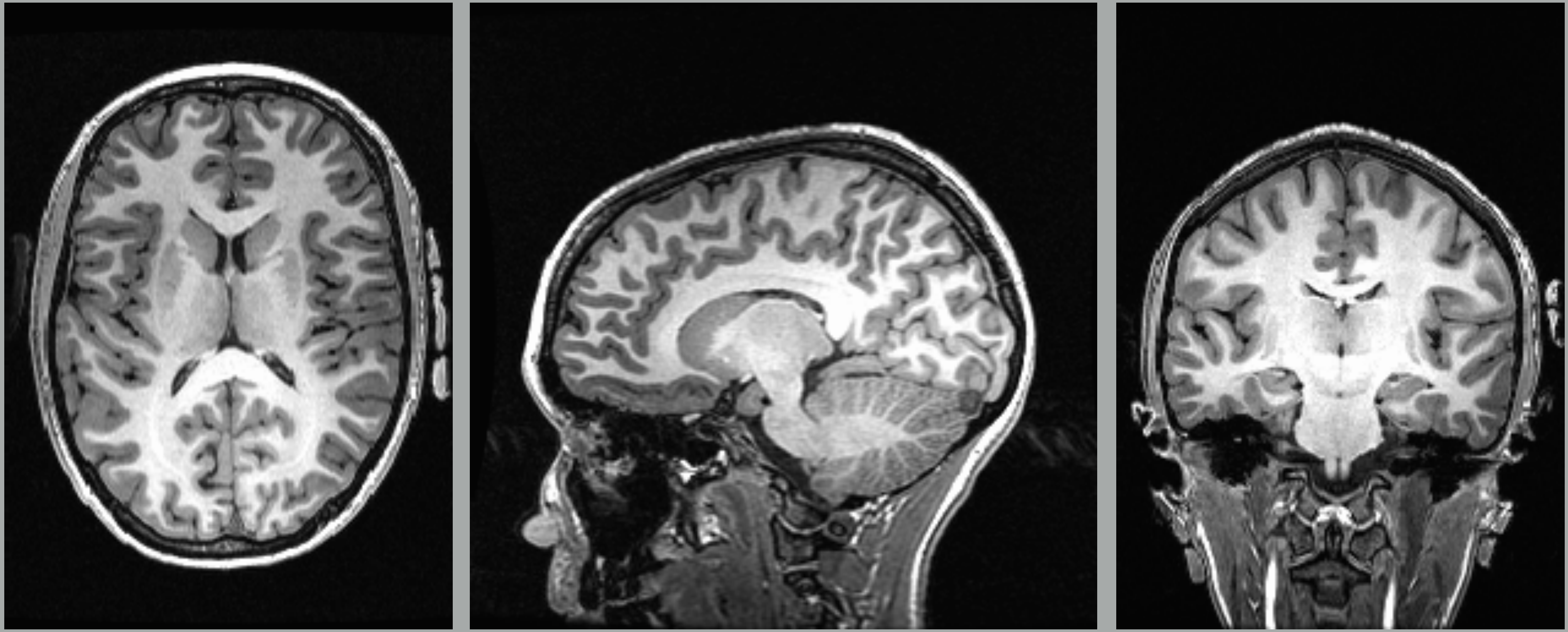
k-space trajectory



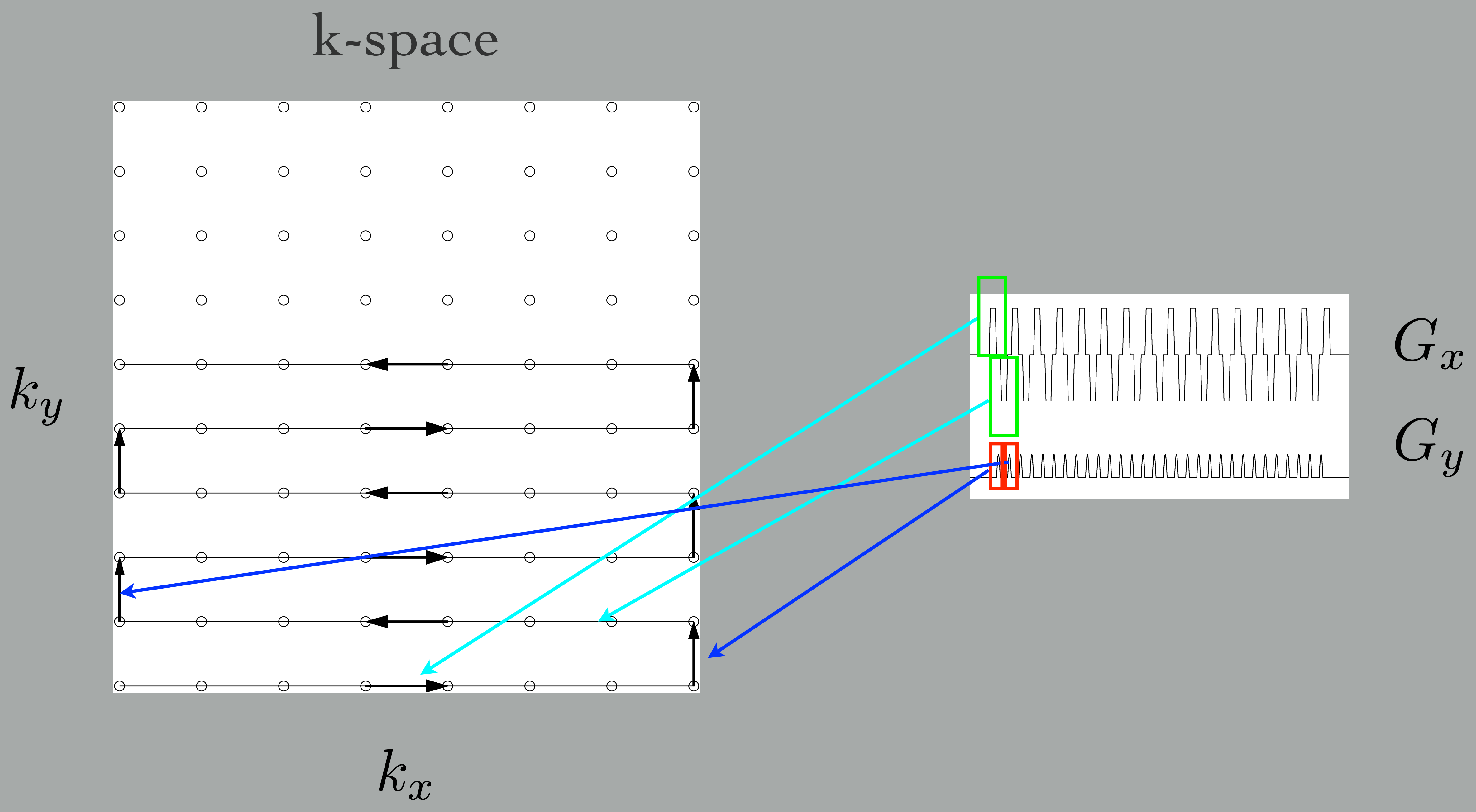
“Phase encoding”

“Frequency encoding”

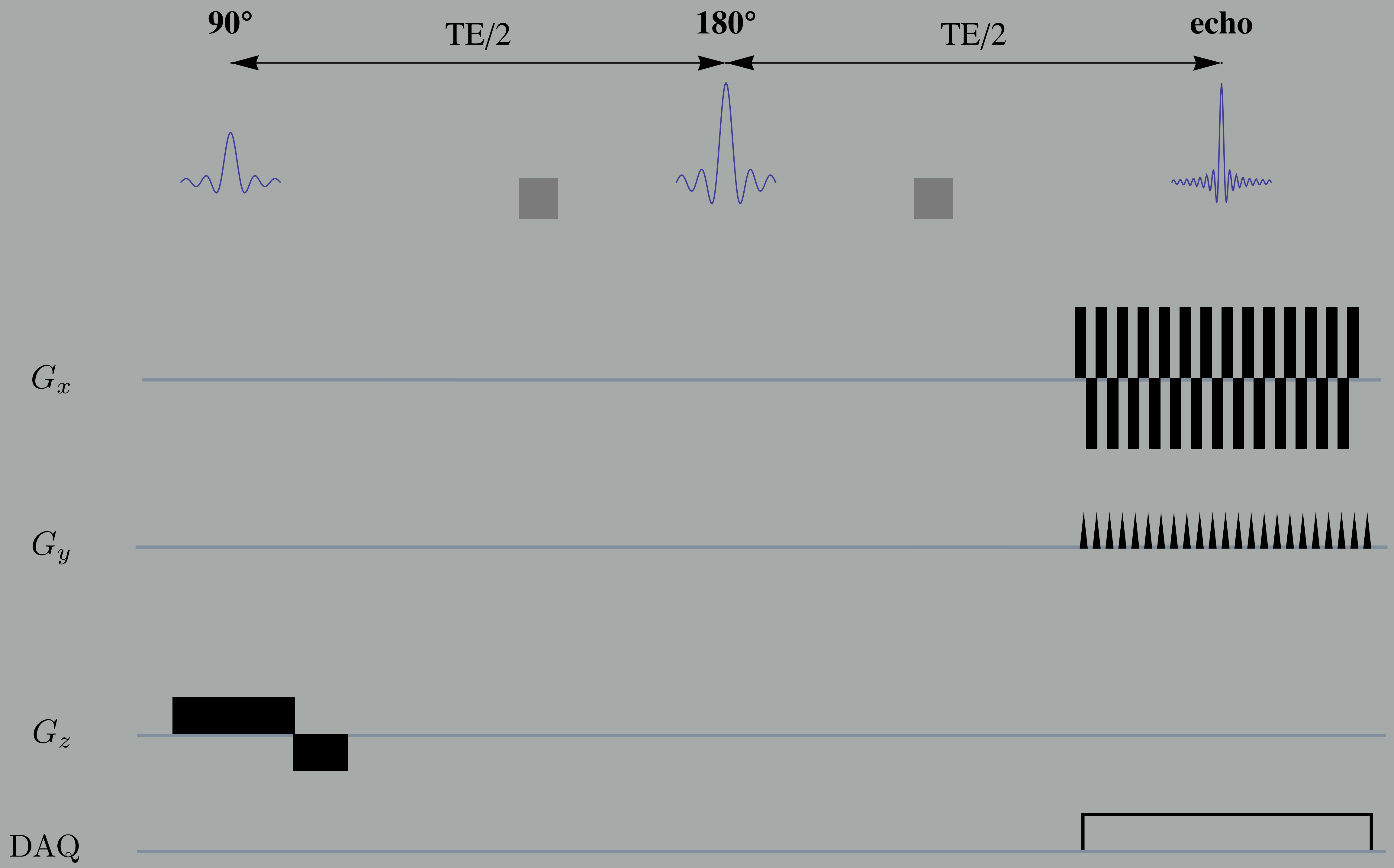
High resolution anatomical acquisition



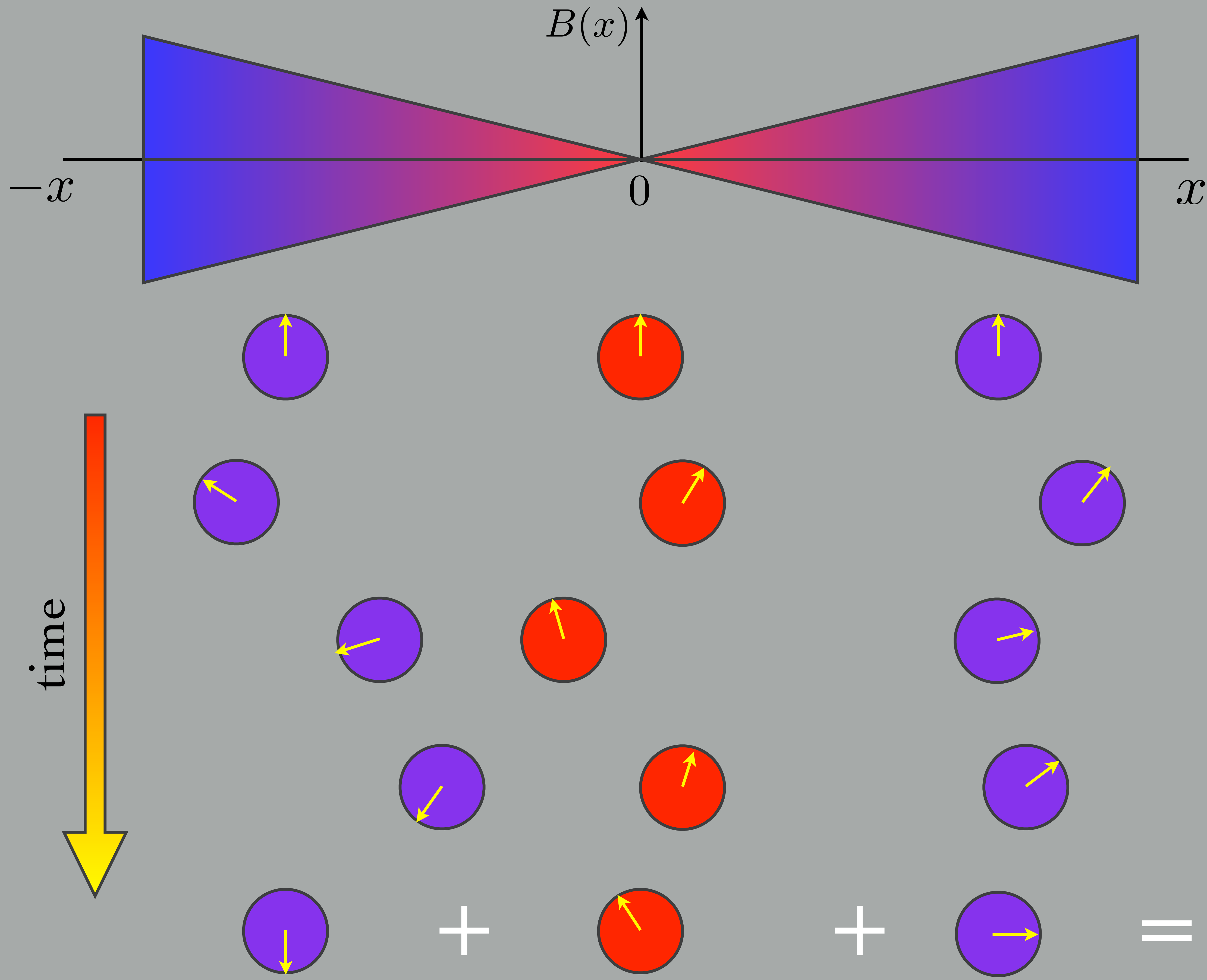
Basic EPI acquisition



Basic EPI acquisition



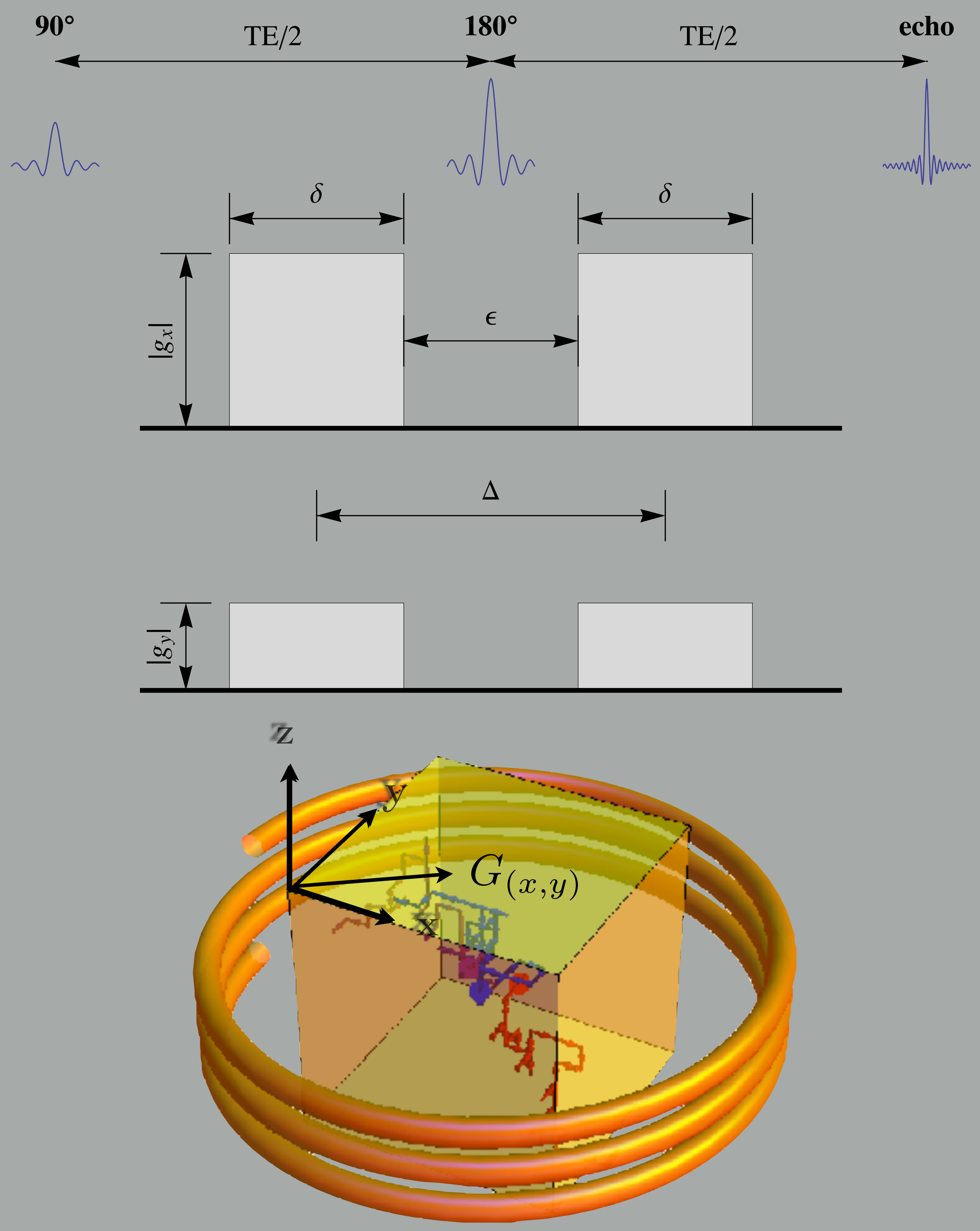
Diffusing Spins in Bipolar Pulse



Key Fact

Only diffusion along the direction of the applied gradient has an effect

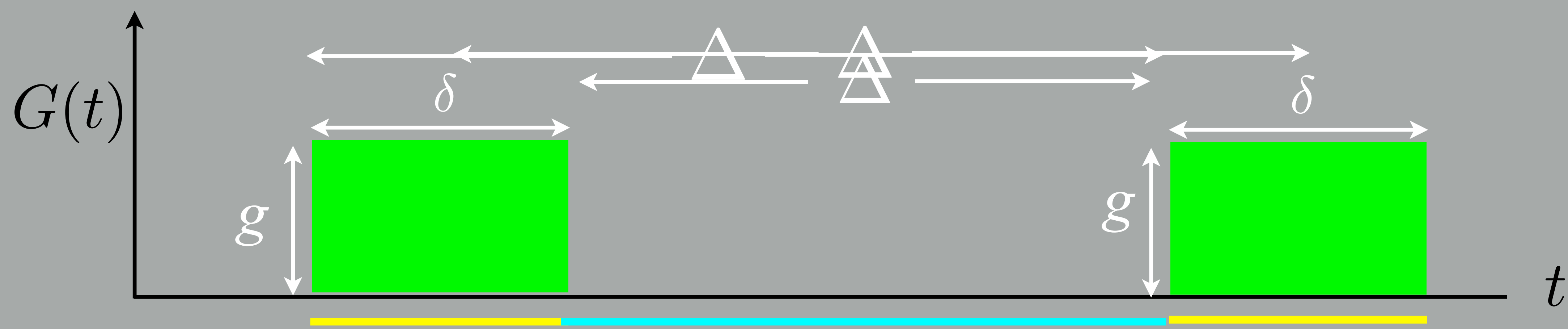
Directional Diffusion Encoding



The b-value

$$b = g^2\delta^2\left(\Delta + \frac{2}{3}\delta\right)$$

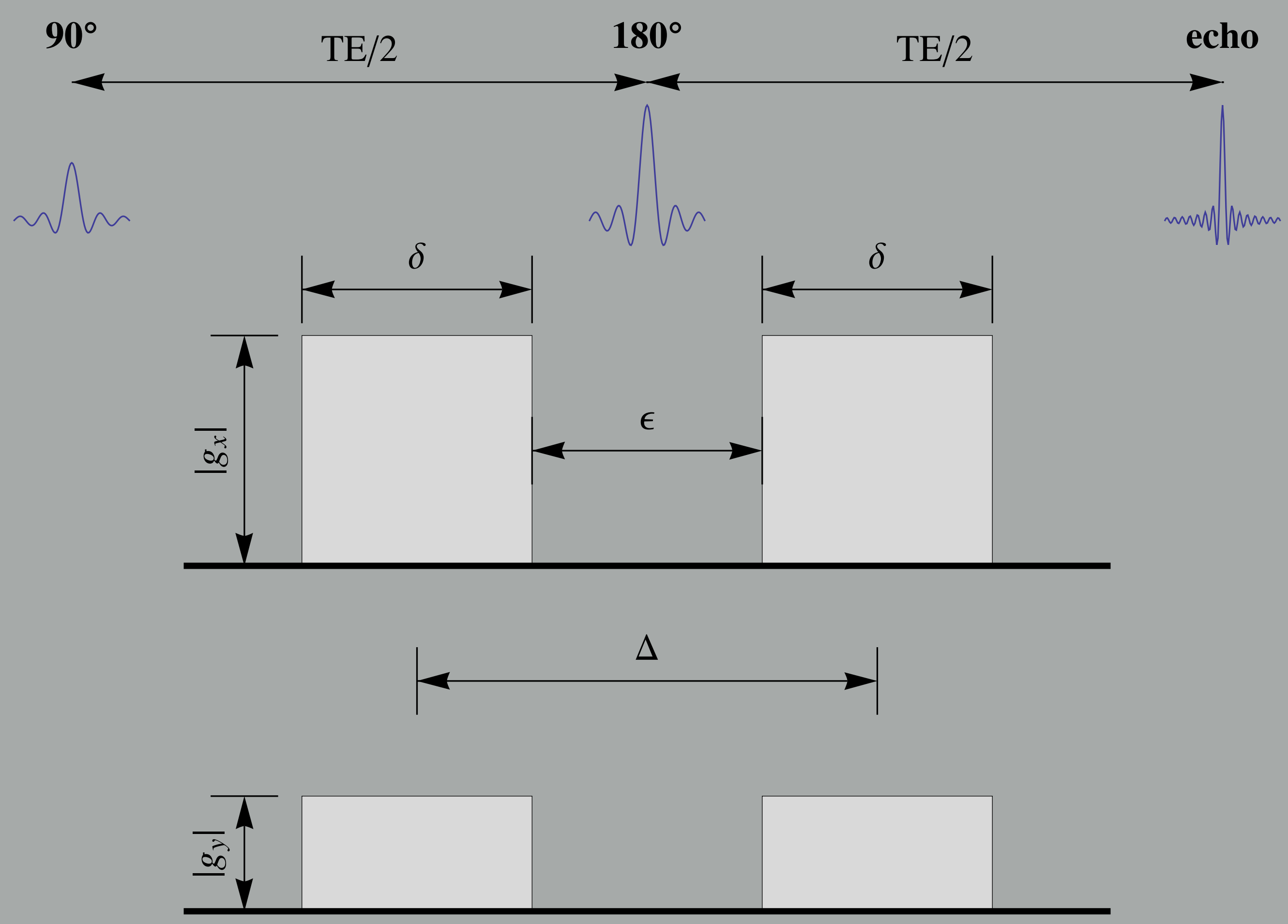
$$b = g^2\delta^2\left(\Delta - \frac{1}{3}\delta\right)$$



$$\int k^2\, dt = g^2 \int_0^{\delta} t^2\, dt + g^2\delta^2 \int_0^{\Delta} dt + g^2 \int_0^{\delta} t^2\, dt$$

$$b = g^2\frac{\delta^3}{3} + g^2\delta^2\Delta + g^2\frac{\delta^3}{3}$$

B-MATRIX



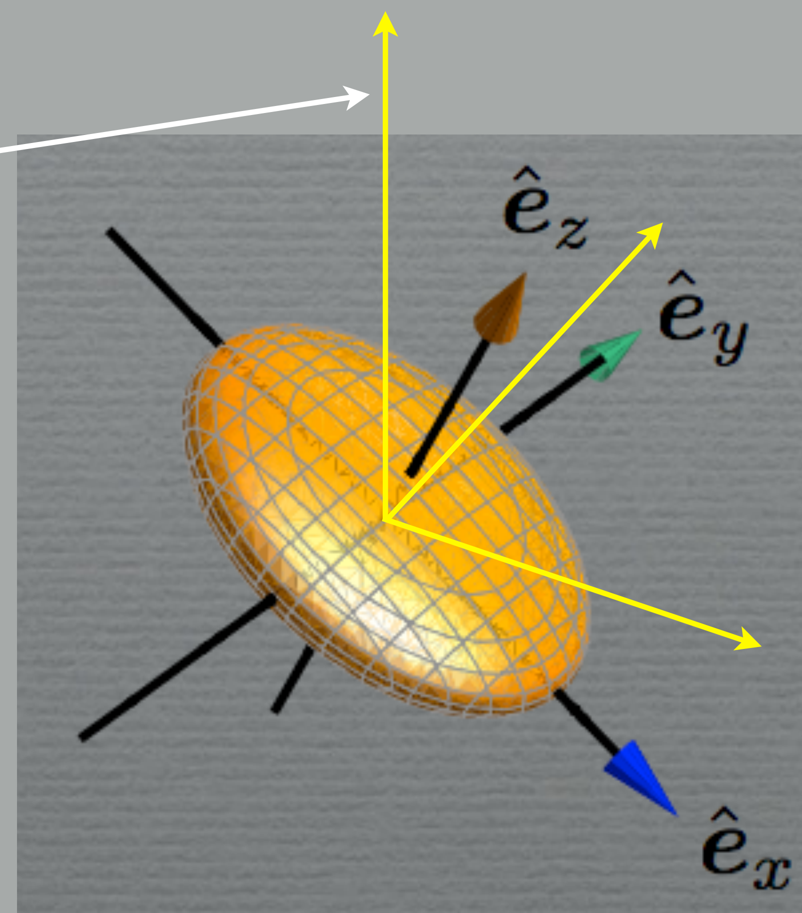
$$b_{ij} = \underbrace{G_i G_j \delta^2}_{q_i q_j} \underbrace{(\Delta - \delta/3)}_{\tau}$$

The 3D Gaussian Distribution:

$$P(\boldsymbol{r}|\boldsymbol{r}_0, \tau) \sim N(\boldsymbol{r}_0, \boldsymbol{\Sigma})$$

$$\boldsymbol{r} = \{x, y, z\}$$

scanner coordinate system



Diffusion Tensor

Covariance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 & \sigma_{xz}^2 \\ \sigma_{yx}^2 & \sigma_{yy}^2 & \sigma_{yz}^2 \\ \sigma_{zx}^2 & \sigma_{zy}^2 & \sigma_{zz}^2 \end{pmatrix} = 6\tau \underbrace{\begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}}_{\boldsymbol{D}}$$

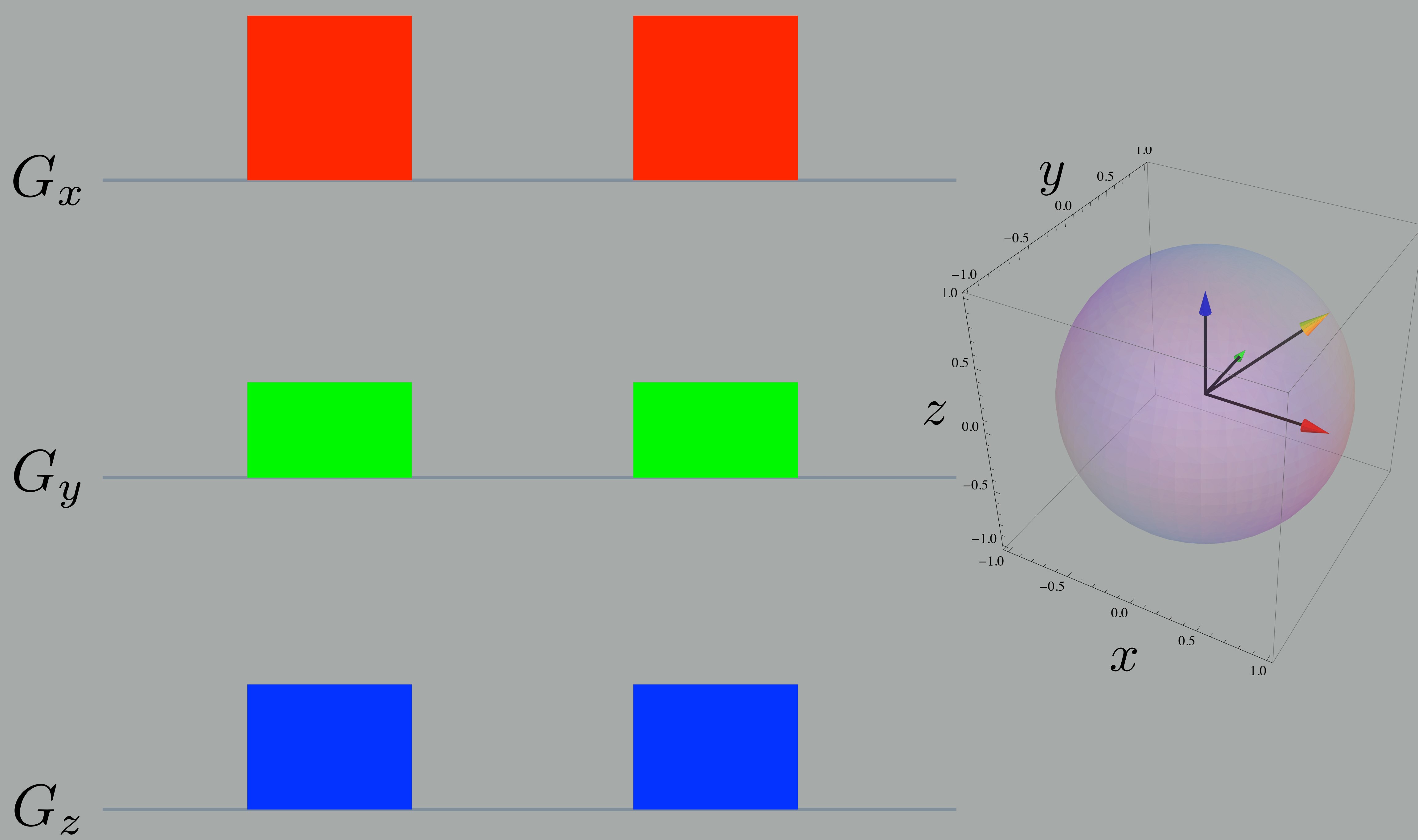
Estimating the diffusion tensor

3D Gaussian diffusion

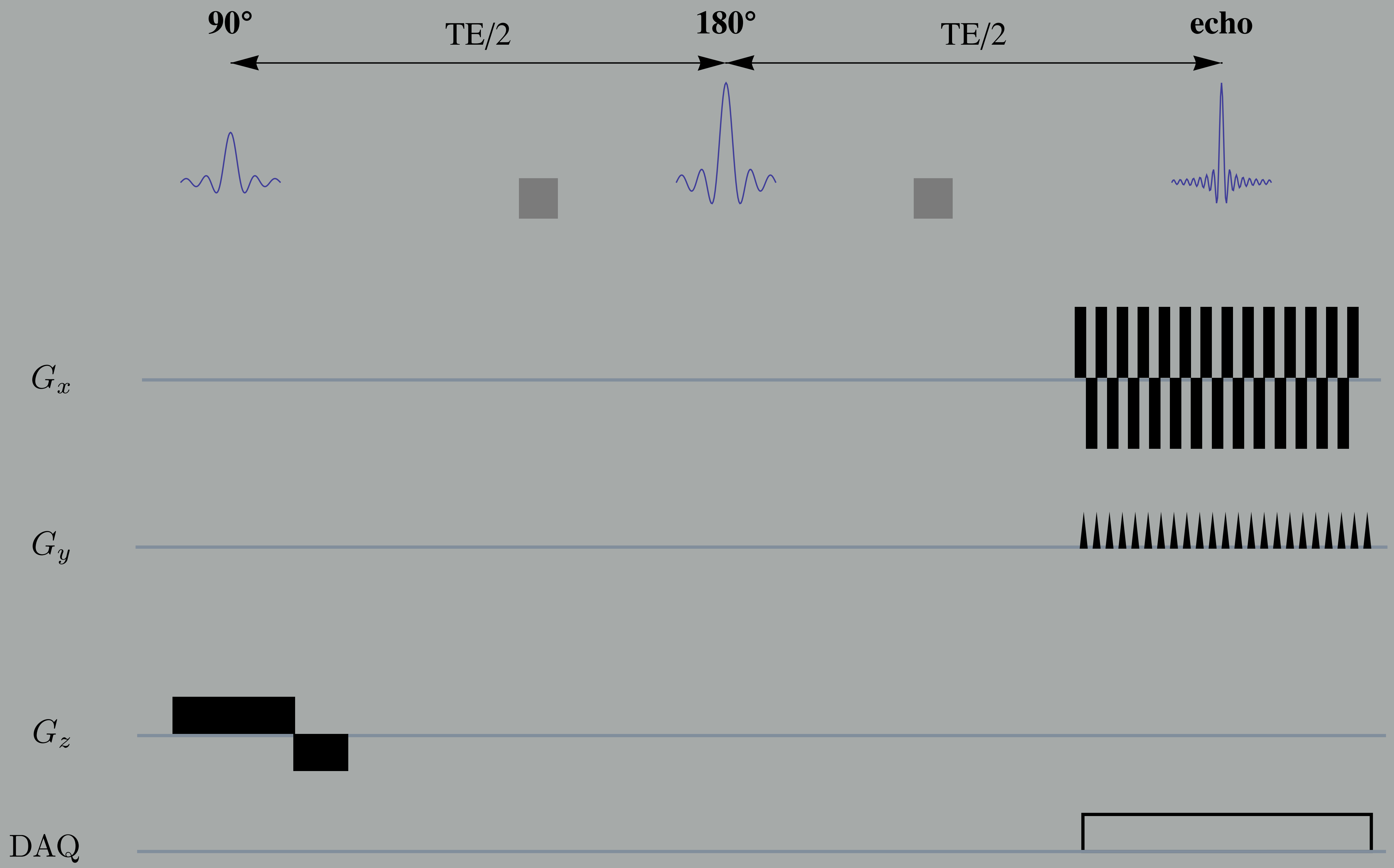
$$s(b_{ij}) = s(0)\exp\left(-\sum_{i}^{3}\sum_{j}^{3} b_{ij} D_{ij}\right)$$

$$b_{ij} = \underbrace{G_i G_j \delta^2}_{q_i q_j} \underbrace{(\Delta - \delta/3)}_{\tau}$$

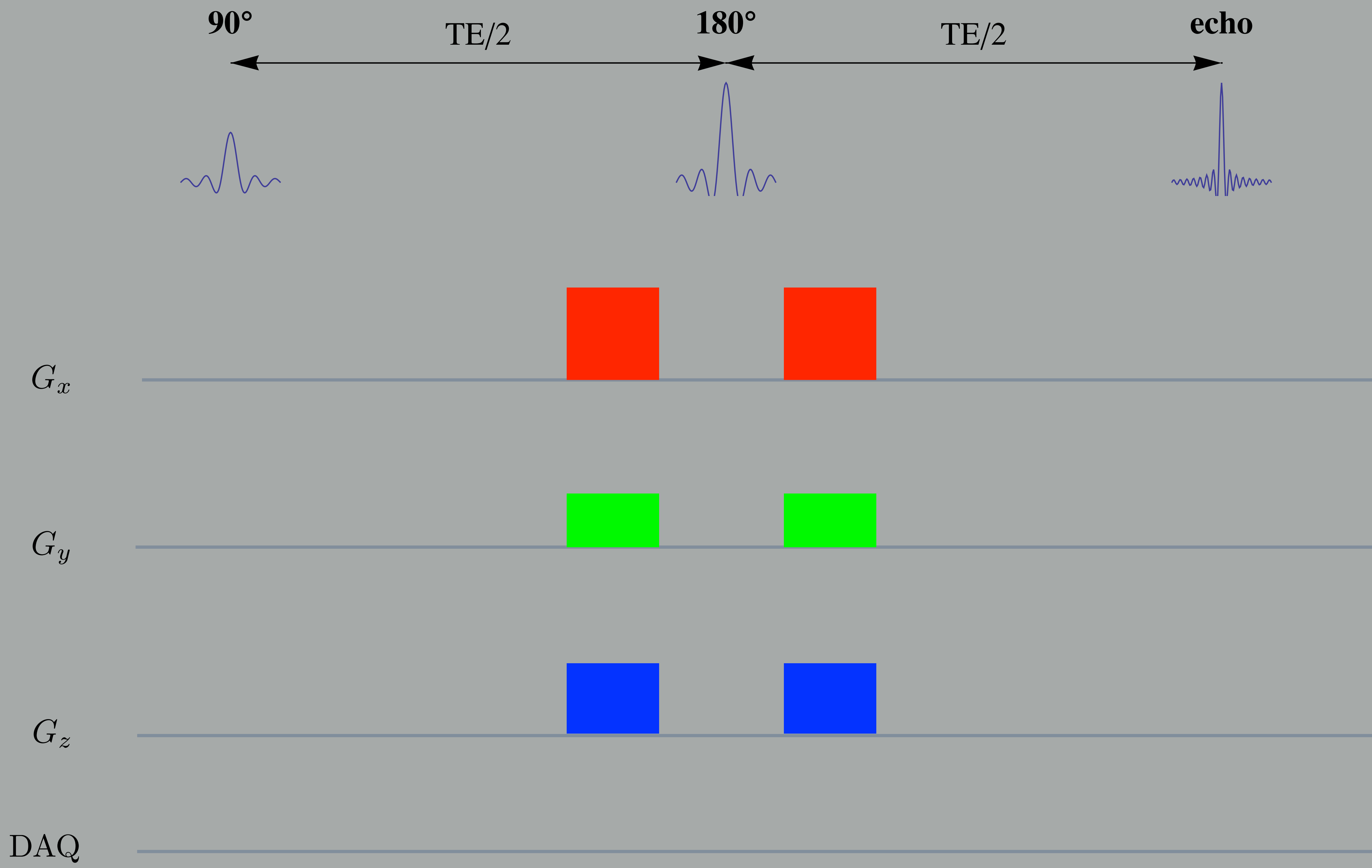
A single diffusion-weighting direction



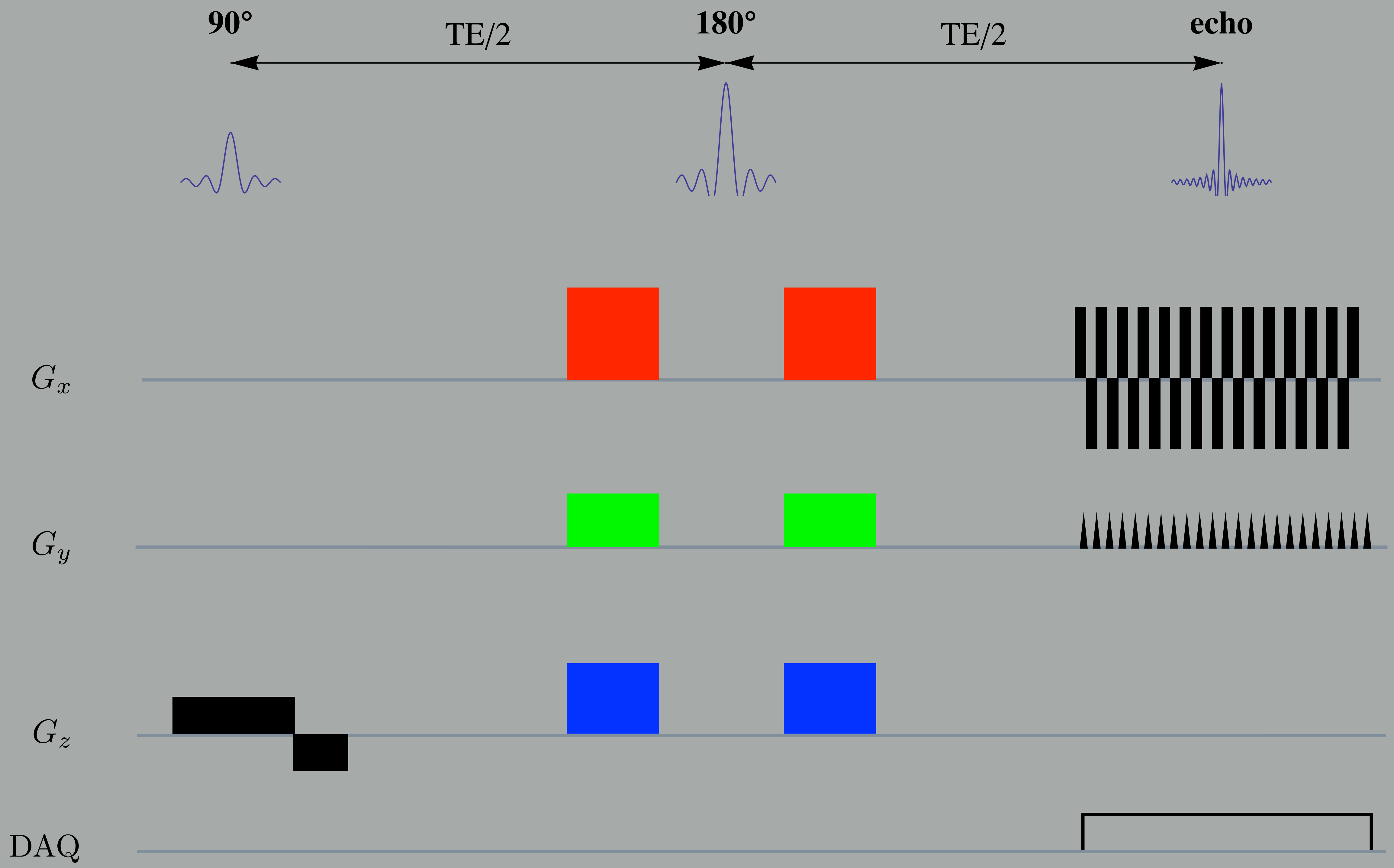
Basic EPI acquisition ...



... plus diffusion weighting gradients ...



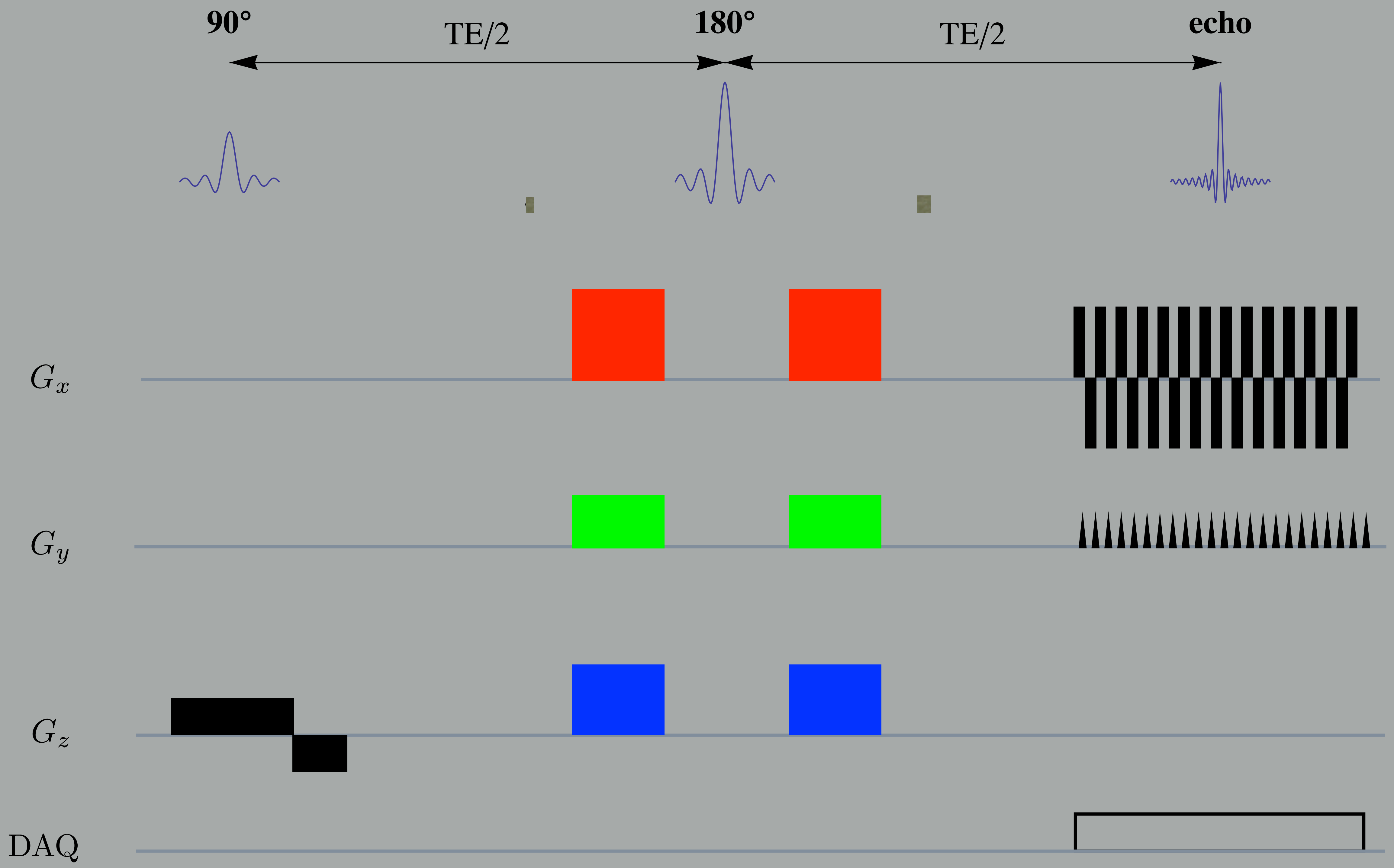
... gives Basic EPI DTI acquisition!



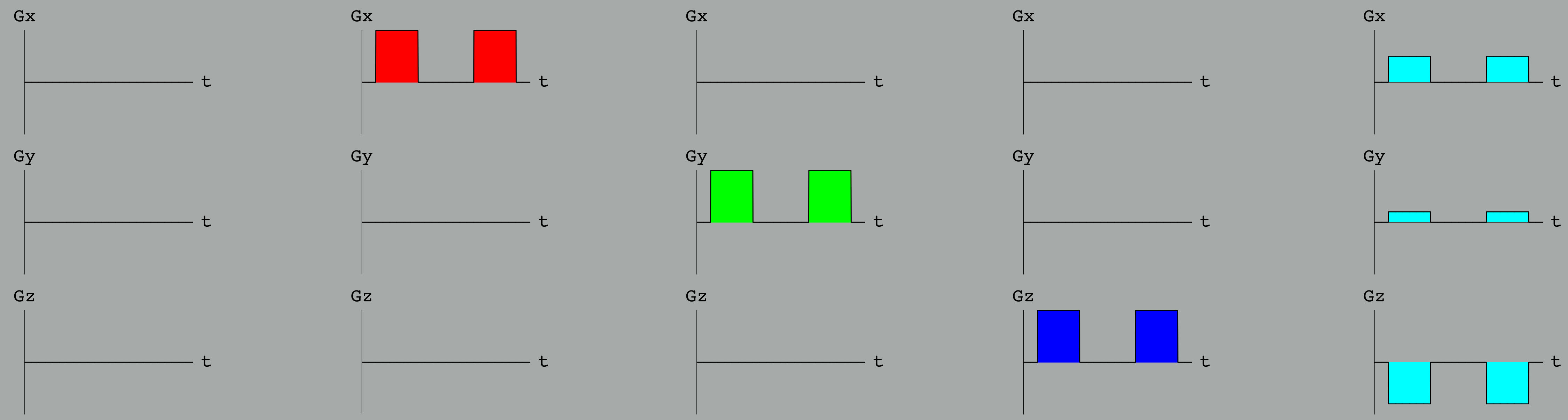
Extension to Imaging

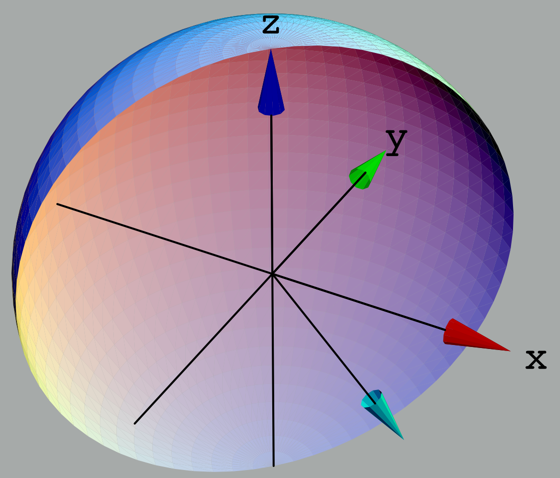
Because the diffusion weighting does not interfere
with the stationary tissue signal,
we can "insert" it into a
standard imaging procedure

Extension to Imaging

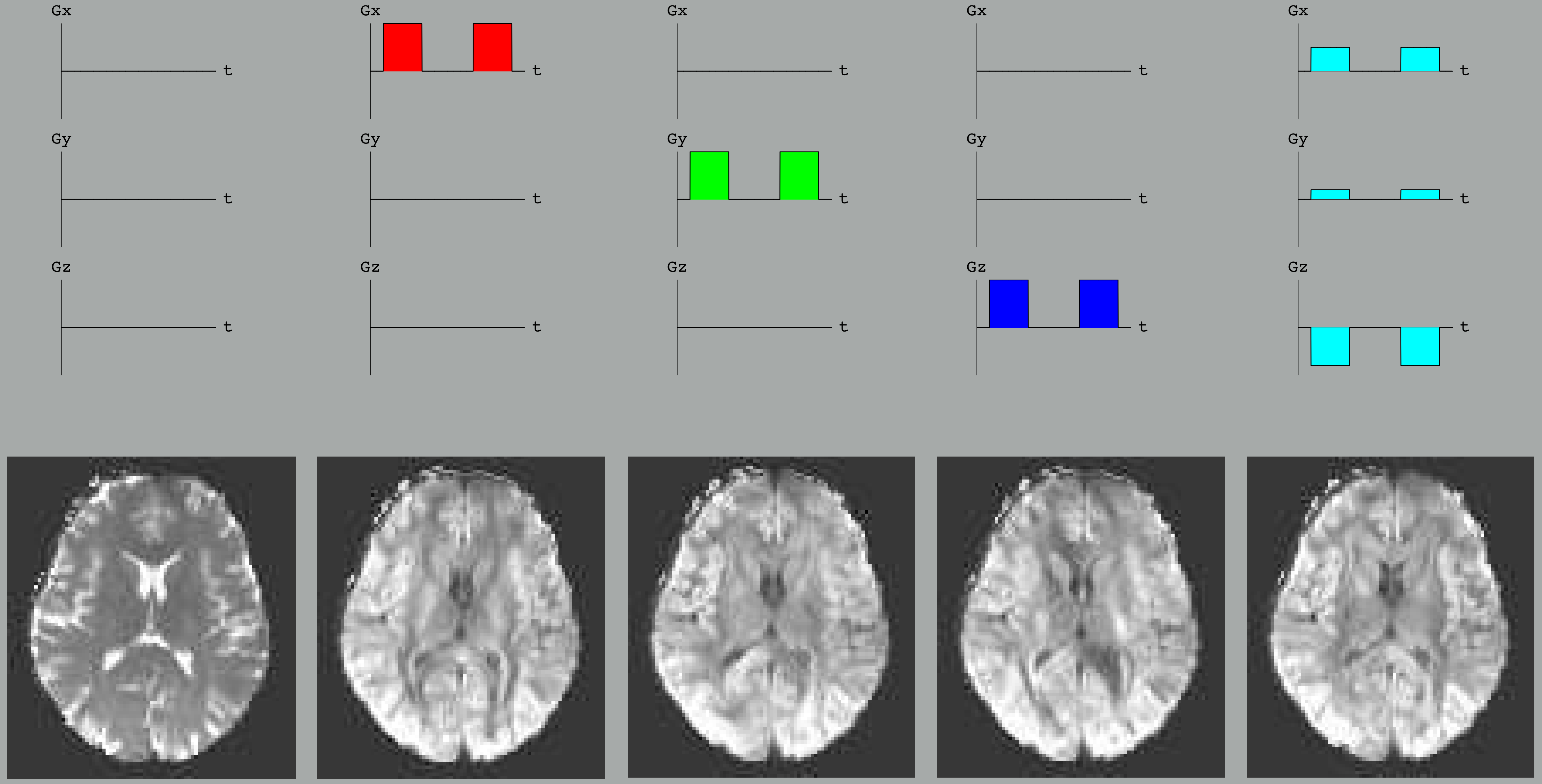


Directional diffusion encoding

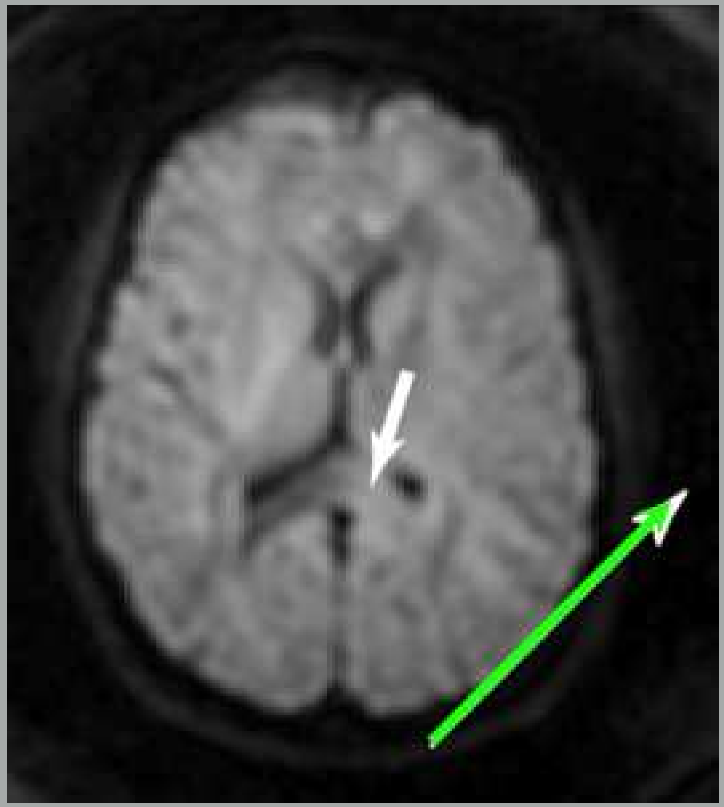


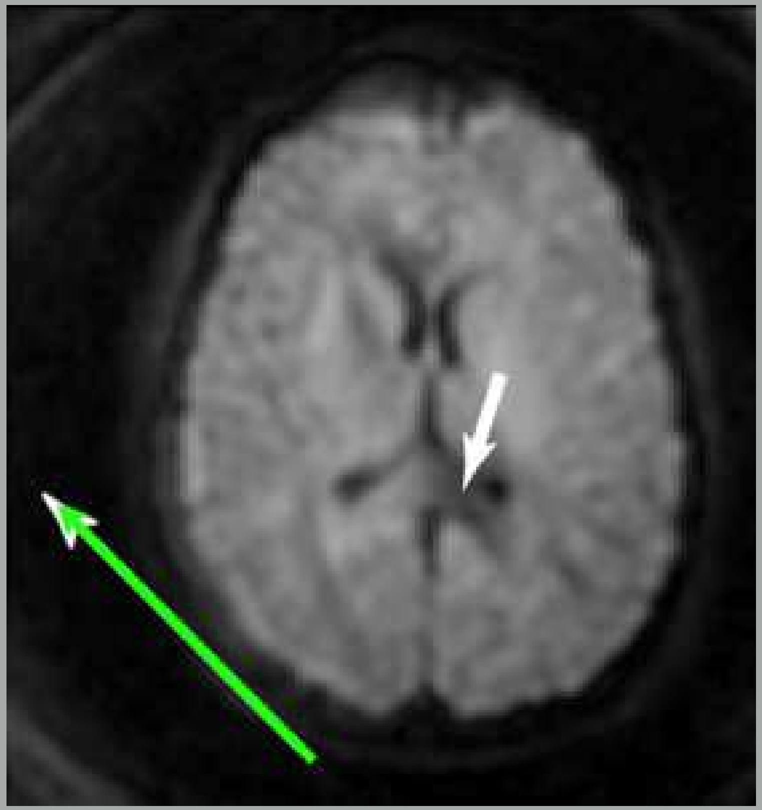


Directional diffusion encoding

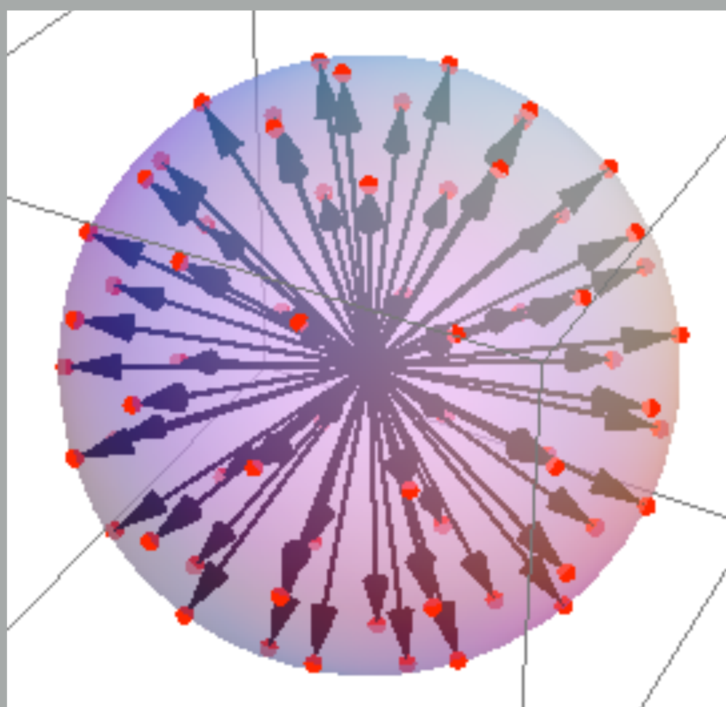


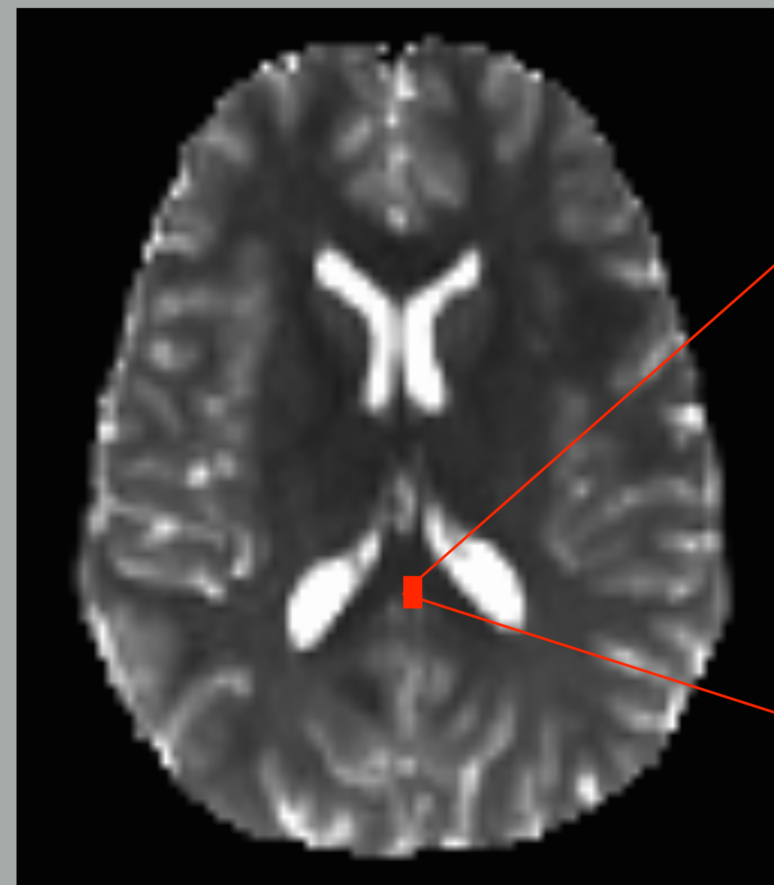
Directional diffusion encoding



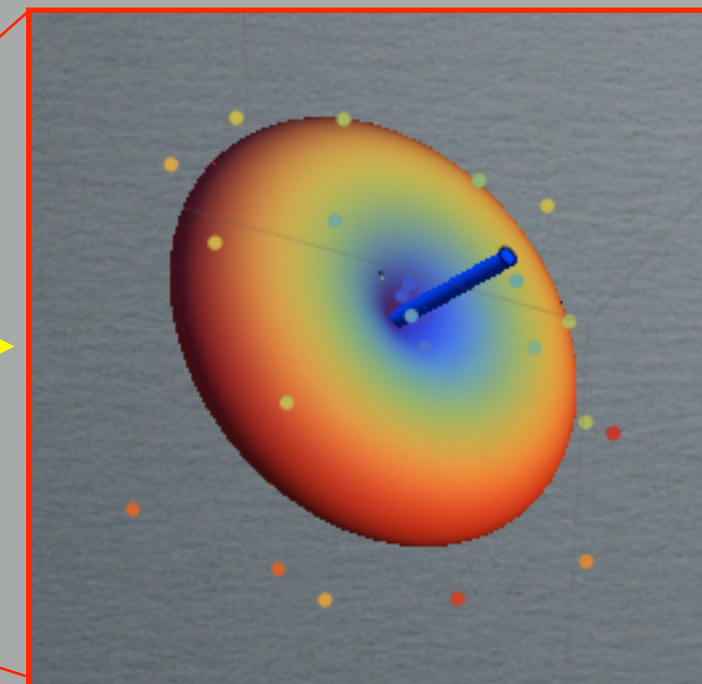


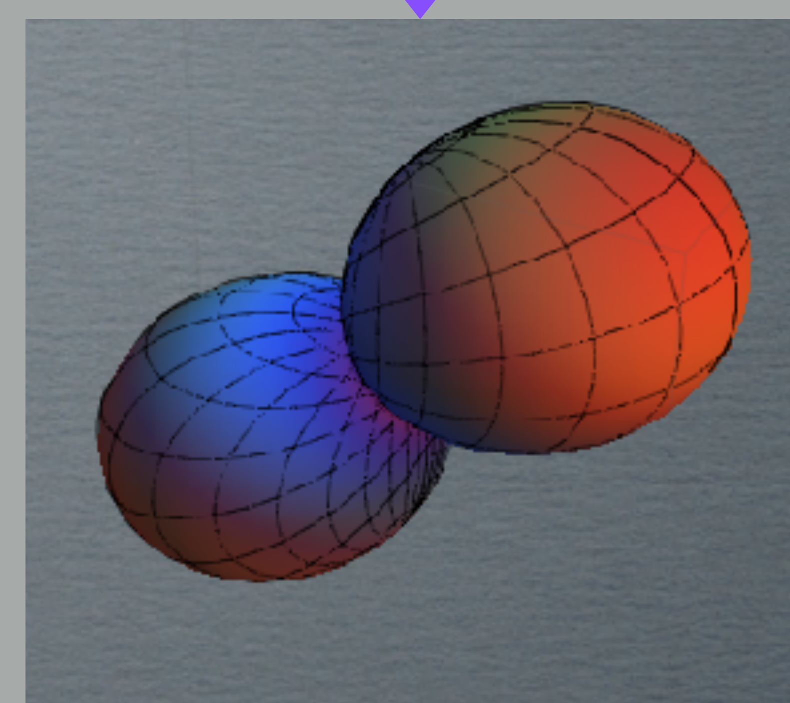
DTI





voxel signal
from multiple images at
different directions

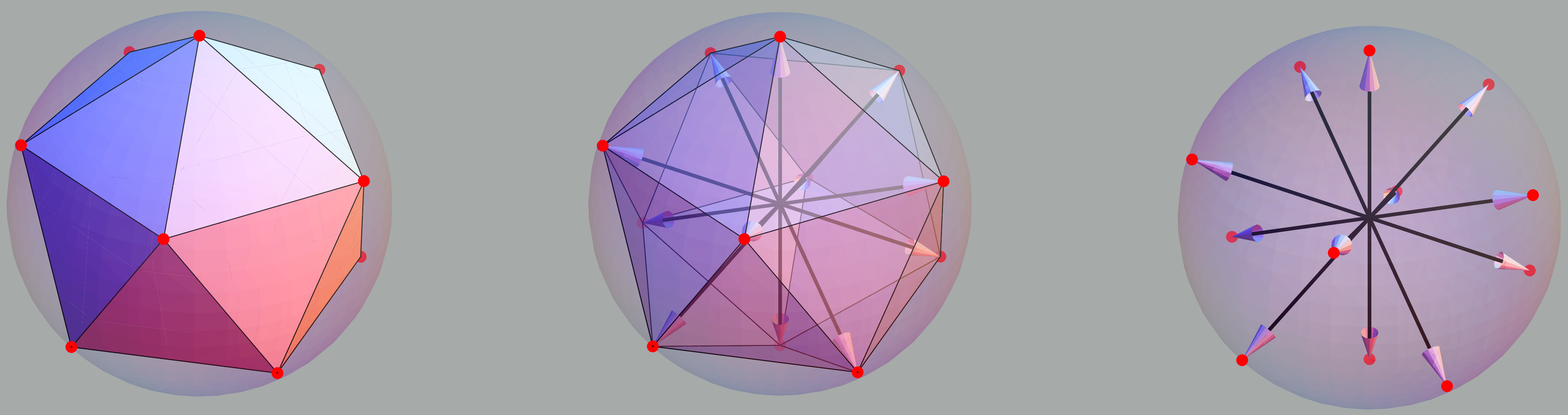




voxel apparent diffusion coefficient
from multiple images at different
directions

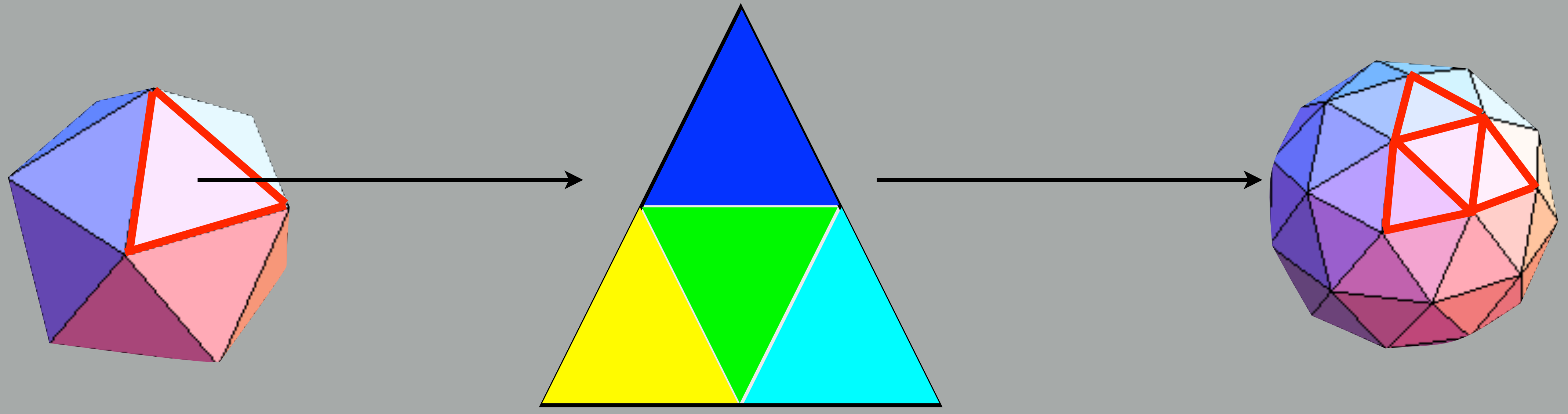
Sampling schemes for angular measurements

Vertices of Platonic Solids



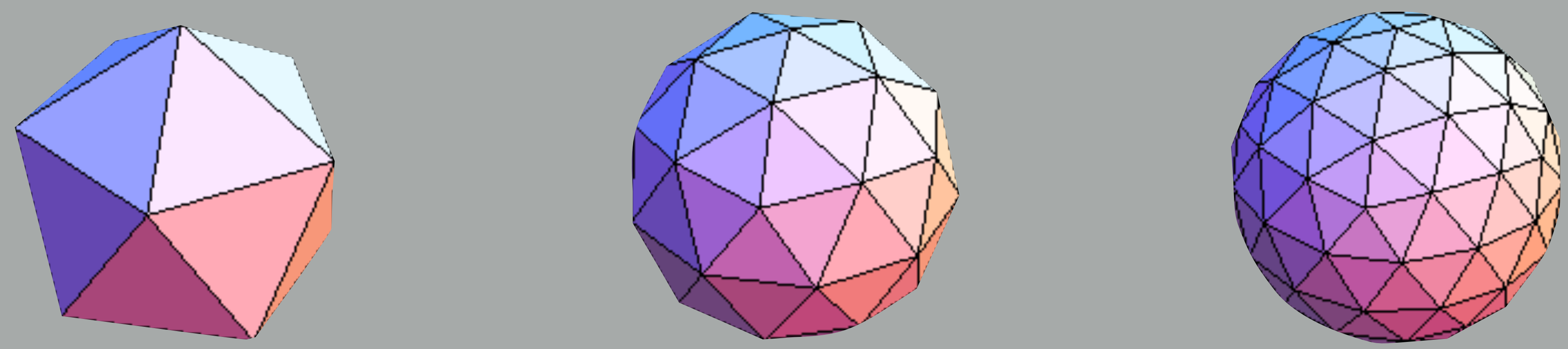
Sampling schemes for angular measurements

Tessellation of the icosahedron



Sampling schemes for angular measurements

Tessellation of the icosahedron

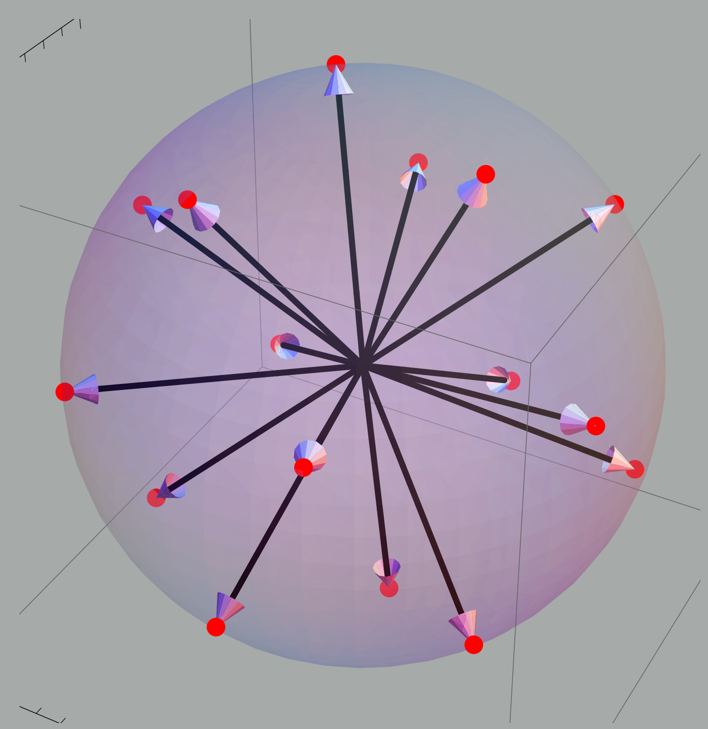


Not very flexible - only a few fixed (and large!) number of directions available

Sampling schemes for angular measurements

Electrostatic Repulsion Model

Minimize energy of charged points on a sphere



Very flexible - can choose any number of directions
(This is used on the CFMRI GE scanners)

The Shape of (Gaussian) Diffusion

$$s(b) = s(0)e^{-b\tilde{D}}$$

where

$$\tilde{D} = \boldsymbol{u}^t \cdot \boldsymbol{D} \cdot \boldsymbol{u}$$

$\boldsymbol{u}$ is the measurement direction

What is the meaning of $\tilde{\boldsymbol{D}}$?

$$\tilde{\boldsymbol{D}} \equiv \boldsymbol{u}^t \cdot \boldsymbol{D} \cdot \boldsymbol{u}$$

The *measured* quantity is the *projection* of the ellipsoid onto the diffusion-sensitized axis

Thus, the measured diffusion coefficient along an arbitrary direction is proportional to the variance of the projection of the spin displacement onto the measurement axis

The Sampling Vector

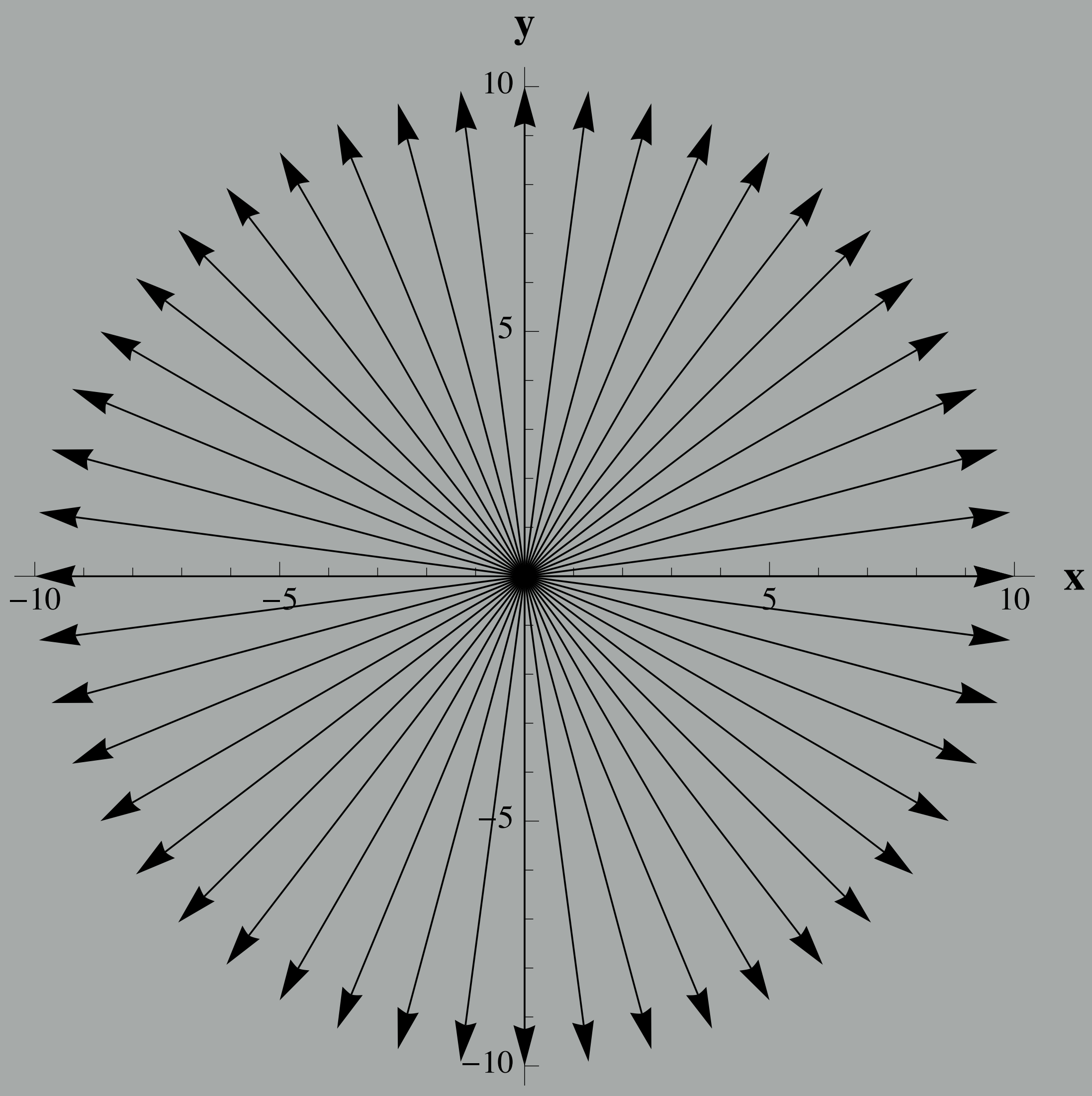
$$\boldsymbol{u} = \boldsymbol{R}\boldsymbol{v}$$

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\boldsymbol{u} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

Sampling Directions

$$\boldsymbol{u}_i = \begin{pmatrix} \cos\theta_i \\ \sin\theta_i \end{pmatrix}$$



The Apparent Diffusion Coefficient

$$\boldsymbol{u}^t = \begin{pmatrix} \cos\theta & \sin\theta \end{pmatrix} \qquad \boldsymbol{D} = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix} \qquad \boldsymbol{u} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$\tilde{D} = \boldsymbol{u}^t \cdot \boldsymbol{D} \cdot \boldsymbol{u}$$

$$D(\theta) = d_1 \cos^2\theta + d_2 \sin^2\theta$$

The Shape of (Gaussian) Diffusion

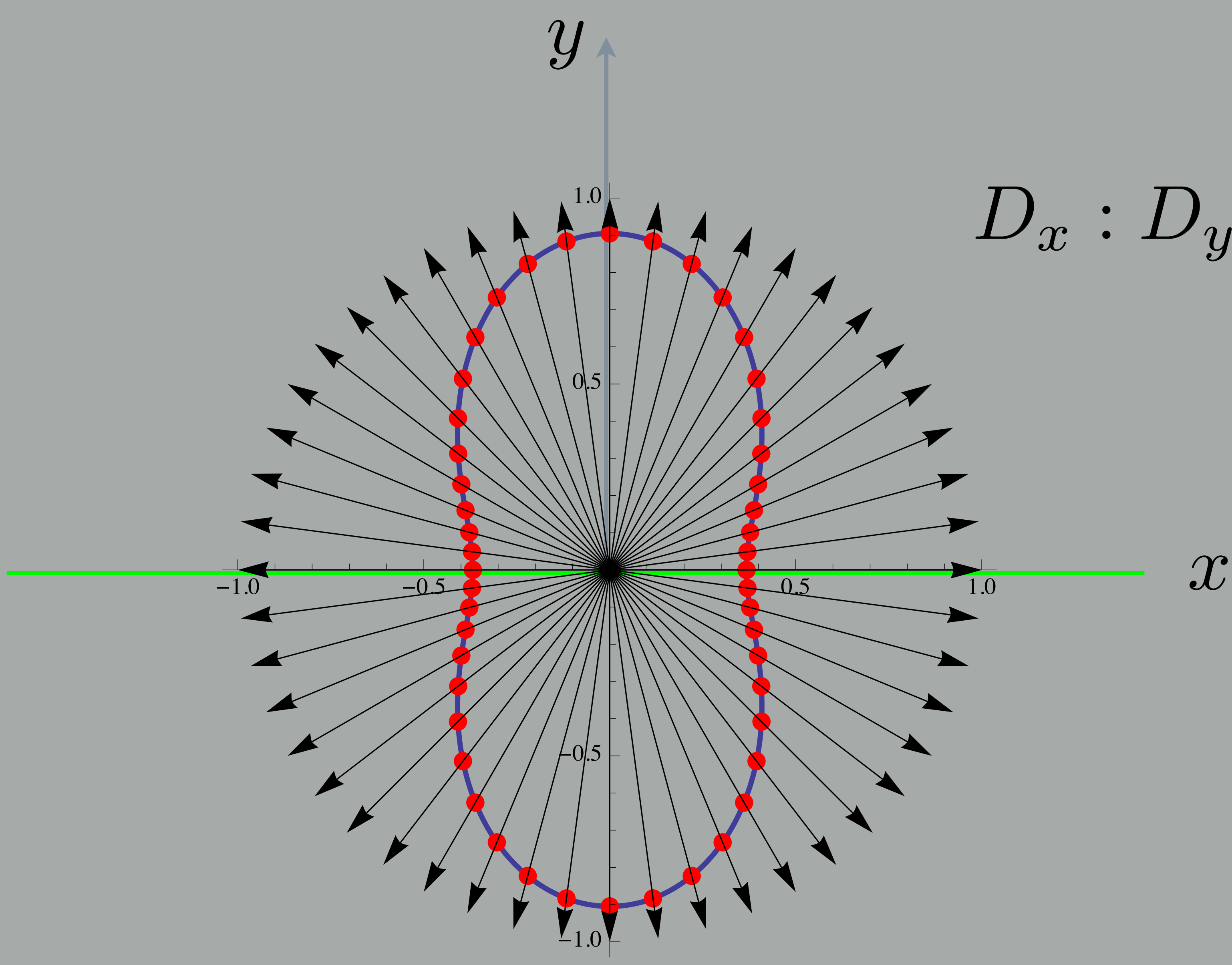
$$s(b) = s(0)e^{-b\tilde{D}}$$

where

$$\tilde{D} = \boldsymbol{u}^t \cdot \boldsymbol{D} \cdot \boldsymbol{u}$$

$$s(b) = s(0)e^{-b(d_2 \sin^2 \theta + d_1 \cos^2 \theta)}$$

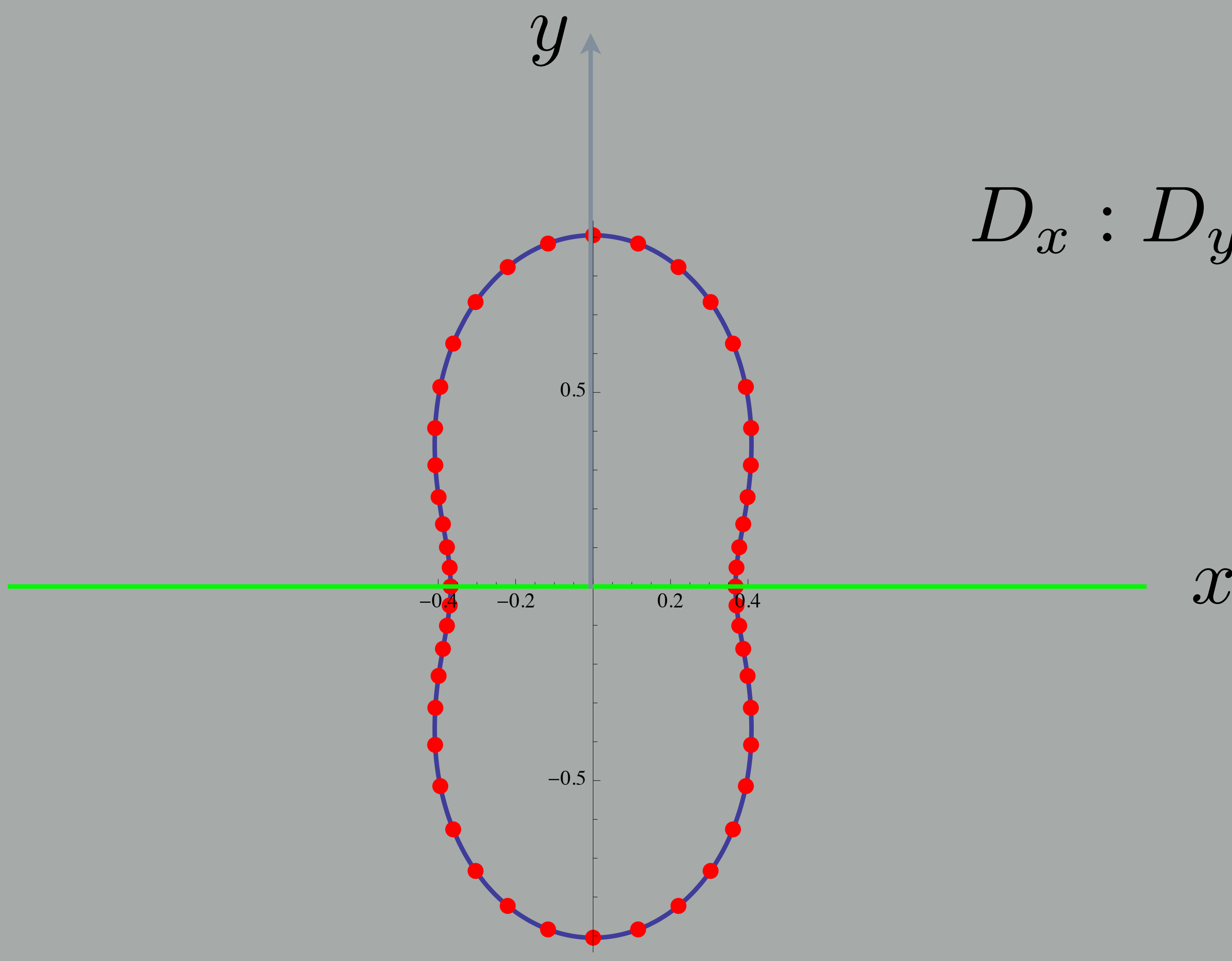
The Shape of the 2D Diffusion Signal



$$D_x : D_y = 10 : 1$$

$$s(b) = s(0)e^{-b(d_2 \sin^2 \theta + d_1 \cos^2 \theta)}$$

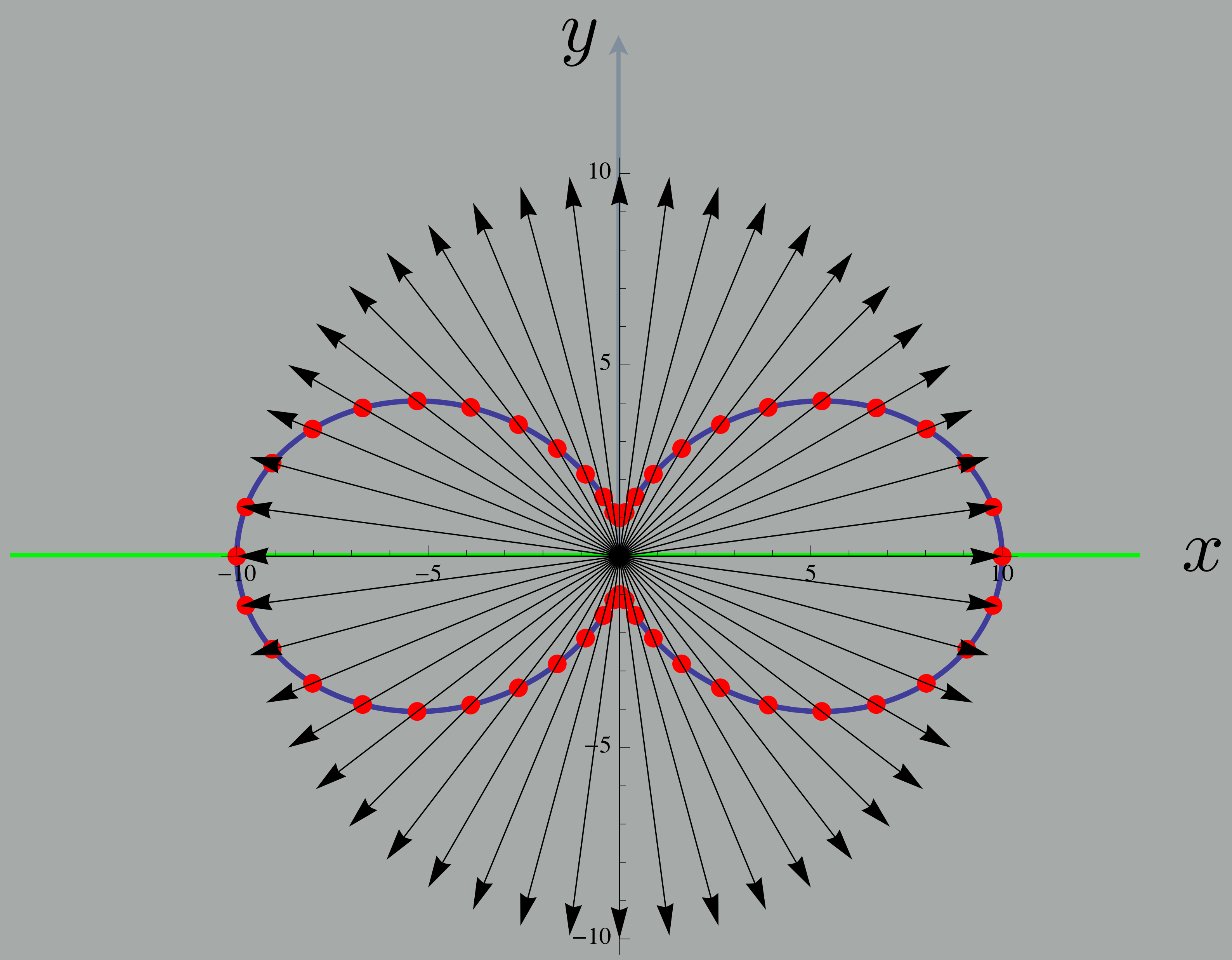
The Shape of the 2D Diffusion Signal



$D_x : D_y = 10 : 1$

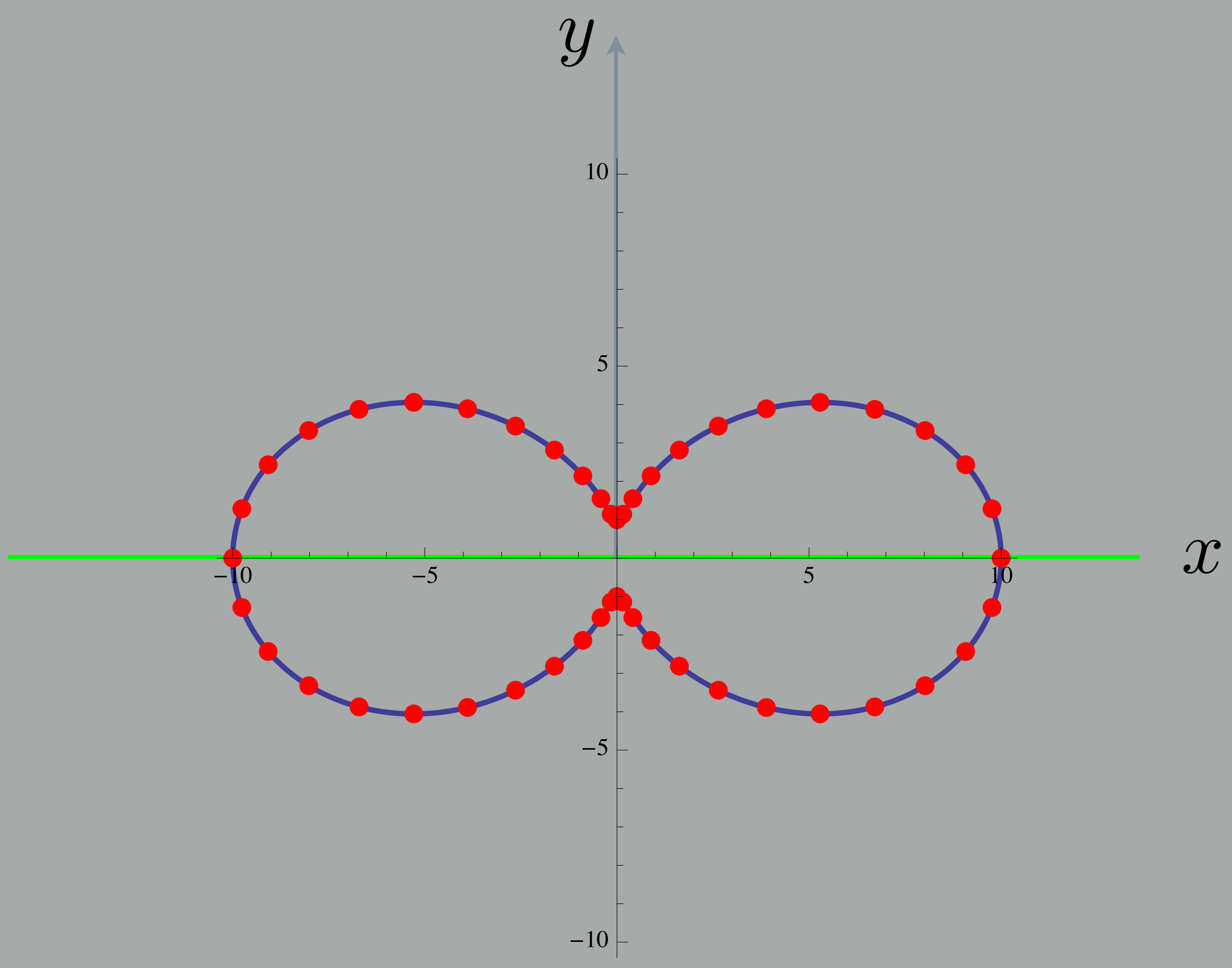
$$s(b) = s(0)e^{-b(d_2 \sin^2 \theta + d_1 \cos^2 \theta)}$$

The Apparent Diffusion Coefficient



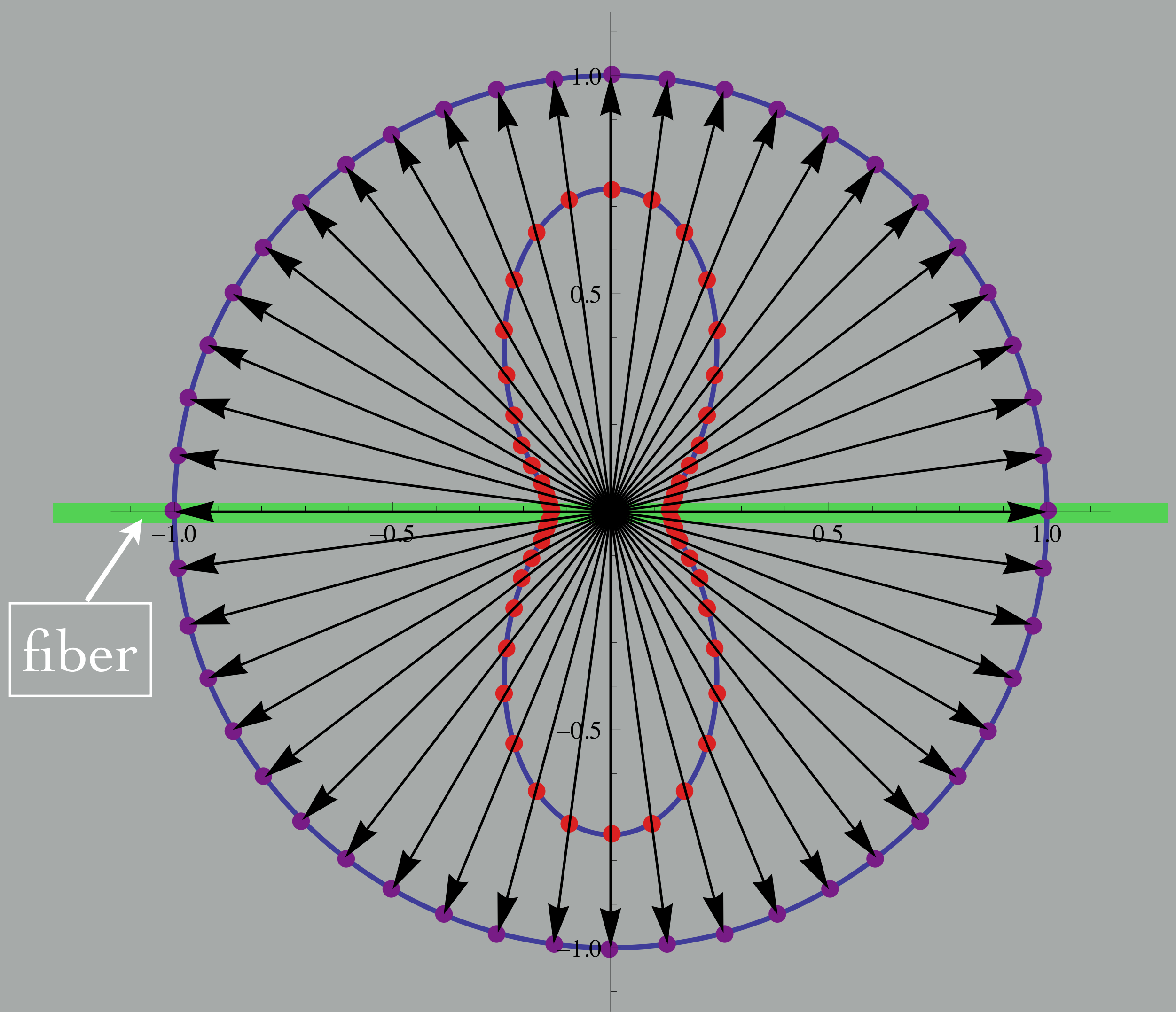
$$D_{app}(\theta) = -\frac{1}{b}\log\frac{s(b)}{s(0)}$$

The Apparent Diffusion Coefficient

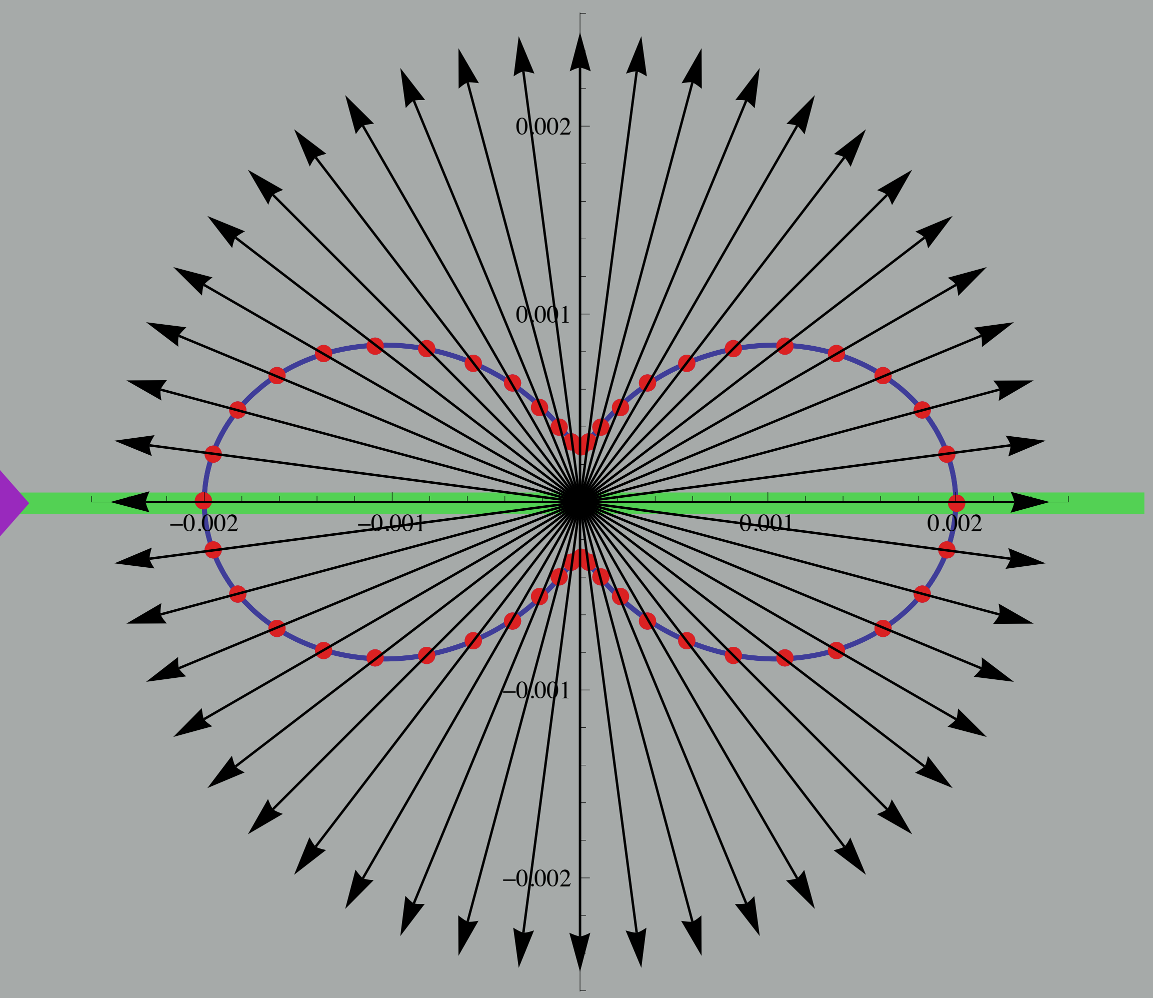


$$D_{app}(\theta) = -\frac{1}{b} \log \frac{s(b)}{s(0)}$$

The Shape of Diffusion

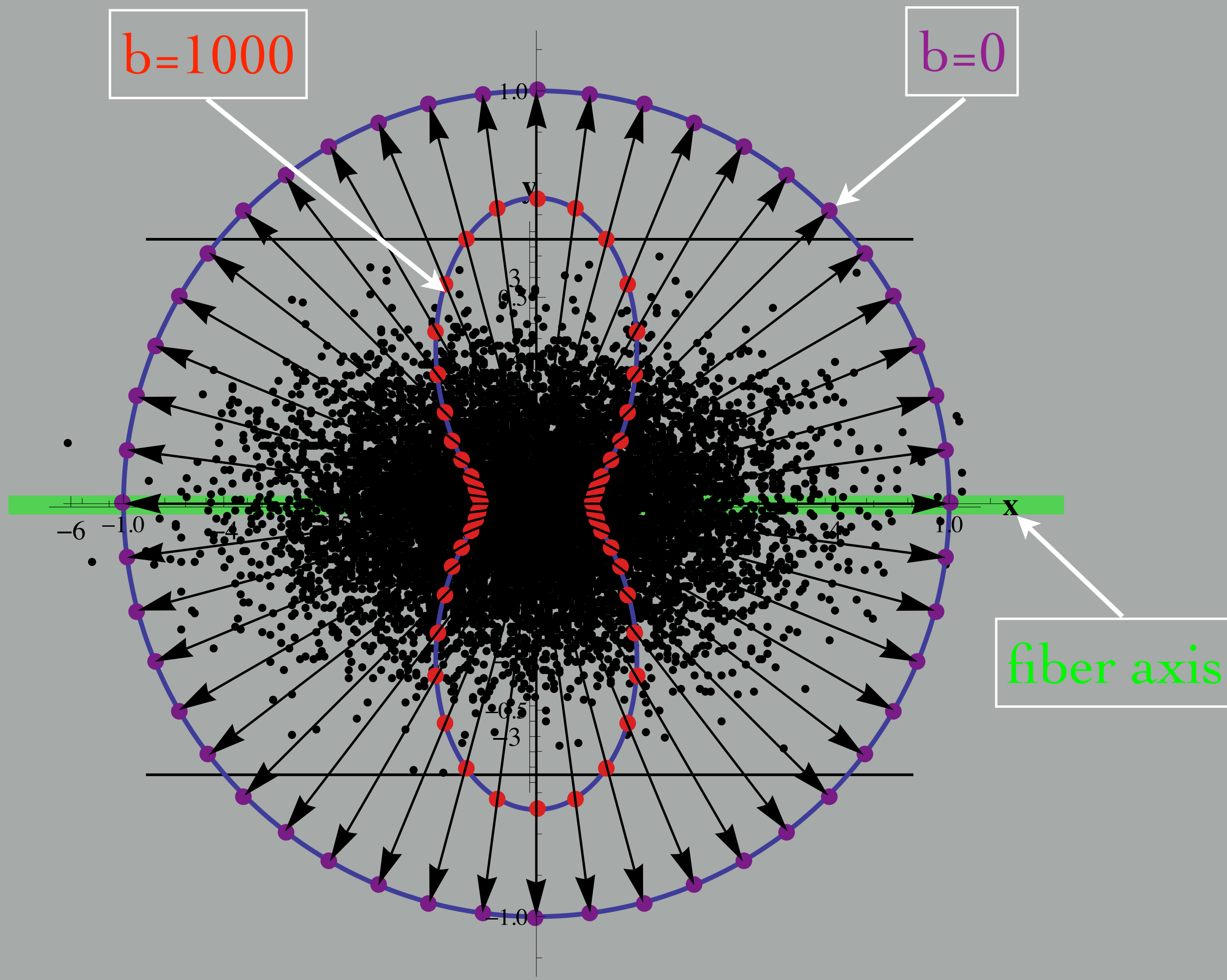


signal $S_b(\theta)$



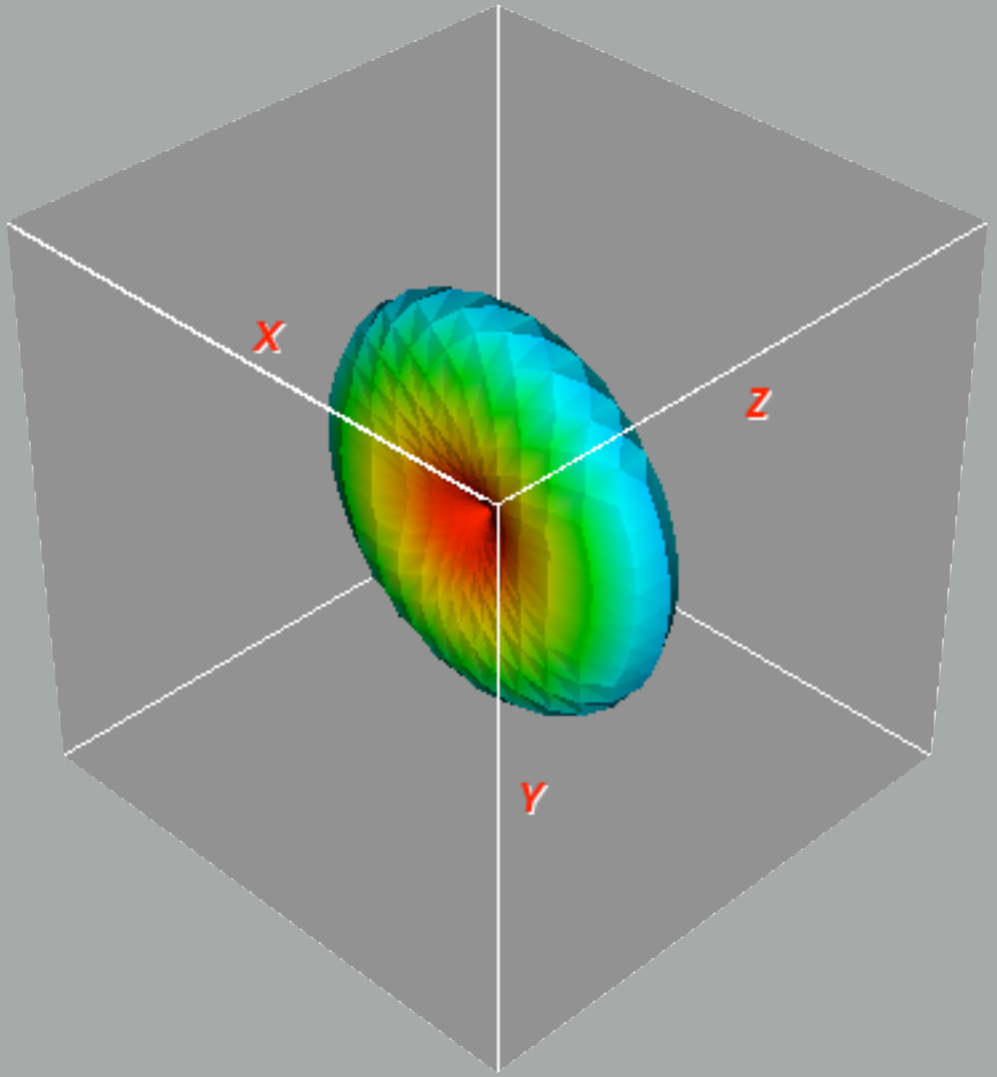
$$D_{app}(\theta) = -\frac{1}{b}\log\left(\frac{S_b}{S_0}\right)$$

The shape of diffusion



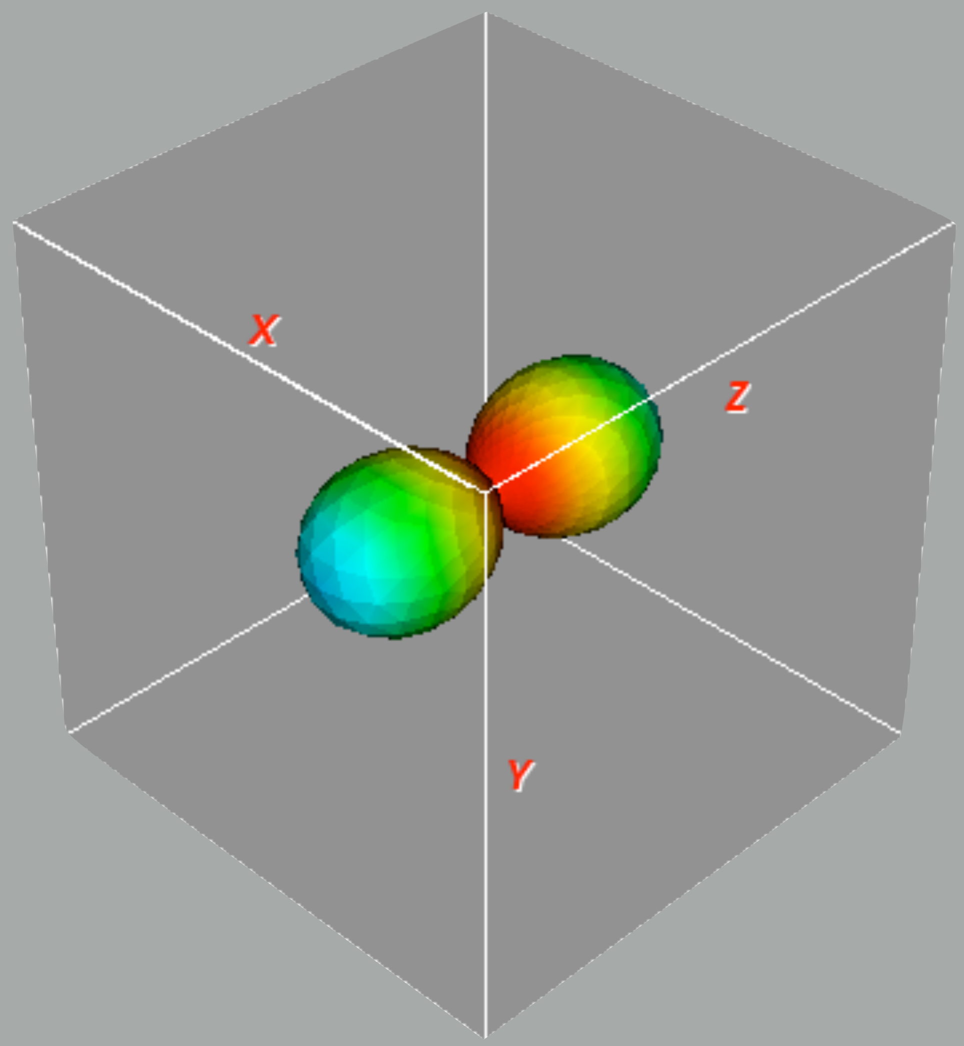
Recall what the "fiber" is underneath

The Shape of the Diffusion Signal



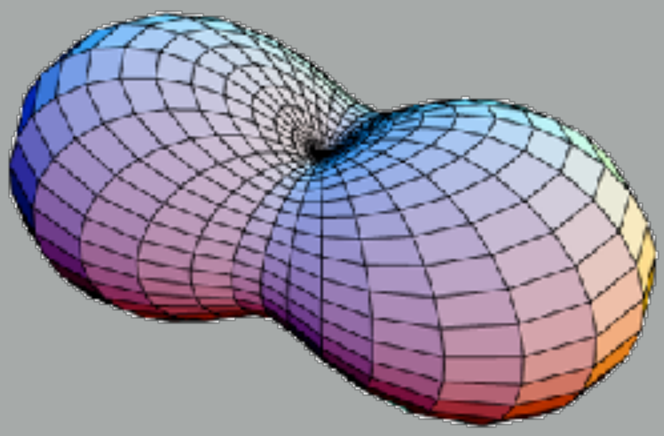
$$D_{xx} : D_{yy} : D_{zz} = 1 : 1 : 10$$

The Apparent Diffusion Coefficient

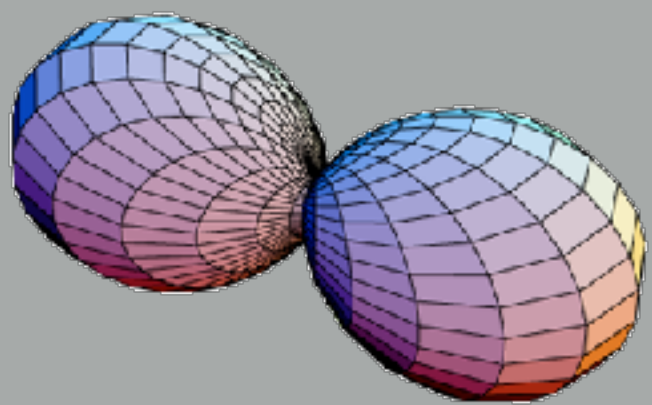


$$D_{xx} : D_{yy} : D_{zz} = 1 : 1 : 10$$

The Shape of the Diffusion



$$D_{xx} : D_{yy} : D_{zz} = 5 : 2 : 1$$



$$D_{xx} : D_{yy} : D_{zz} = 10 : 1 : 1$$

Estimating the Diffusion Tensor

tensor dimensions →

$$
-\begin{pmatrix} \log s(b_1) \\ \log s(b_2) \\ \vdots \\ \log s(b_n) \end{pmatrix} = \begin{pmatrix} \hat{q}_{1,x}^2 & \hat{q}_{1,y}^2 & \hat{q}_{1,z}^2 & \hat{q}_{1,x}\hat{q}_{1,y} & \hat{q}_{1,x}\hat{q}_{1,z} & \hat{q}_{1,y}\hat{q}_{1,z} & 1 \\ \hat{q}_{2,x}^2 & \hat{q}_{2,y}^2 & \hat{q}_{2,z}^2 & \hat{q}_{2,x}\hat{q}_{1,y} & \hat{q}_{2,x}\hat{q}_{2,z} & \hat{q}_{2,y}\hat{q}_{2,z} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{q}_{n,x}^2 & \hat{q}_{n,y}^2 & \hat{q}_{n,z}^2 & \hat{q}_{n,x}\hat{q}_{1,y} & \hat{q}_{n,x}\hat{q}_{n,z} & \hat{q}_{n,y}\hat{q}_{n,z} & 1 \end{pmatrix} \begin{pmatrix} D_{xx} \\ D_{yy} \\ D_{zz} \\ D_{xy} \\ D_{xz} \\ D_{yz} \\ -\log s(0) \end{pmatrix}
$$

data — b-matrix — tensor

↑ gradient directions

$$q_{j,k} = g_k \delta \quad j\text{'th direction}$$

$$\tau = \Delta - \delta/3$$

Estimating the Diffusion Tensor

Matrix equation

$$\boldsymbol{y} = \boldsymbol{B}\ \boldsymbol{d}$$

diffusion tensor elements

data

b-matrix

Matrix solution

$$\boldsymbol{d} = \boldsymbol{B}^{+}\boldsymbol{y}$$

pseudo-inverse

In practice, 3dDWItoDT used to compute ***D***

Reassemble D

reassemble $\boldsymbol{d}$ back into $\boldsymbol{D}$

$$\boldsymbol{d} = \begin{pmatrix} D_{xx} \\ D_{yy} \\ D_{zz} \\ D_{xy} \\ D_{xz} \\ D_{yz} \\ -\log s(0) \end{pmatrix} \Rightarrow \boldsymbol{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}$$

Compute Eigensystem of D

Data gives us:

$$\boldsymbol{D} = \boldsymbol{R}^t \boldsymbol{D}_\Lambda \boldsymbol{R}$$

Output

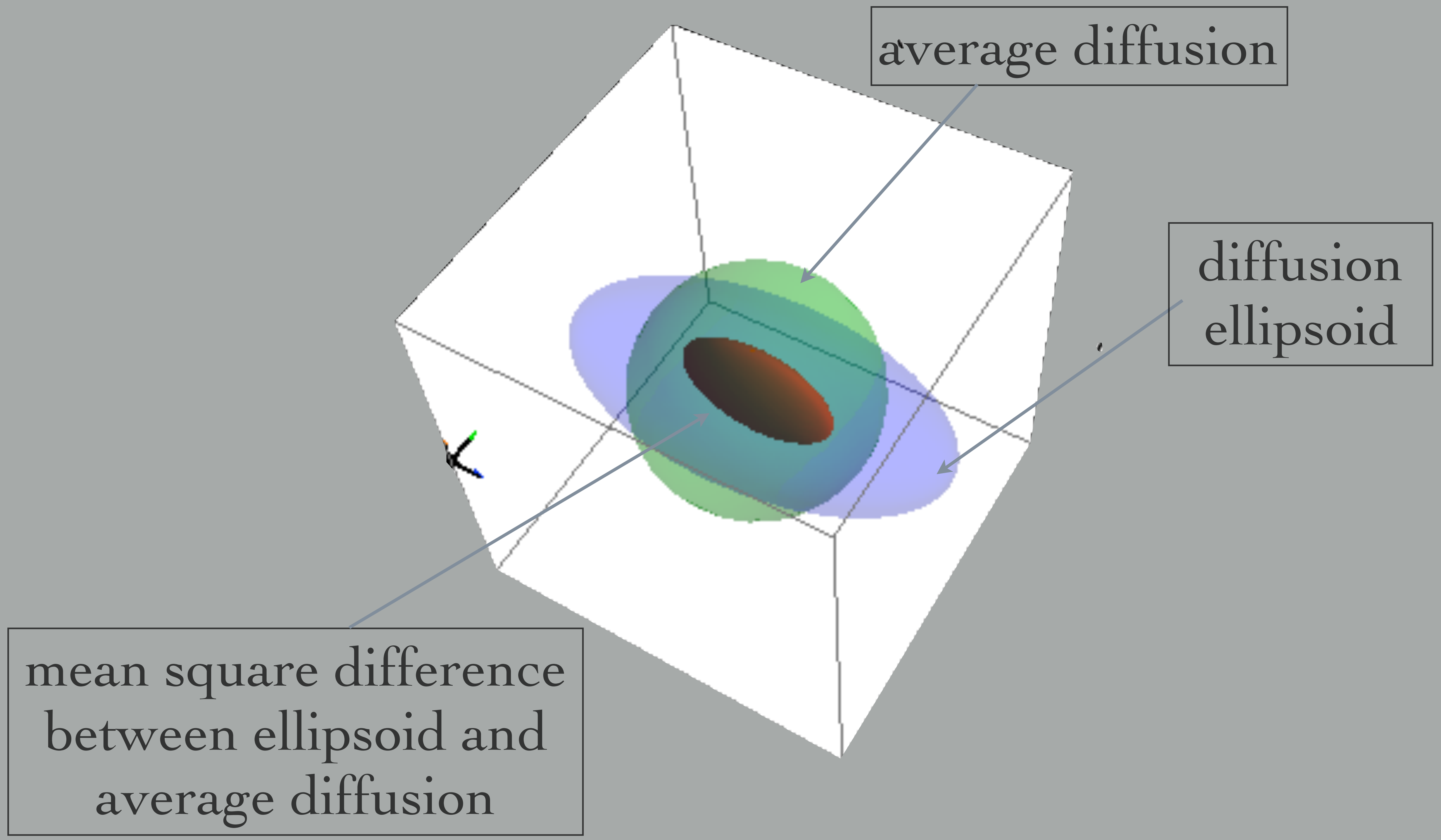
Input

$$[\boldsymbol{\lambda}, \hat{\boldsymbol{e}}] = \mathrm{eig}(\boldsymbol{D})$$

eigenvalues

eigenvectors

Geometric Picture of Diffusion



Trace of the Diffusion Tensor

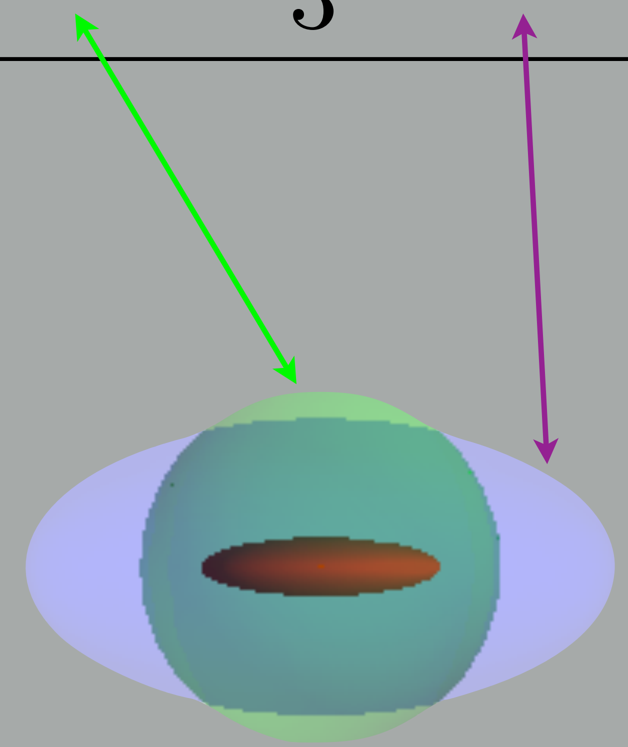
$$\boldsymbol{D}_\Lambda = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$$\mathrm{Tr}(\boldsymbol{D}_\Lambda) = \lambda_1 + \lambda_2 + \lambda_3 = \underbrace{D_{xx} + D_{yy} + D_{zz}}_{3\langle D \rangle}$$

$$\therefore \boxed{\langle D \rangle = \frac{1}{3}\mathrm{Tr}(\boldsymbol{D}_\Lambda)}$$

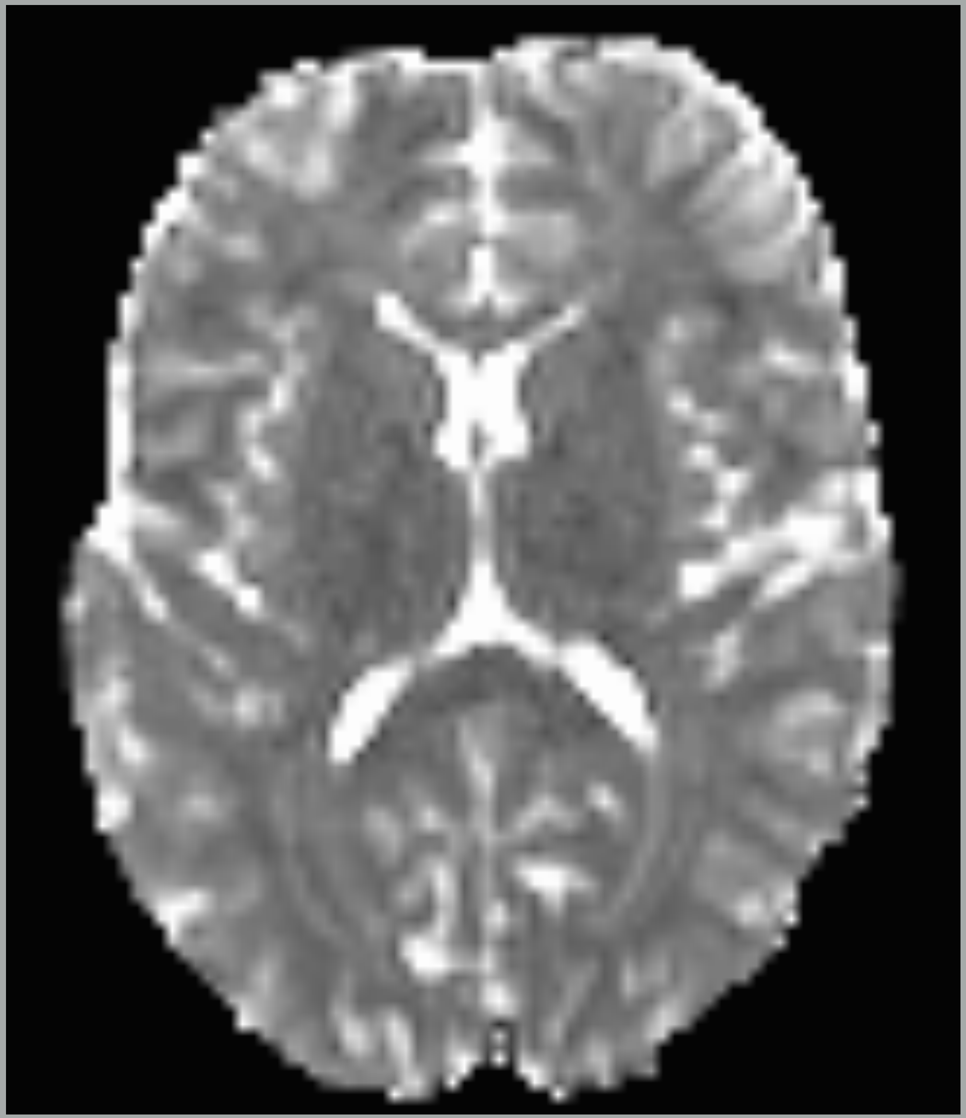
Mean diffusivity

$$\boxed{\langle D \rangle = \frac{1}{3}\mathrm{Tr}(\boldsymbol{D})}$$



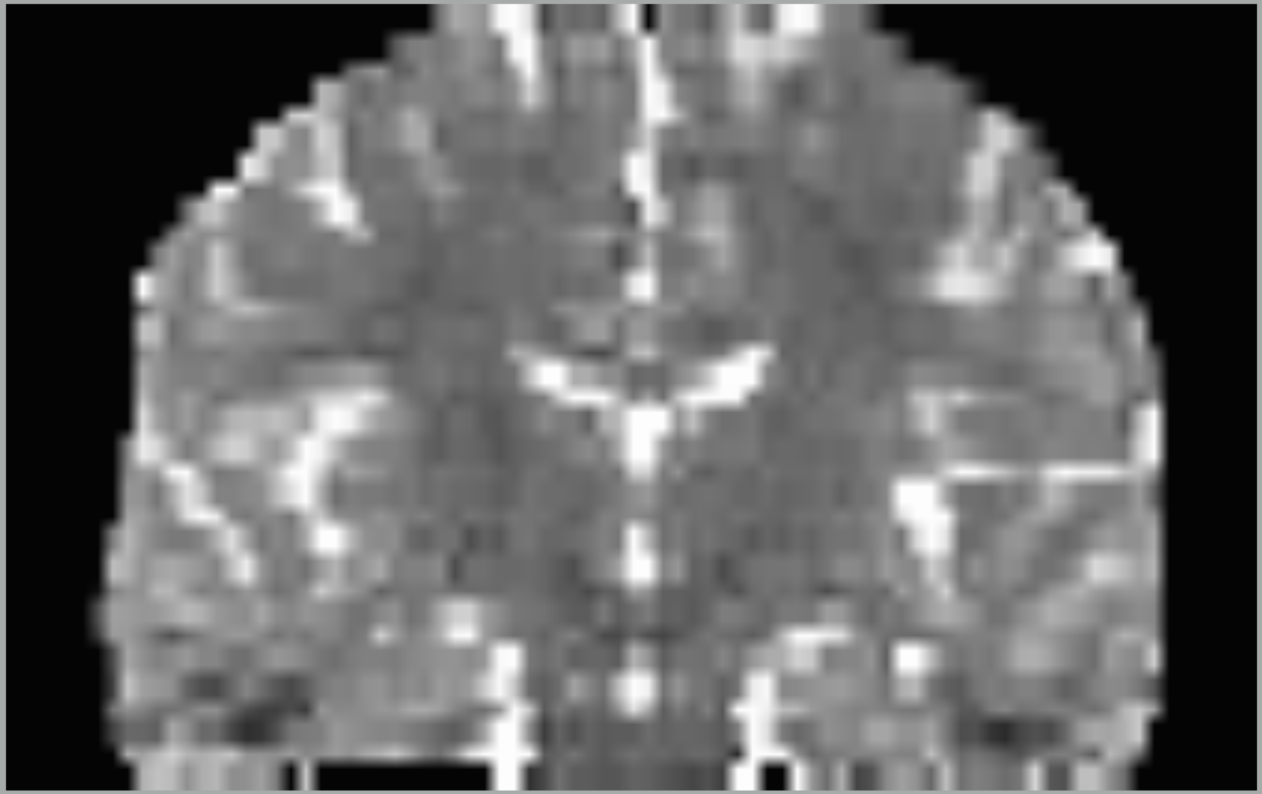
since $\mathrm{Tr}(\boldsymbol{D}) = \mathrm{Tr}(\boldsymbol{D}_\Lambda)$

Mean Diffusivity (MD)



axial

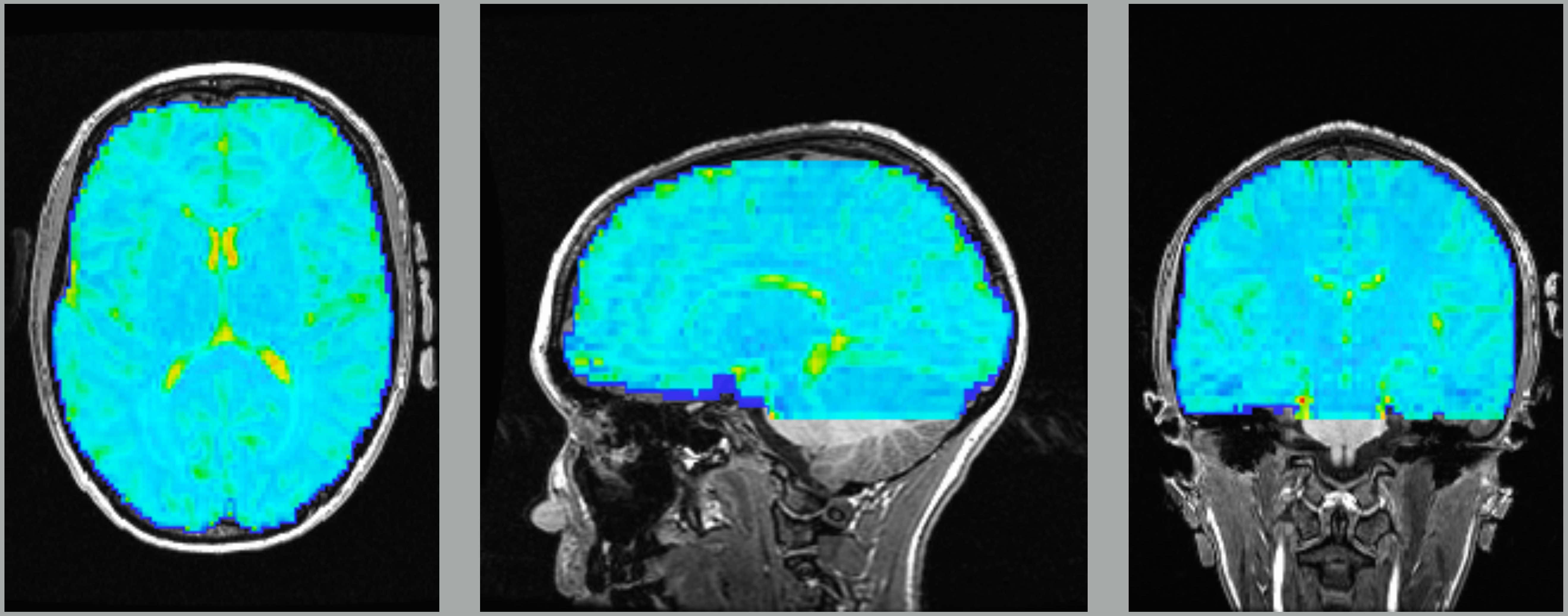
sagittal



coronal

Mean Diffusivity (MD)

AFNI “overlay/underlay” paradigm



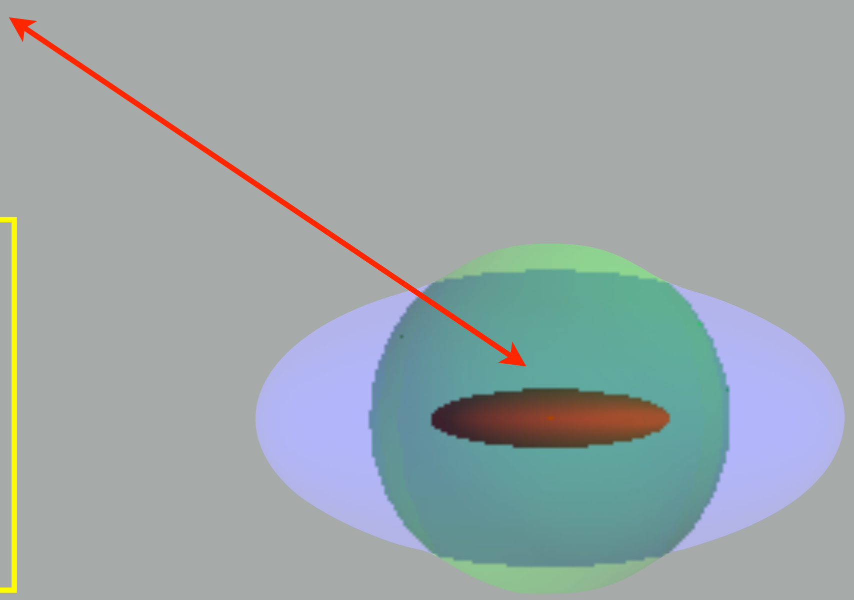
underlay (grayscale) = anatomy
overlay (color) = MD

Diffusion Anisotropy

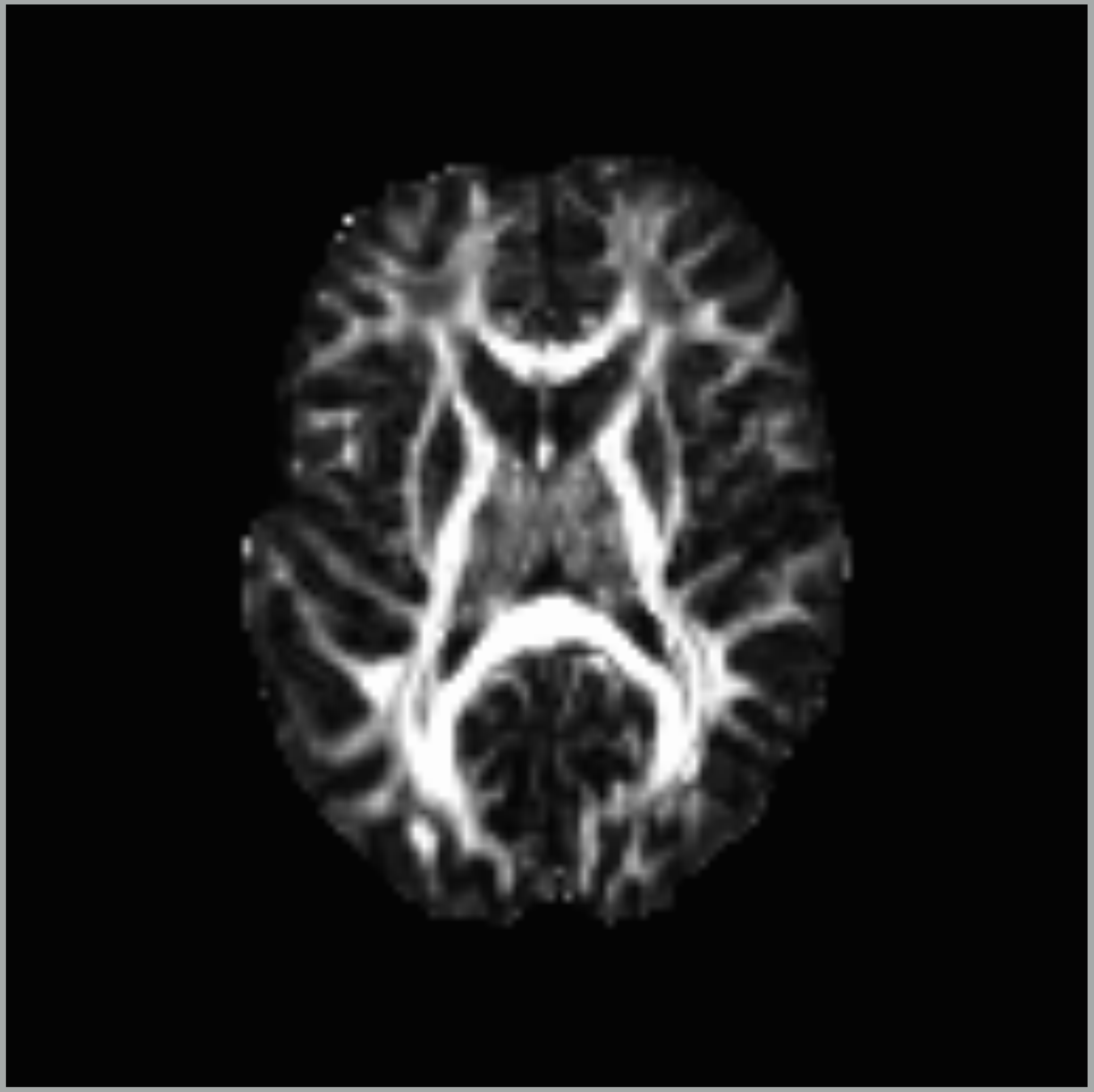
Normalize so that max value = 1
by multiplying by $\sqrt{3/2}$
and define

Fractional Anisotropy

$$FA = \sqrt{\frac{3}{2}\frac{\overline{(\delta\lambda)^2}}{\overline{\lambda^2}}}$$

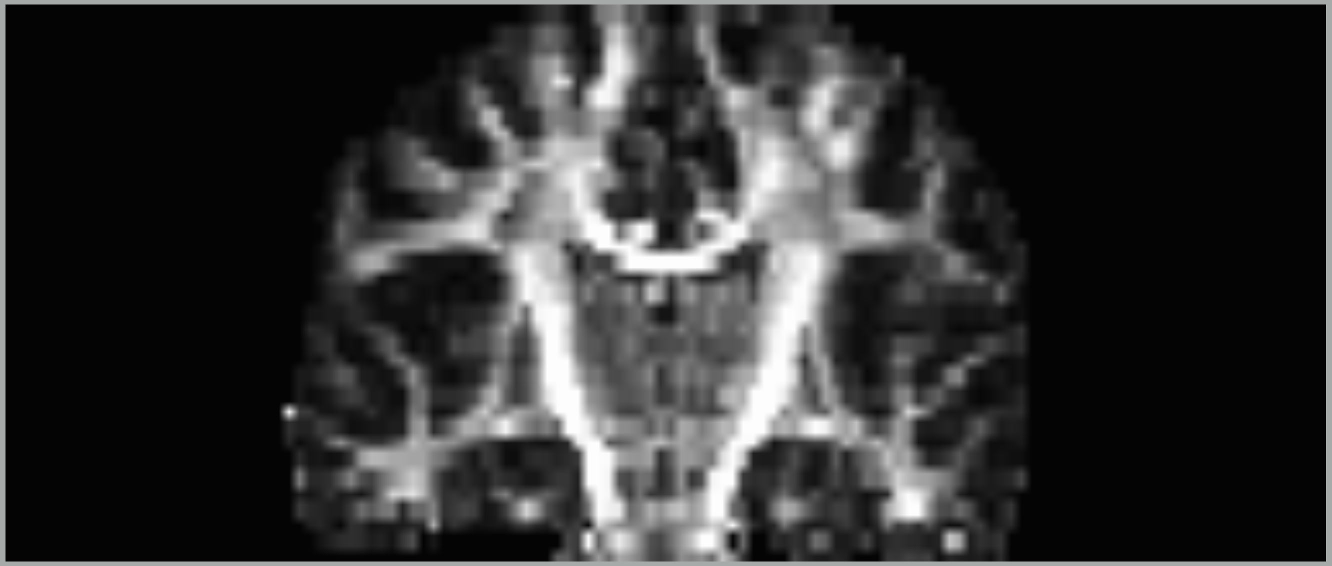


Fractional Anisotropy (FA)



axial

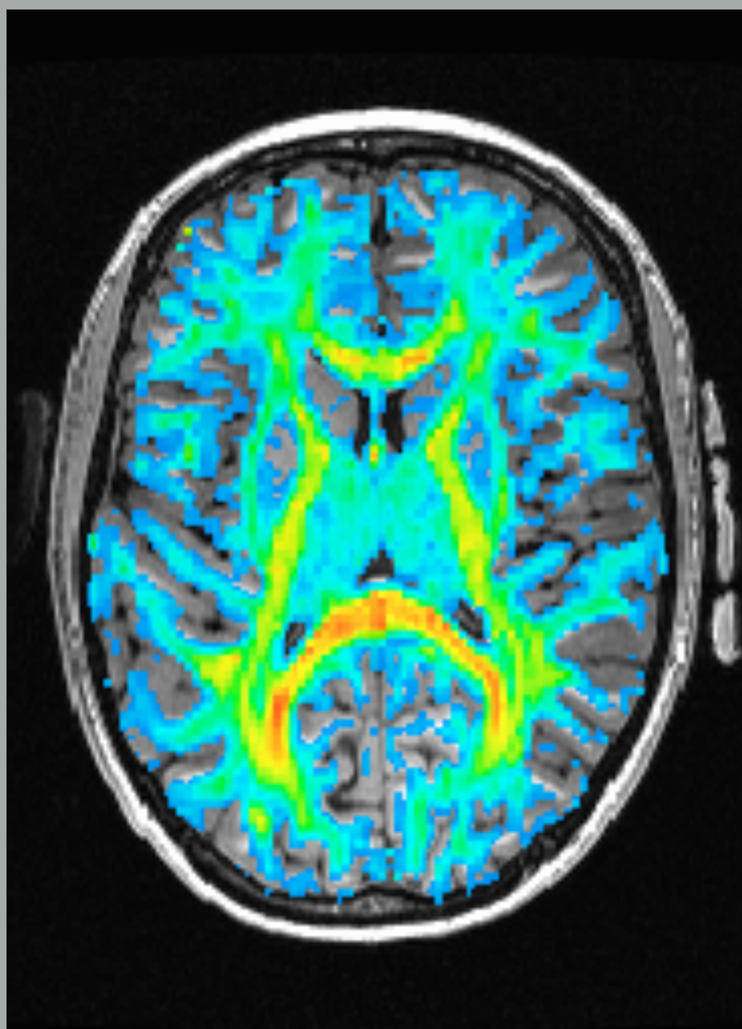
sagittal



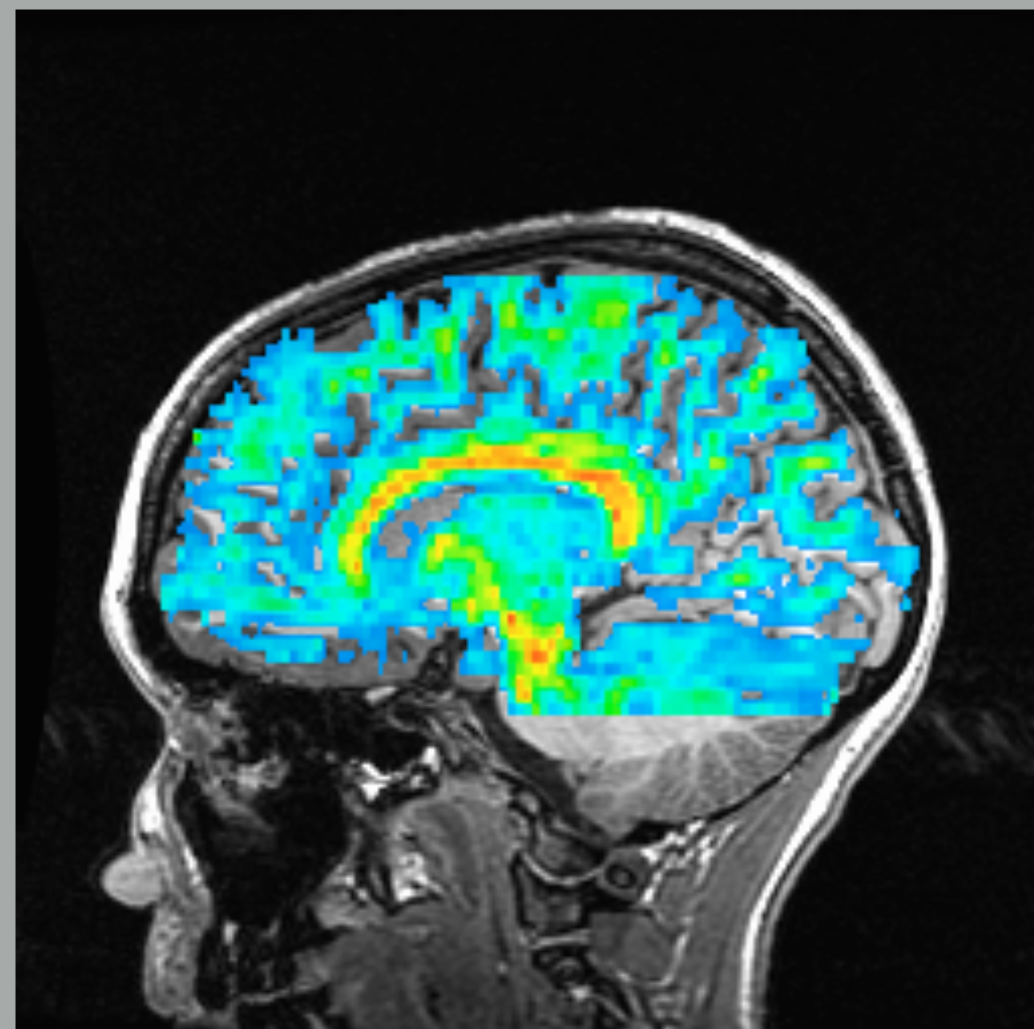
coronal

Fraction Anisotropy (FA)

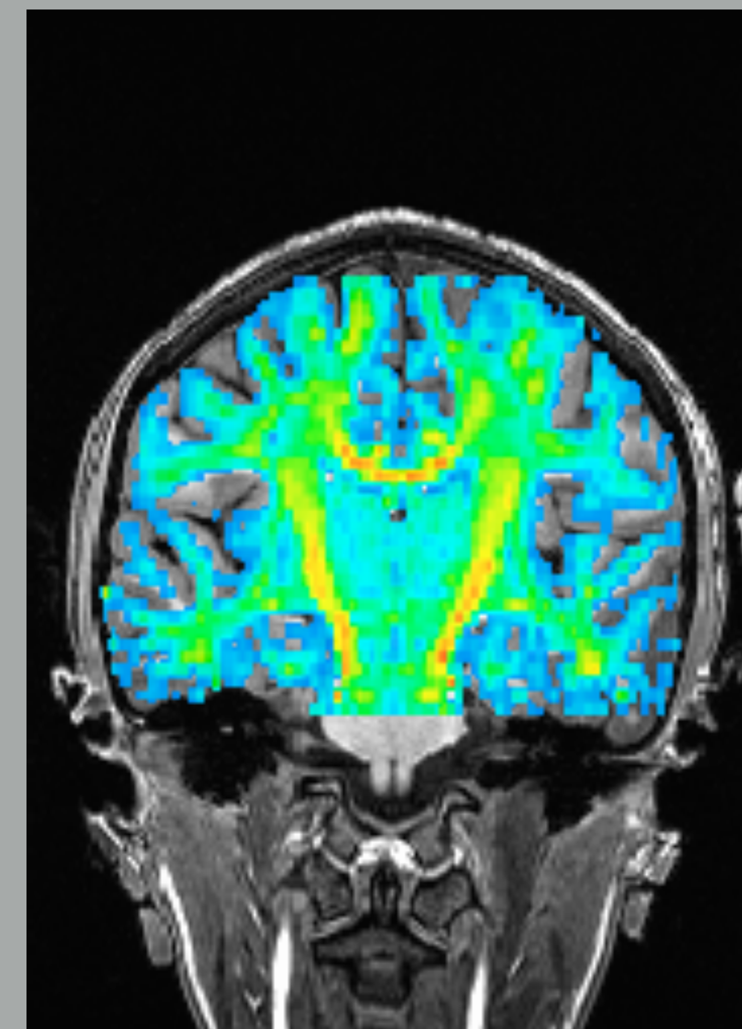
AFNI “overlay/underlay” paradigm



axial



sagittal

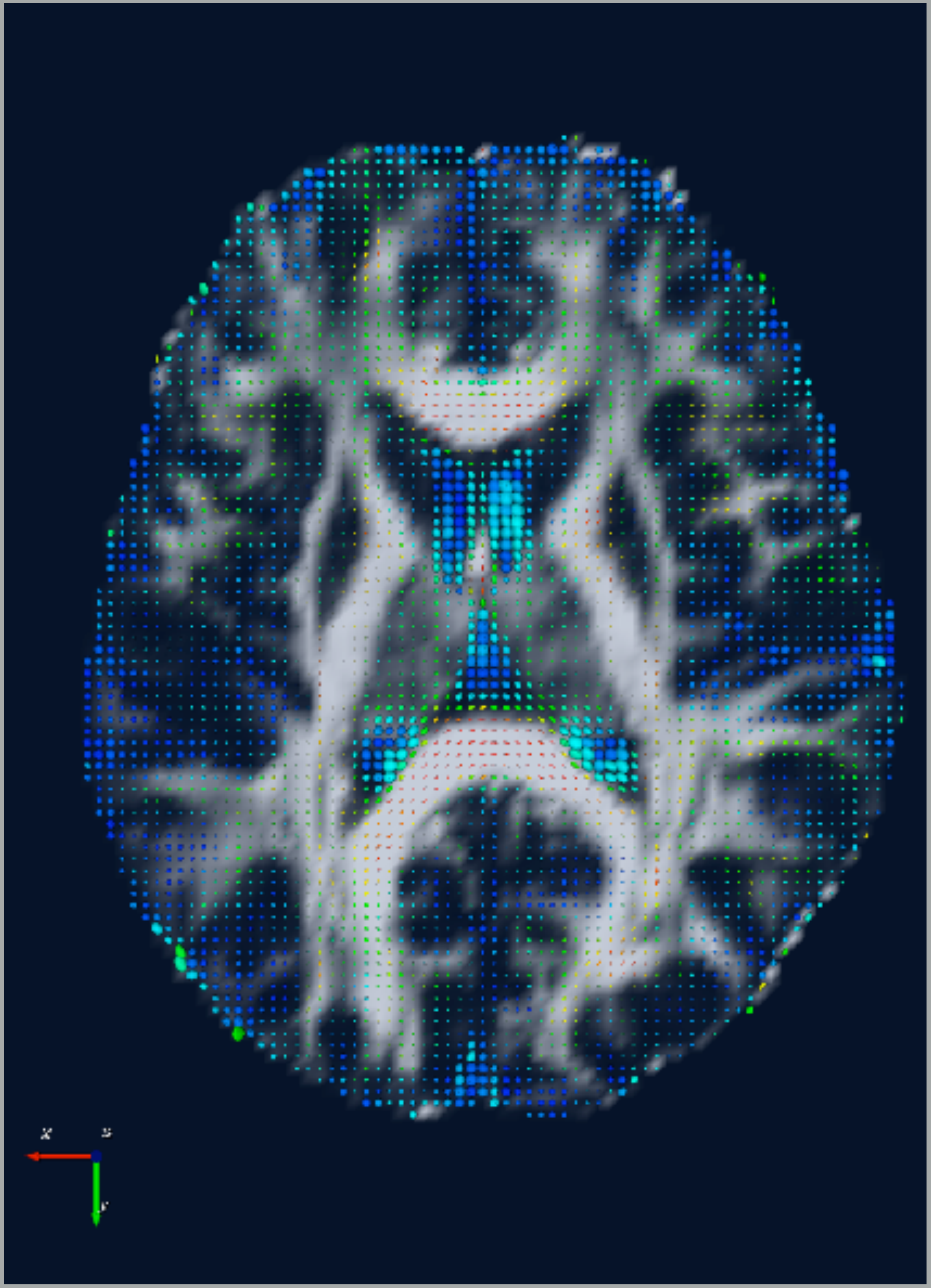


coronal

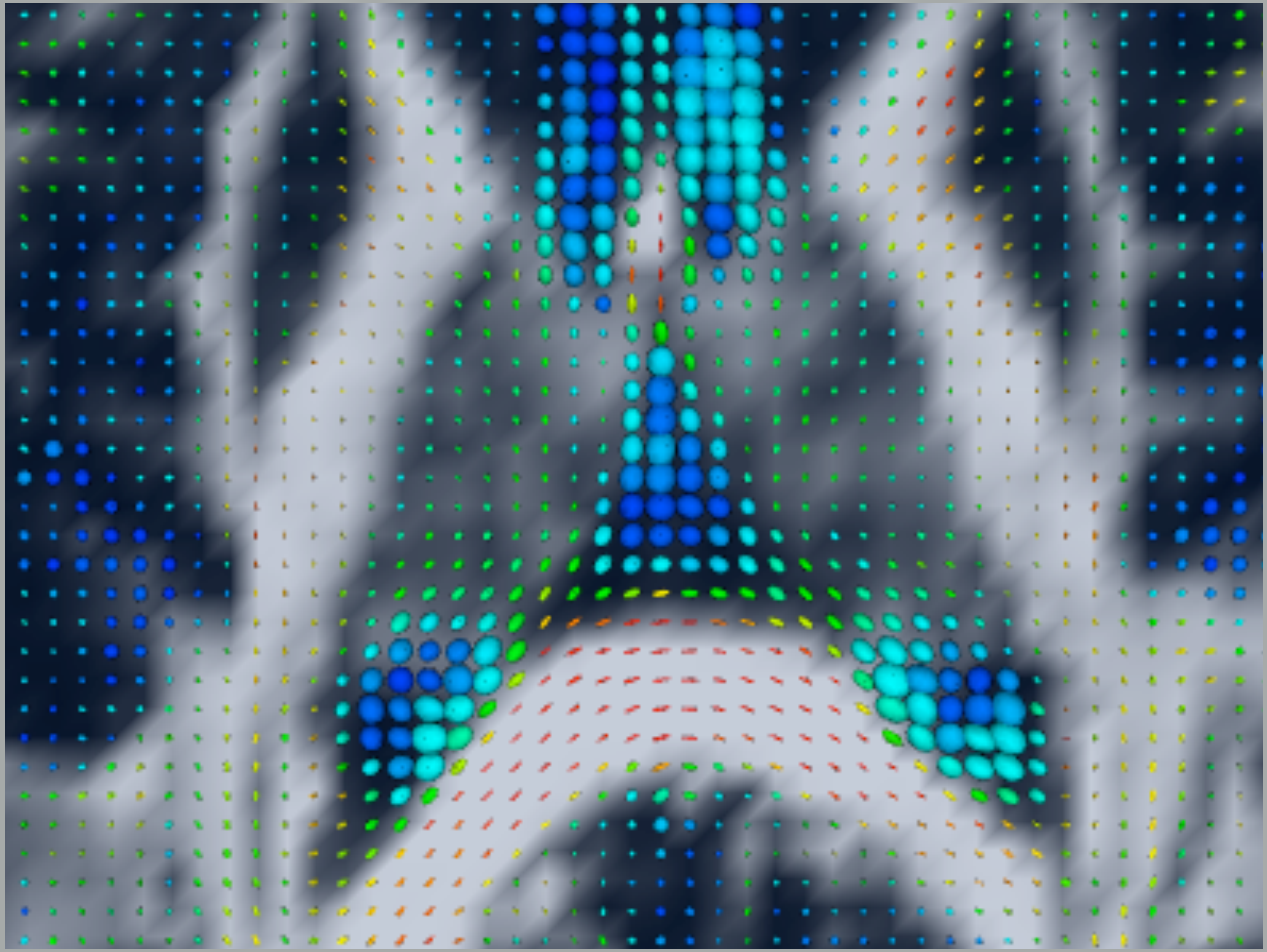
underlay (grayscale) = anatomy

overlay (color) = FA

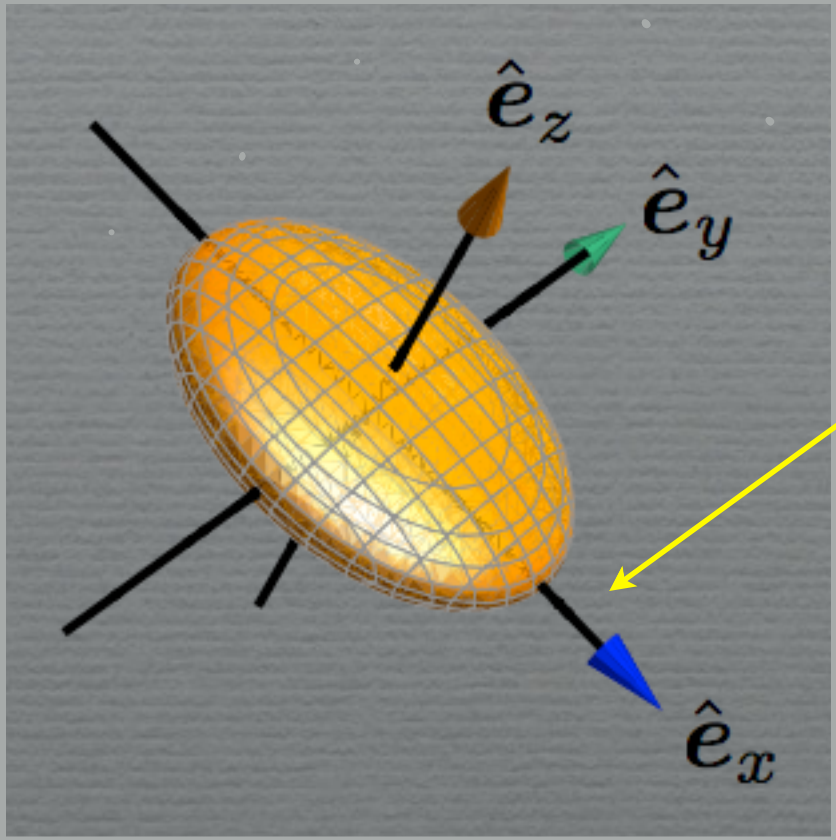
Diffusion ellipsoid overlay



Diffusion ellipsoid overlay



Principal Eigenvector

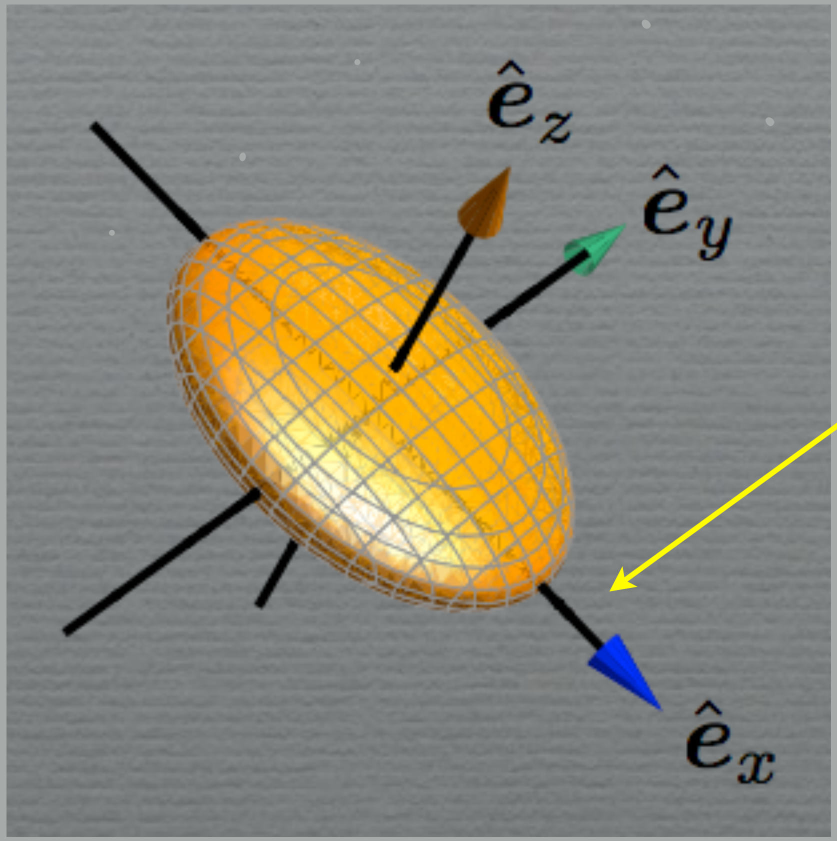


principal eigenvector

The longest eigenvector (i.e., associated with the largest eignvalue) is called the *principal eigenvector*

Principal Eigenvector

Since the longest eigenvector is associated with the highest diffusion, the principal eigenvector is assumed to be in the direction of greatest diffusion

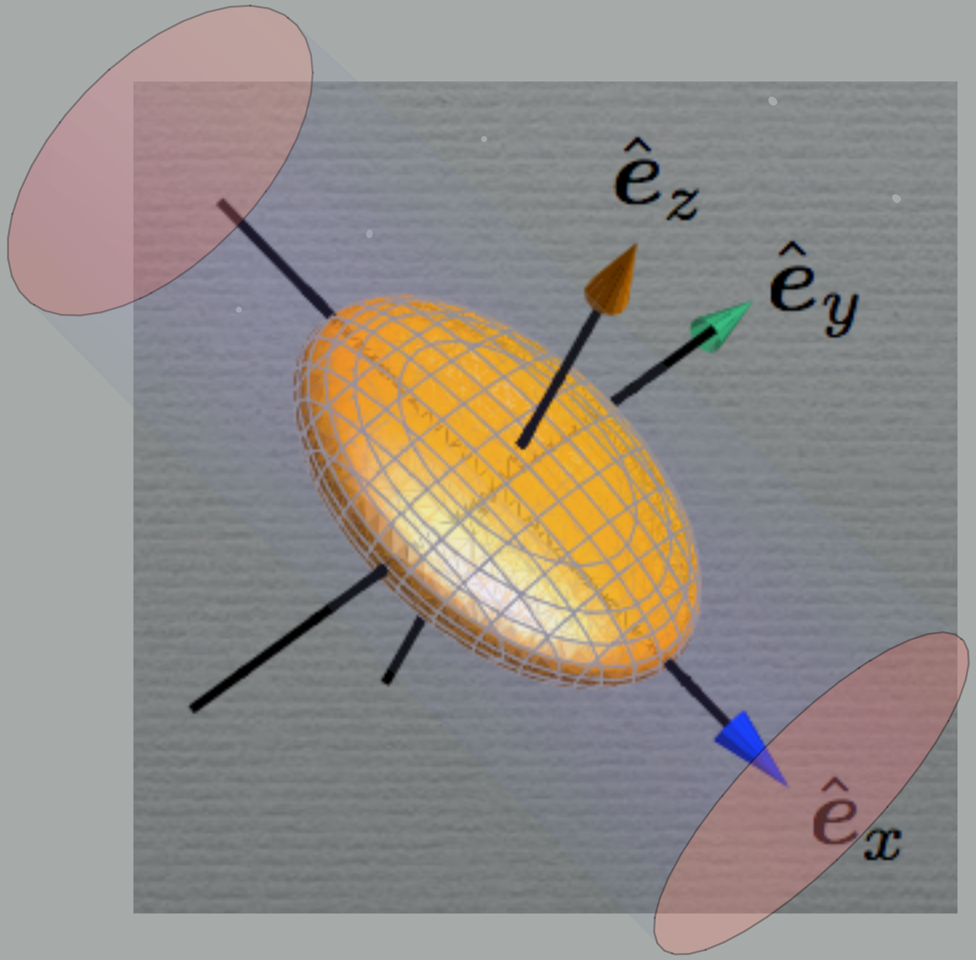


Dxx>Dyy & Dzz

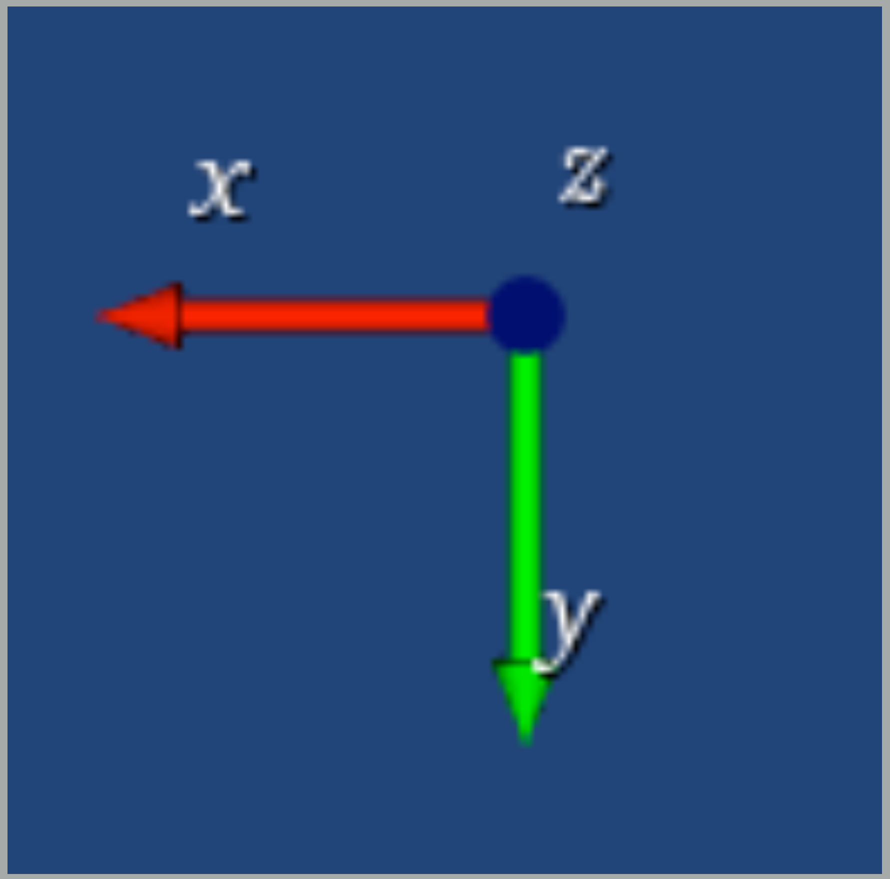
Principal Eigenvector

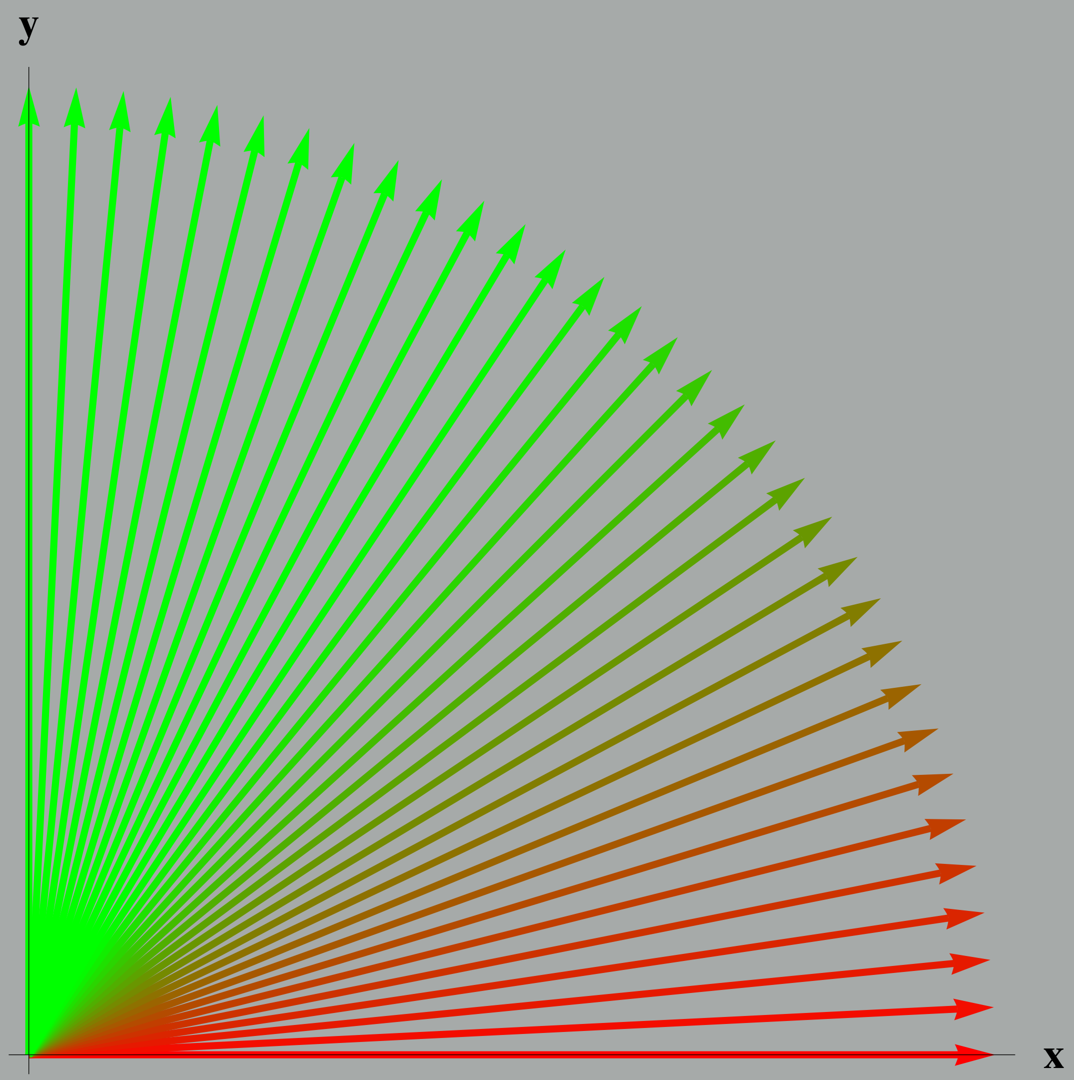
Since direction of greatest diffusion is assumed to lie along the fiber direction,

the principal eigenvector is thus assumed to tell us fiber orientation

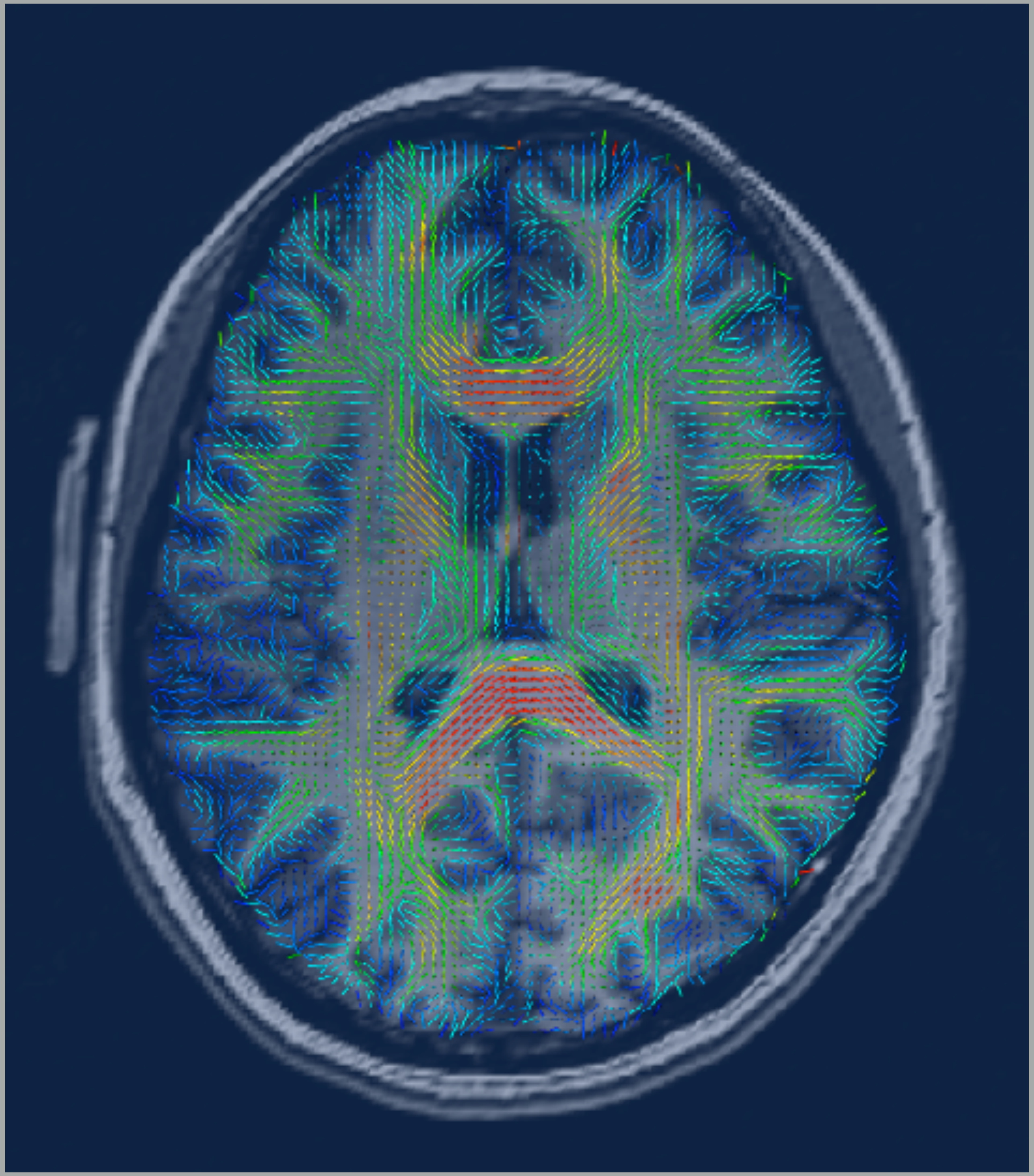


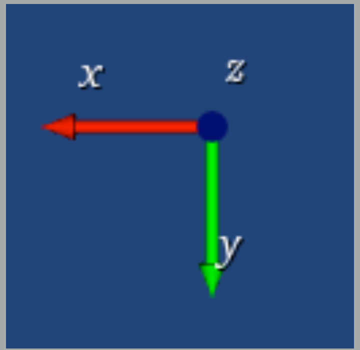
Directional Anisotropy Coloring



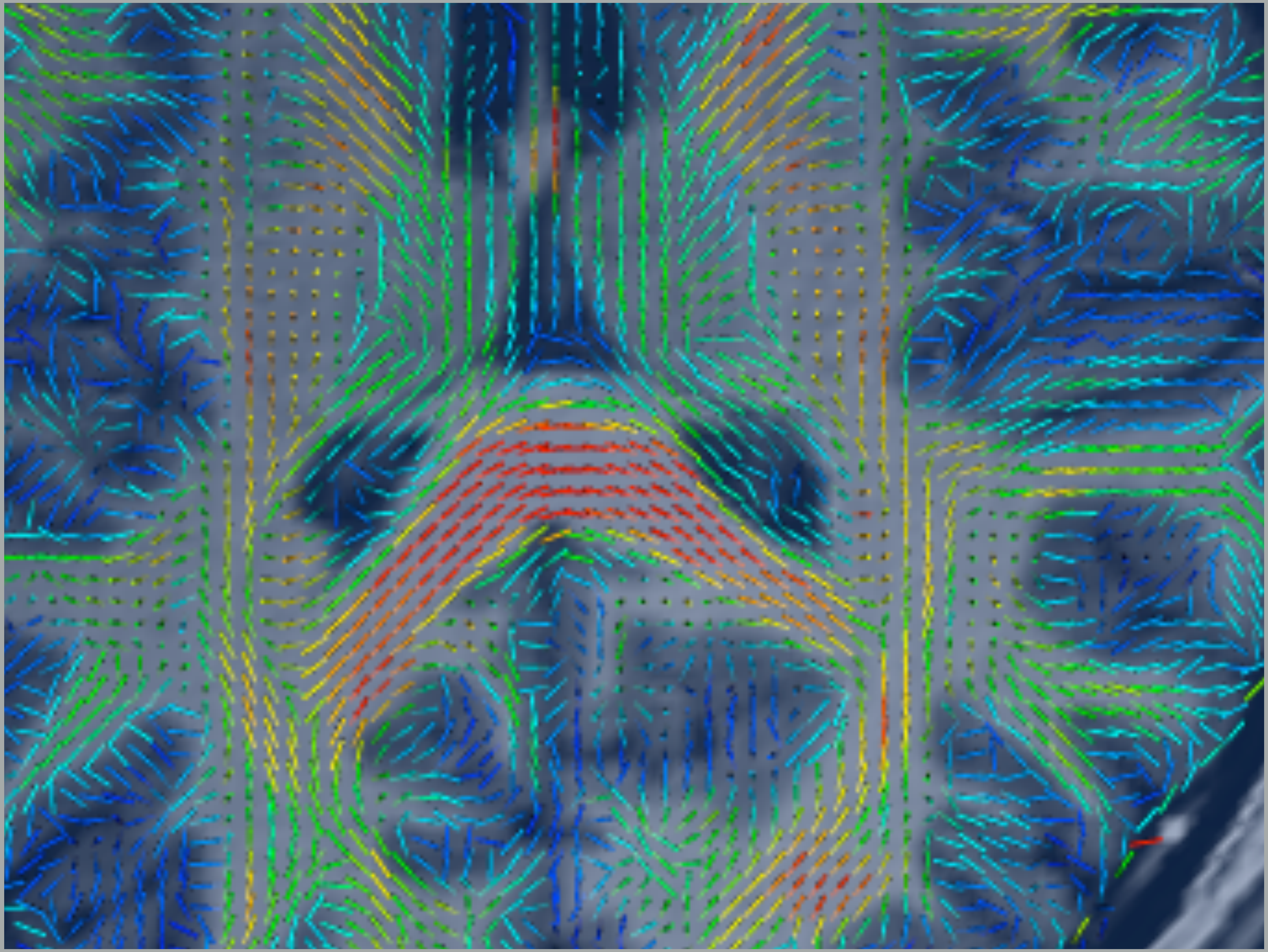


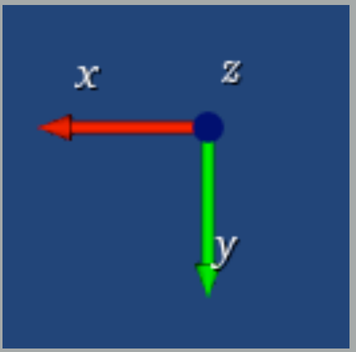
Principal Eigenvector Maps



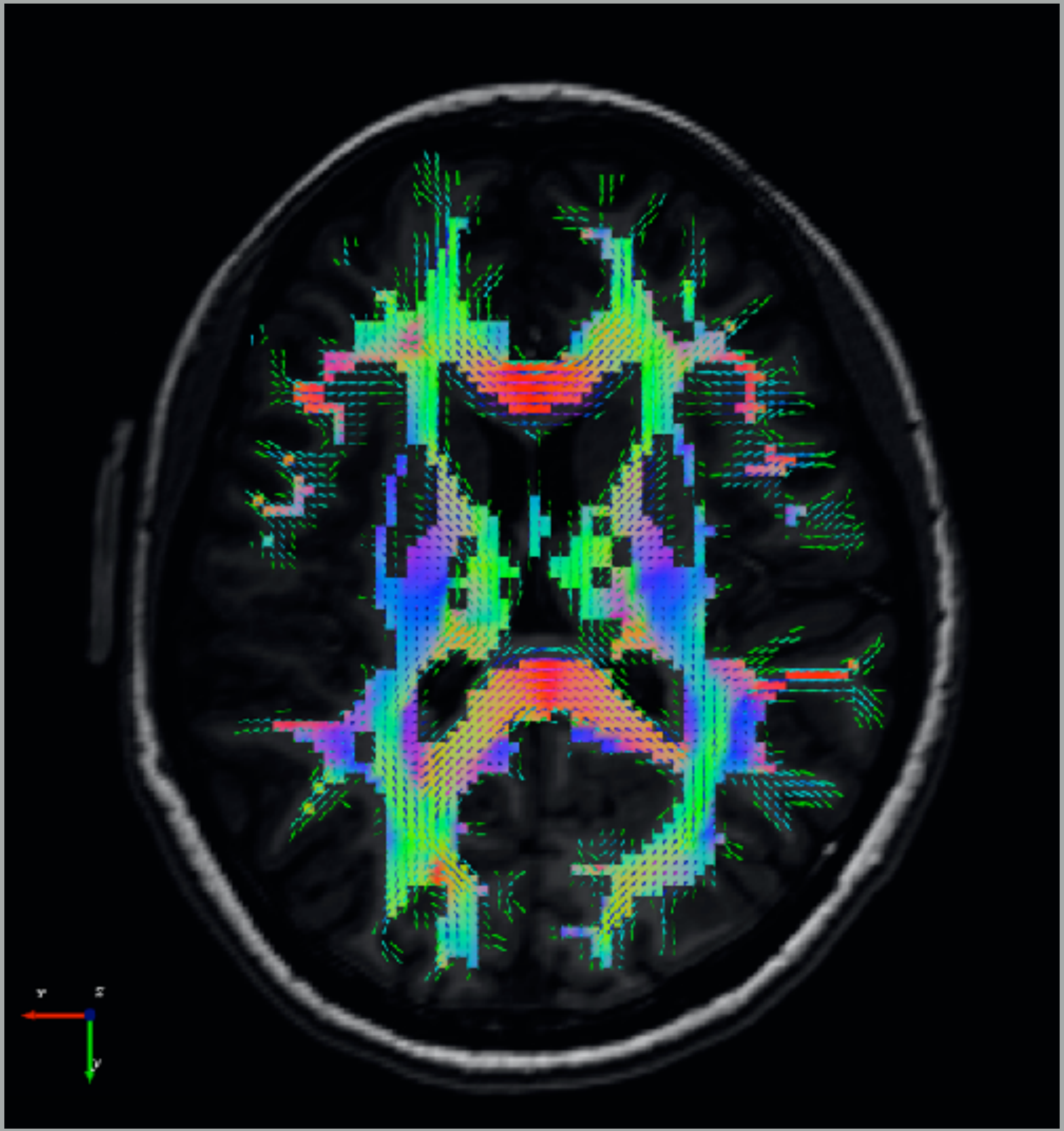


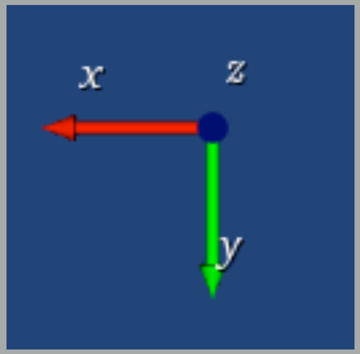
Principal Eigenvector Maps



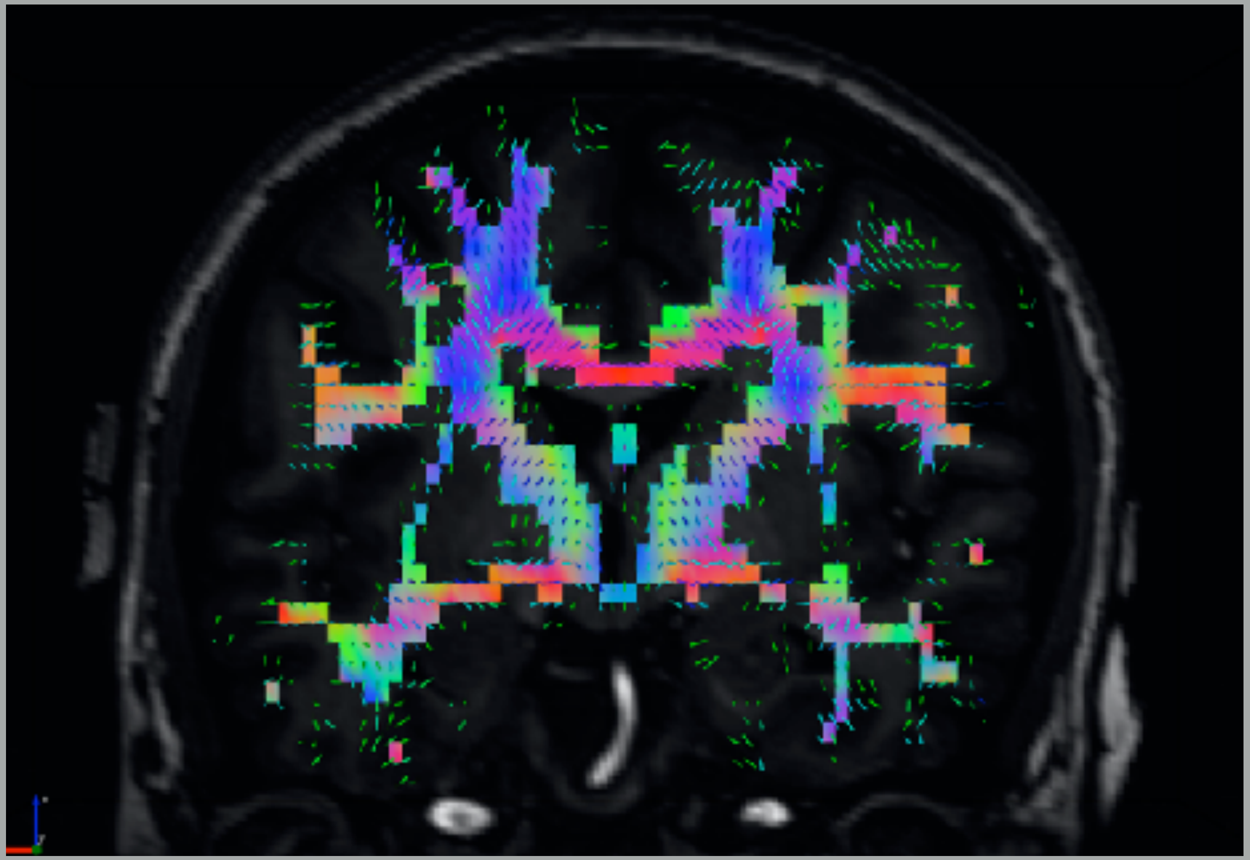


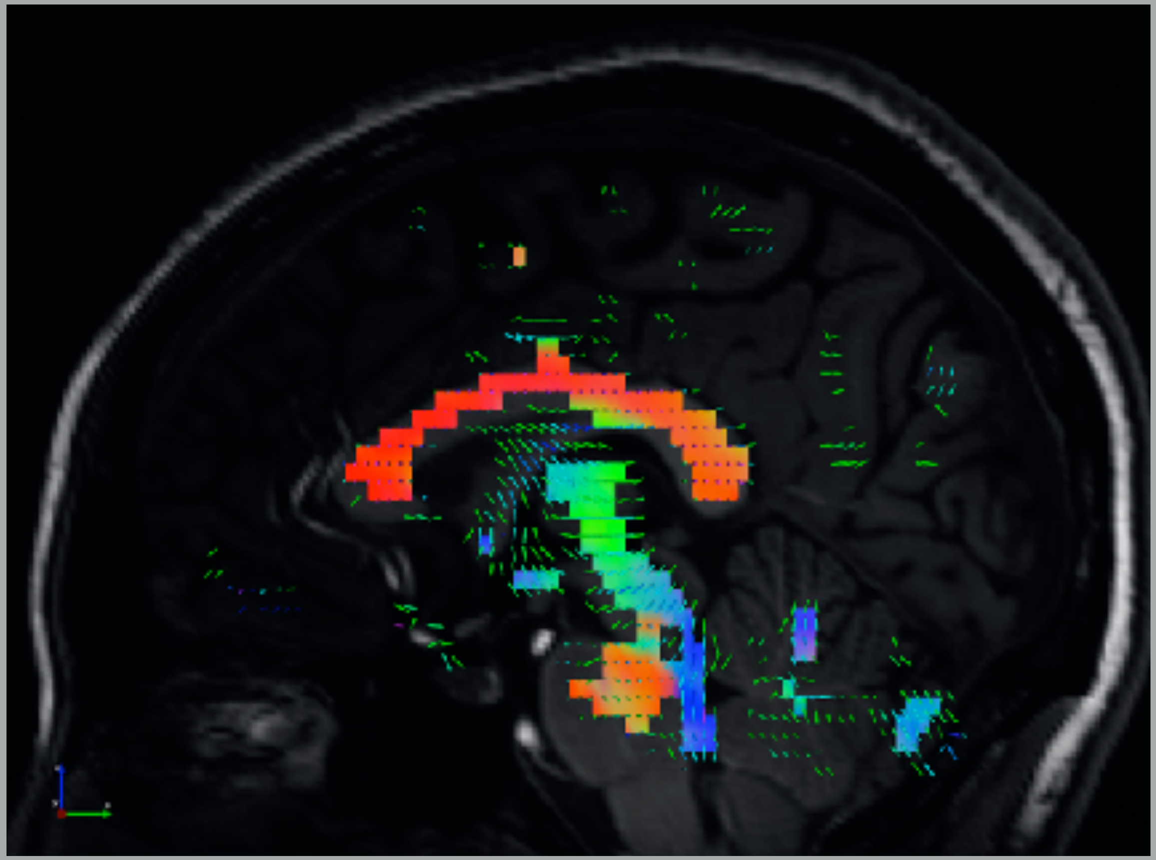
Directional Anisotropy Map

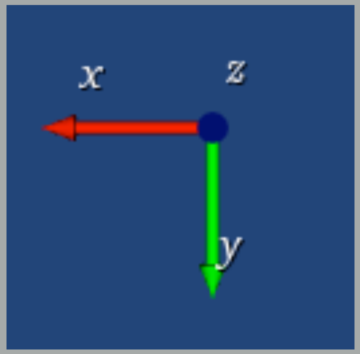




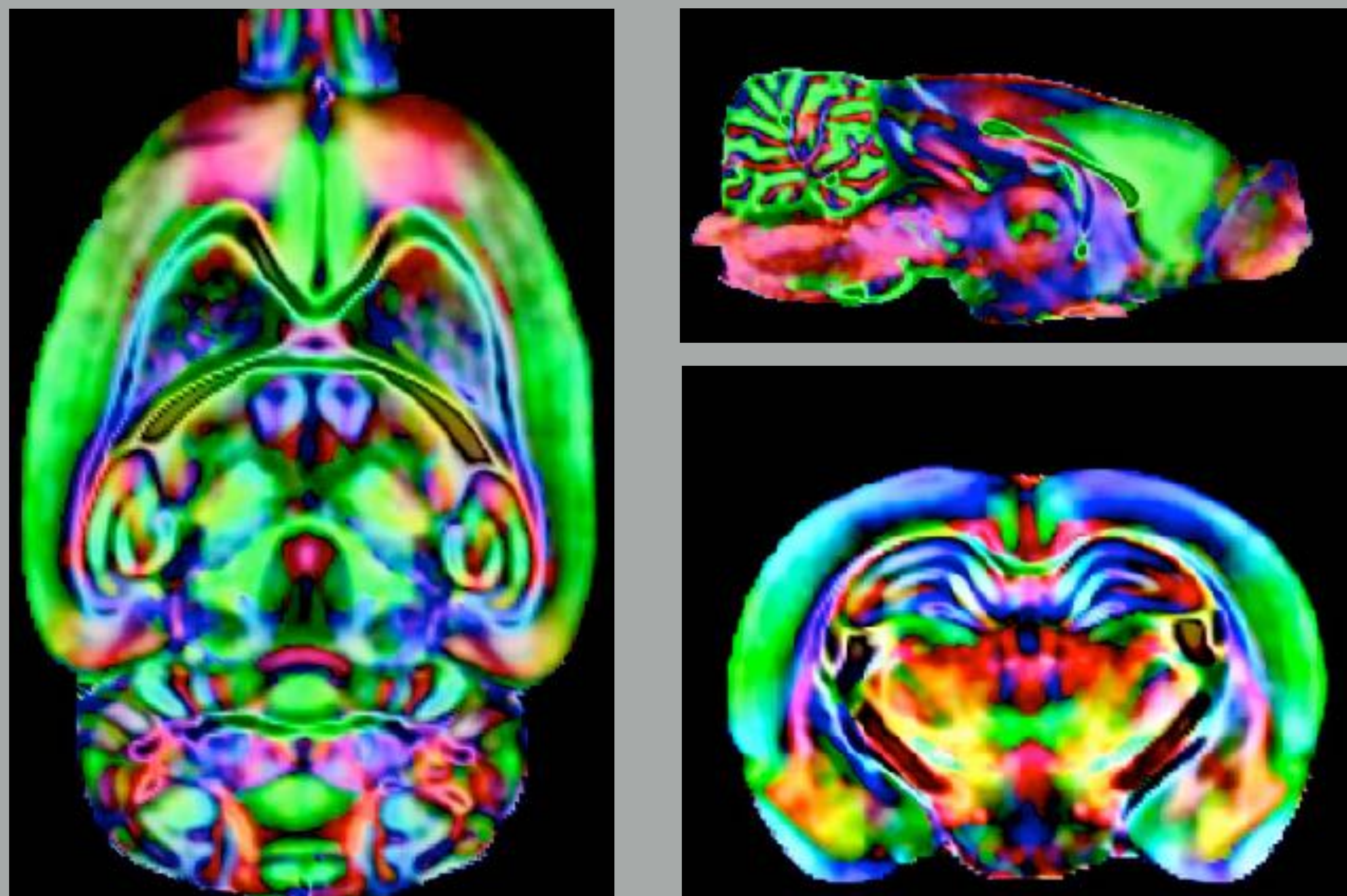
Directional Anisotropy Map







Rat brain at 11.7T



data courtesy J. M. Tyszka, CalTech

Next Lecture

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