Lecture 6
Magnetic Resonance Imaging: Image Formation II

Relaxation Contrast,
Fast imaging, and Artifacts

Lecture Summary

1. The NMR signal
2. The NMR image
3. Review of imaging
4. Creating relaxation contrast
5. Snapshot (EPI, spiral) imaging
6. Image artifacts

The NMR signal

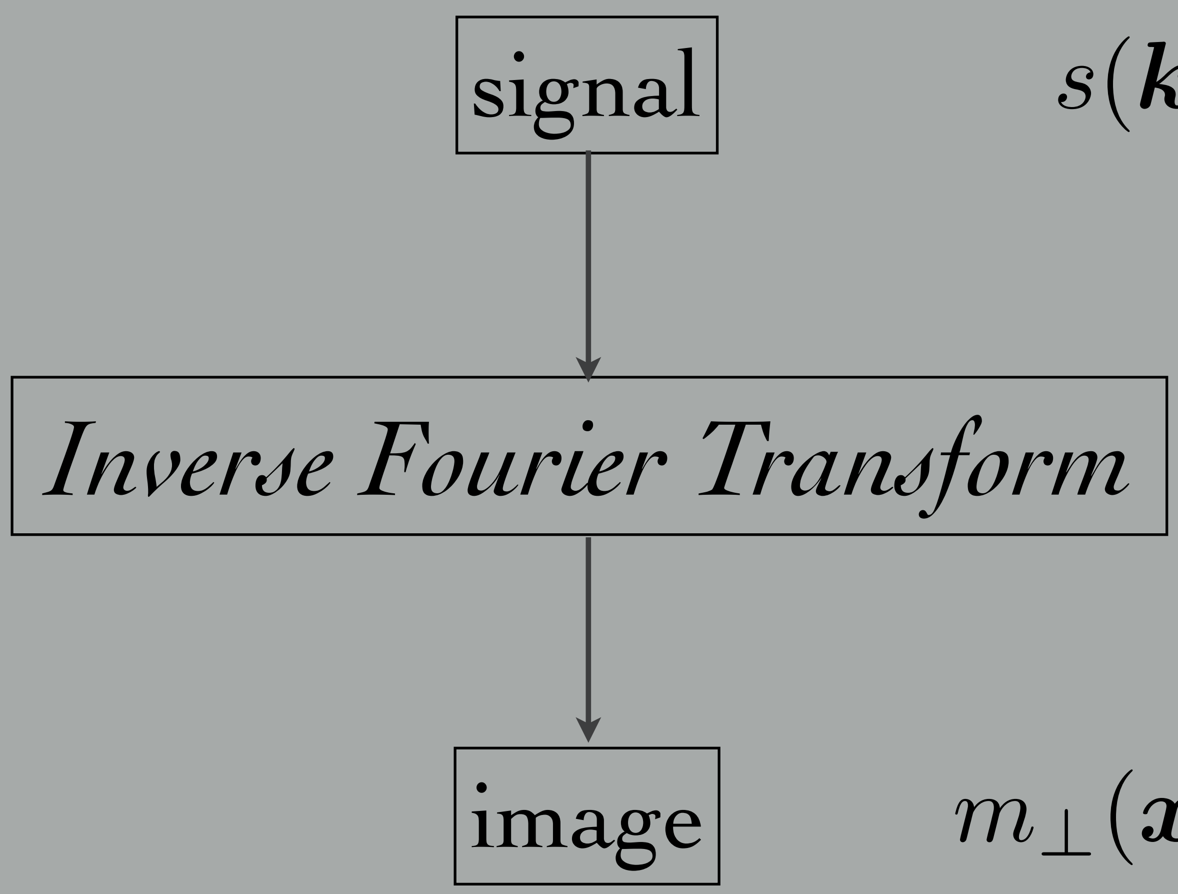
$$s(\boldsymbol{k}) = \int_\Omega m_\perp(\boldsymbol{x}, t) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{x}$$

The signal is the *Fourier Transform*
of the transverse magnetization

For static tissue (and perfect scanner)

$$m_\perp(\boldsymbol{x}, t) = m_\perp(\boldsymbol{x})$$

The Image

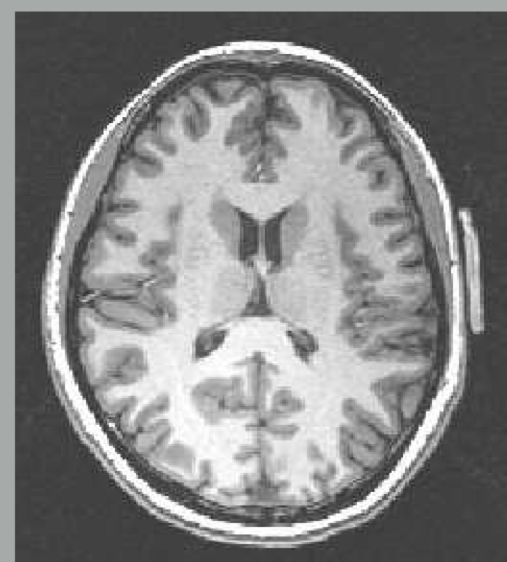
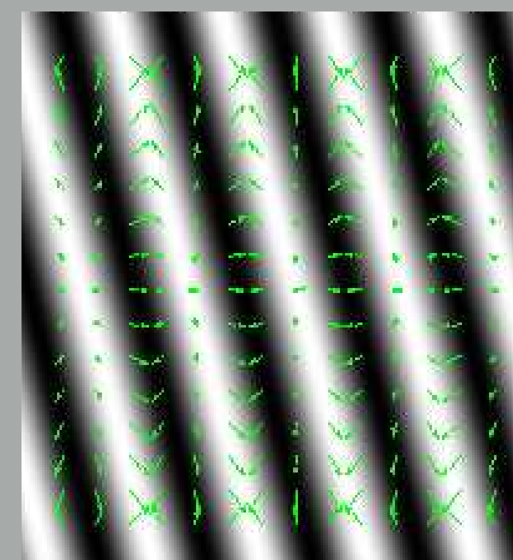


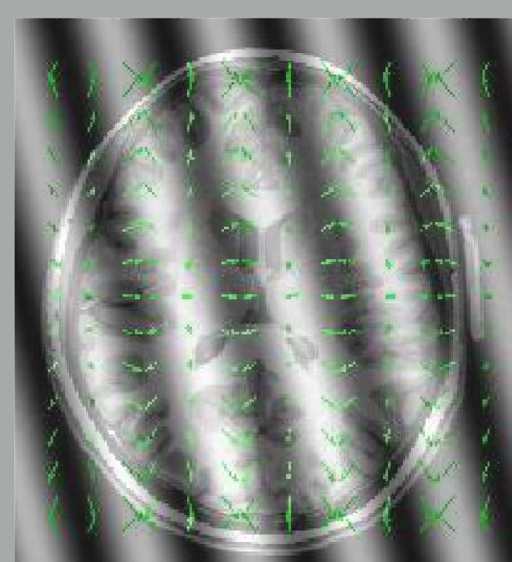
$$s(\boldsymbol{k}) = \int_{\Omega} m_{\perp}(\boldsymbol{x}) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{x}$$

$$m_{\perp}(\boldsymbol{x}) = \int s(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{k}$$

The NMR signal

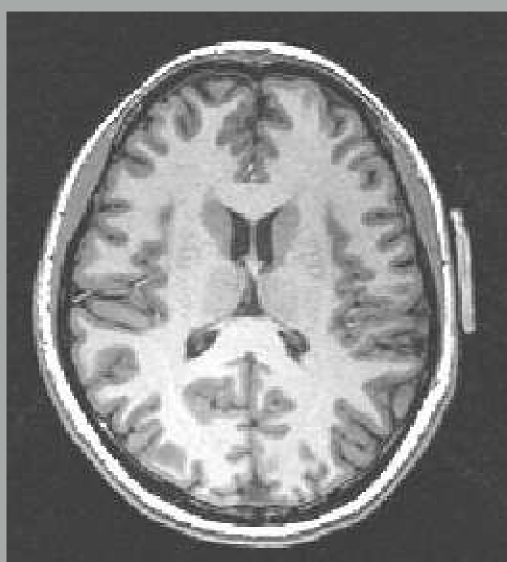
$$s(\boldsymbol{k}) = \int_\Omega m_\perp(\boldsymbol{x}) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{x}$$

$= \int_\Omega$   $d\boldsymbol{x}$

$= \int_\Omega$  $d\boldsymbol{x}$

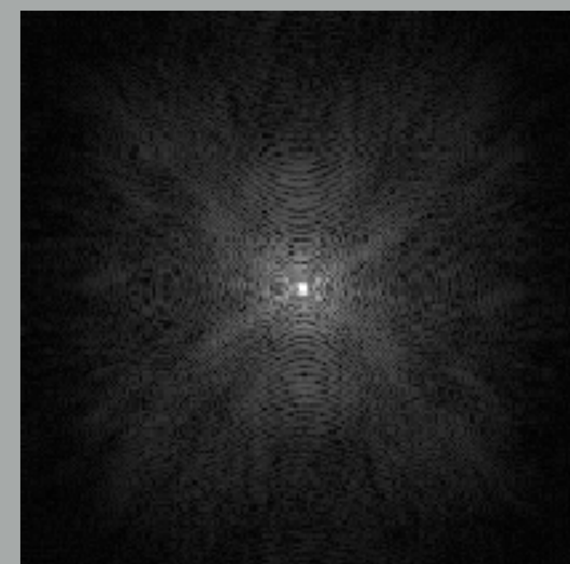
The NMR image

$$m_\perp(\boldsymbol{x}) = \int s(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{k}$$

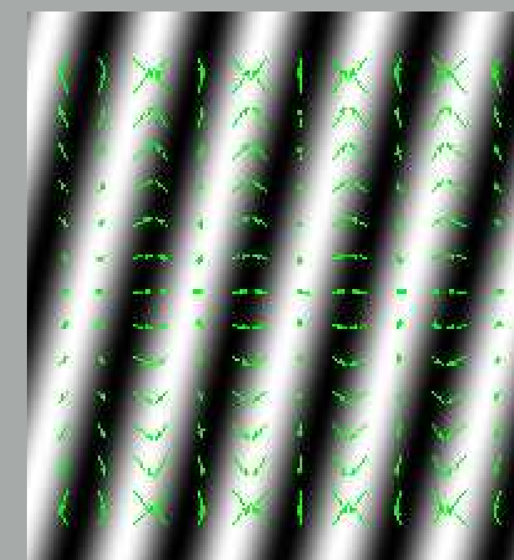


$$= \int_\Omega$$

$\longleftarrow k_y \longrightarrow$



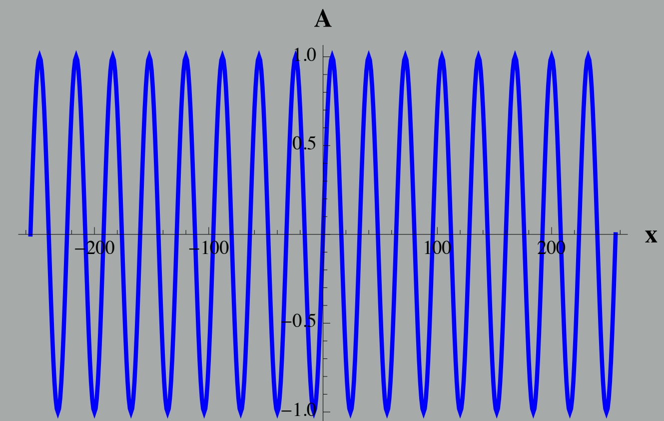
$\longleftarrow k_x \longrightarrow$

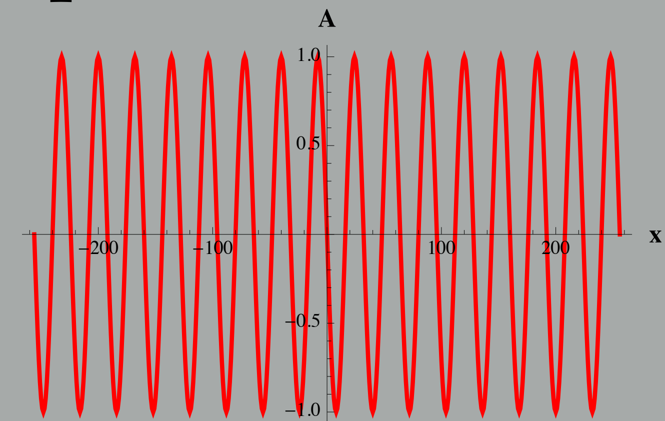


$$d\boldsymbol{x}$$

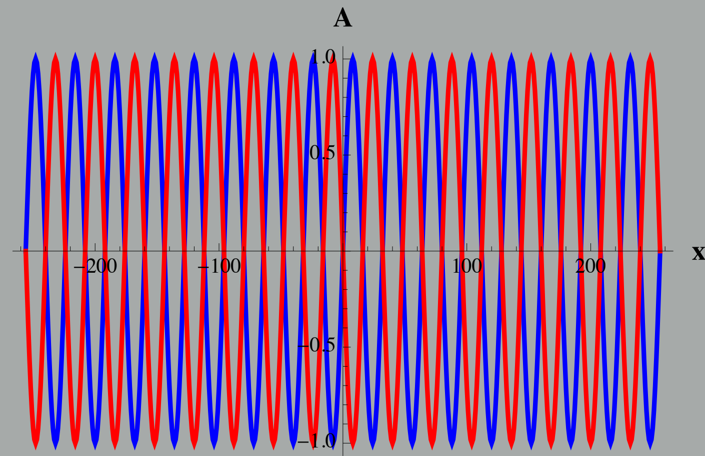
The Inverse Fourier Transform

$$m(\boldsymbol{x}_n) = \sum_{l=1}^{M} s(\boldsymbol{k}_j) e^{i\boldsymbol{k}_l \cdot \boldsymbol{x}_n}$$



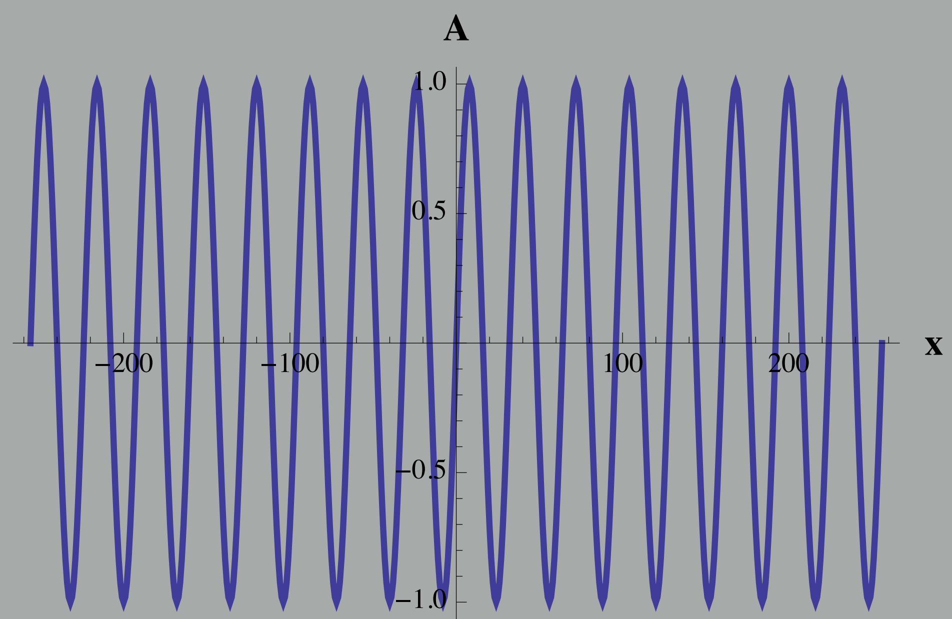


$$s(\boldsymbol{k}_j) e^{i\boldsymbol{k}_l \cdot \boldsymbol{x}}$$

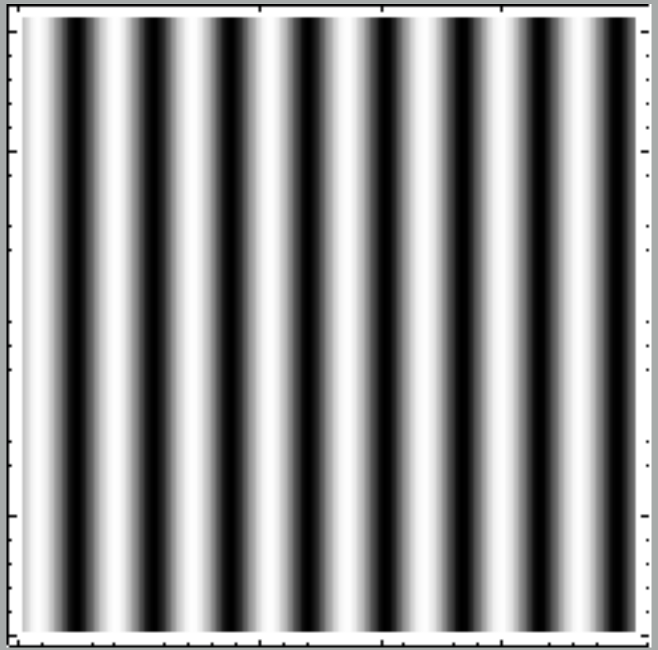


$$\sum_{l=1}^{M} A e^{-i\boldsymbol{k}_j \cdot \boldsymbol{x}_n} e^{i\boldsymbol{k}_l \cdot \boldsymbol{x}_n} = A \sum_{l=1}^{M} e^{-i(\boldsymbol{k}_j - \boldsymbol{k}_l) \cdot \boldsymbol{x}_n} = NA \quad \text{if } j = l$$

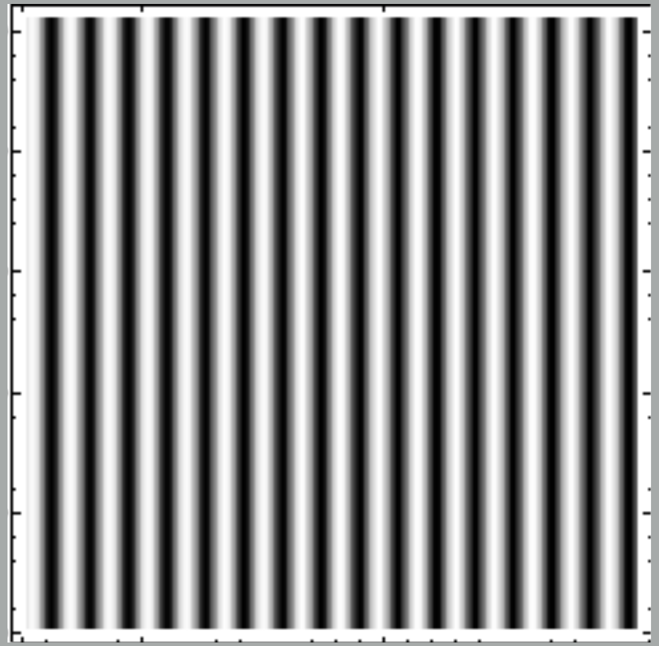
Example: A sinusoidal grating



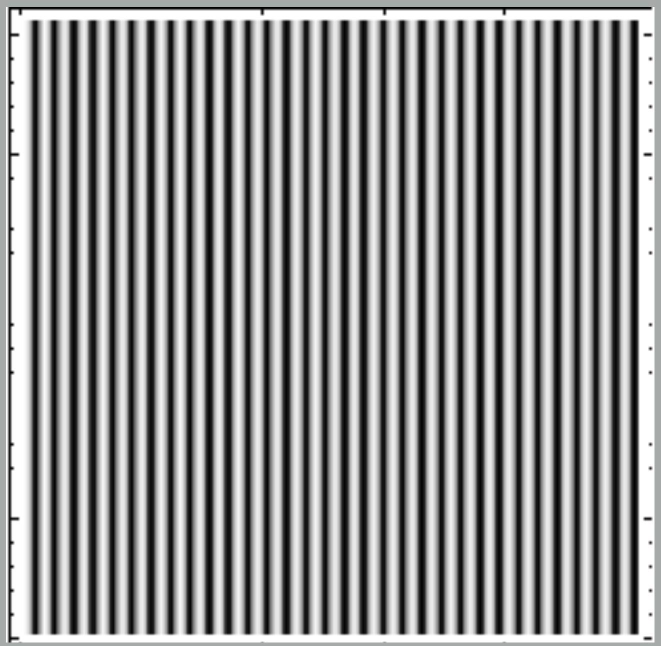
$$m(x) = A \sin(2\pi n x / L_x) \ , \ -L_x \leq x \leq L_x$$



$n = 4$

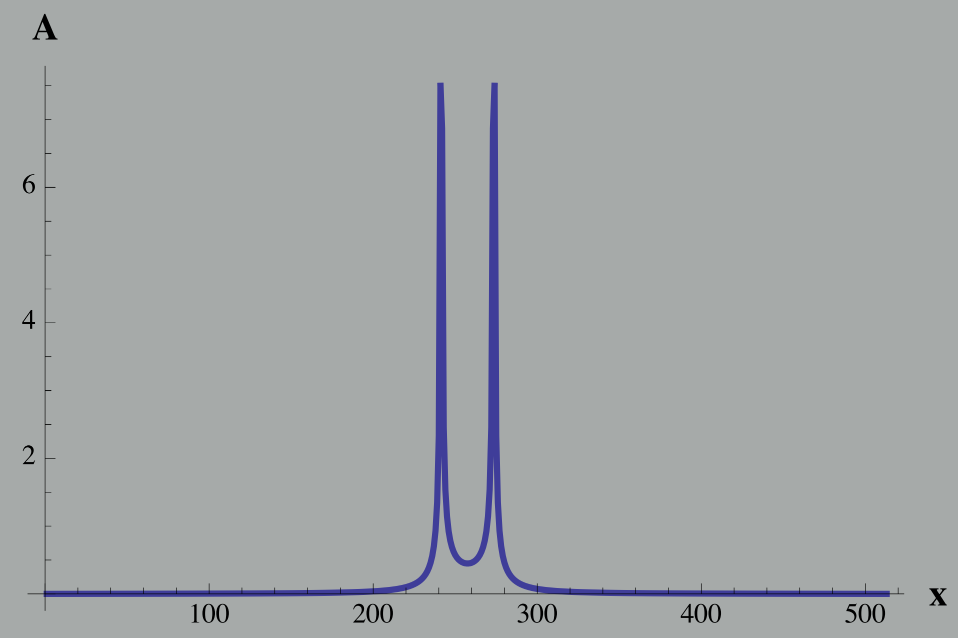


$n = 8$



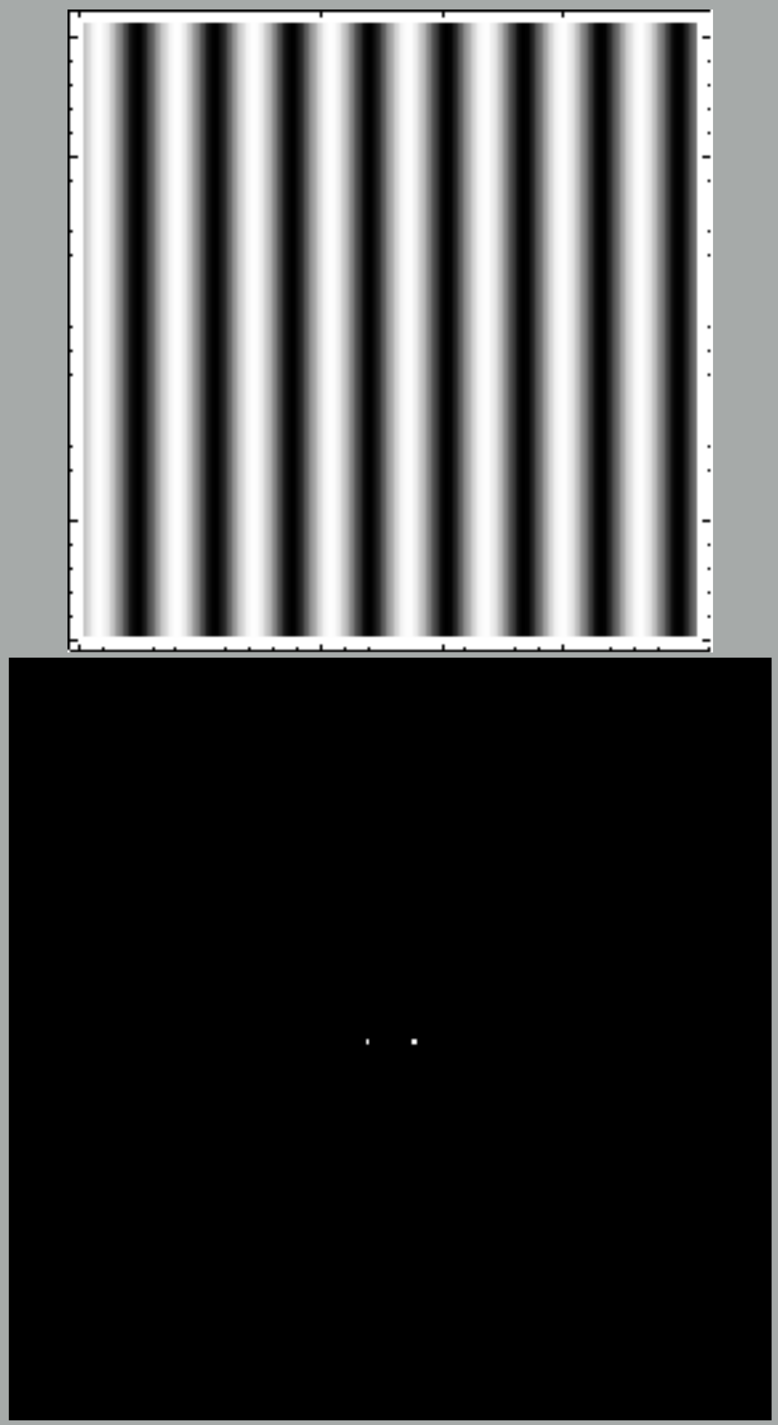
$n = 16$

Example: A sinusoidal grating

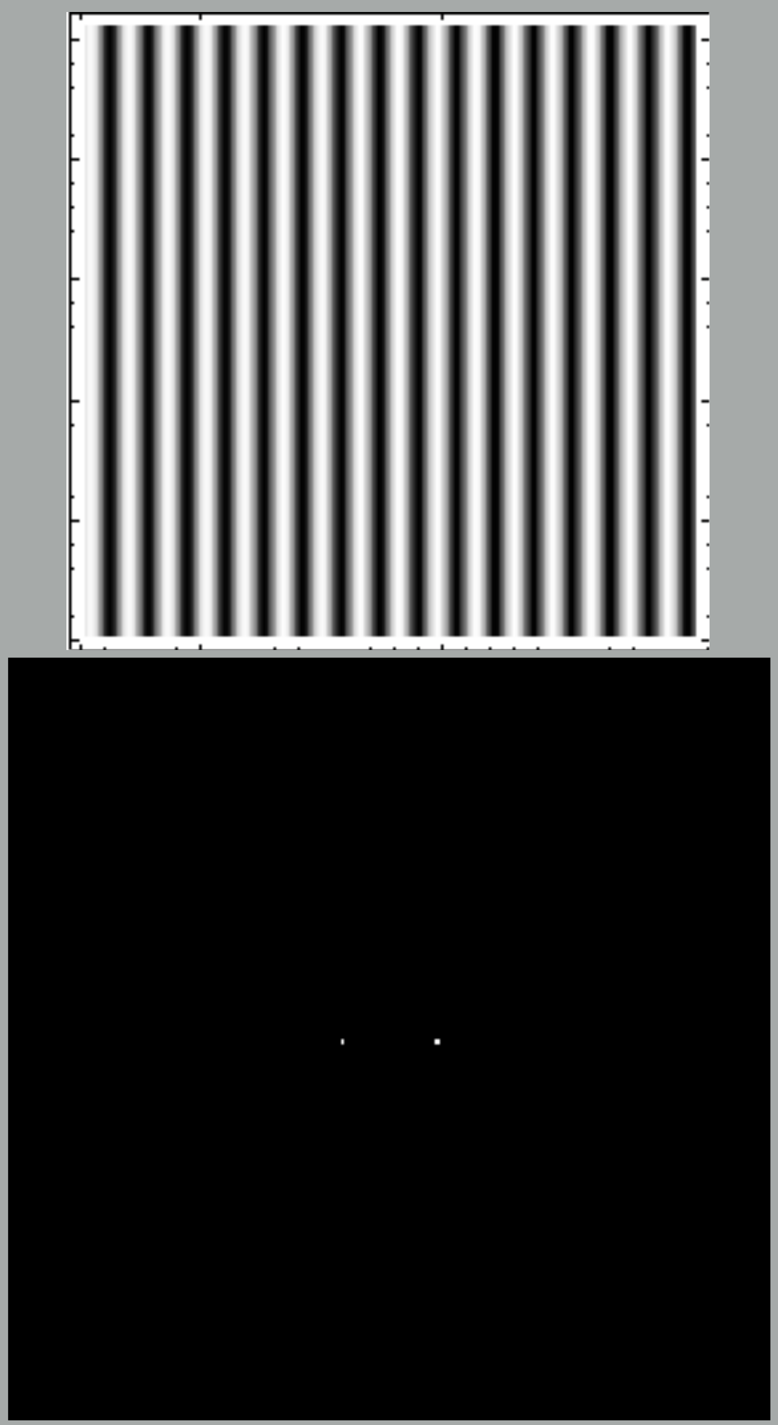


$n = 4$ $n = 8$ $n = 16$

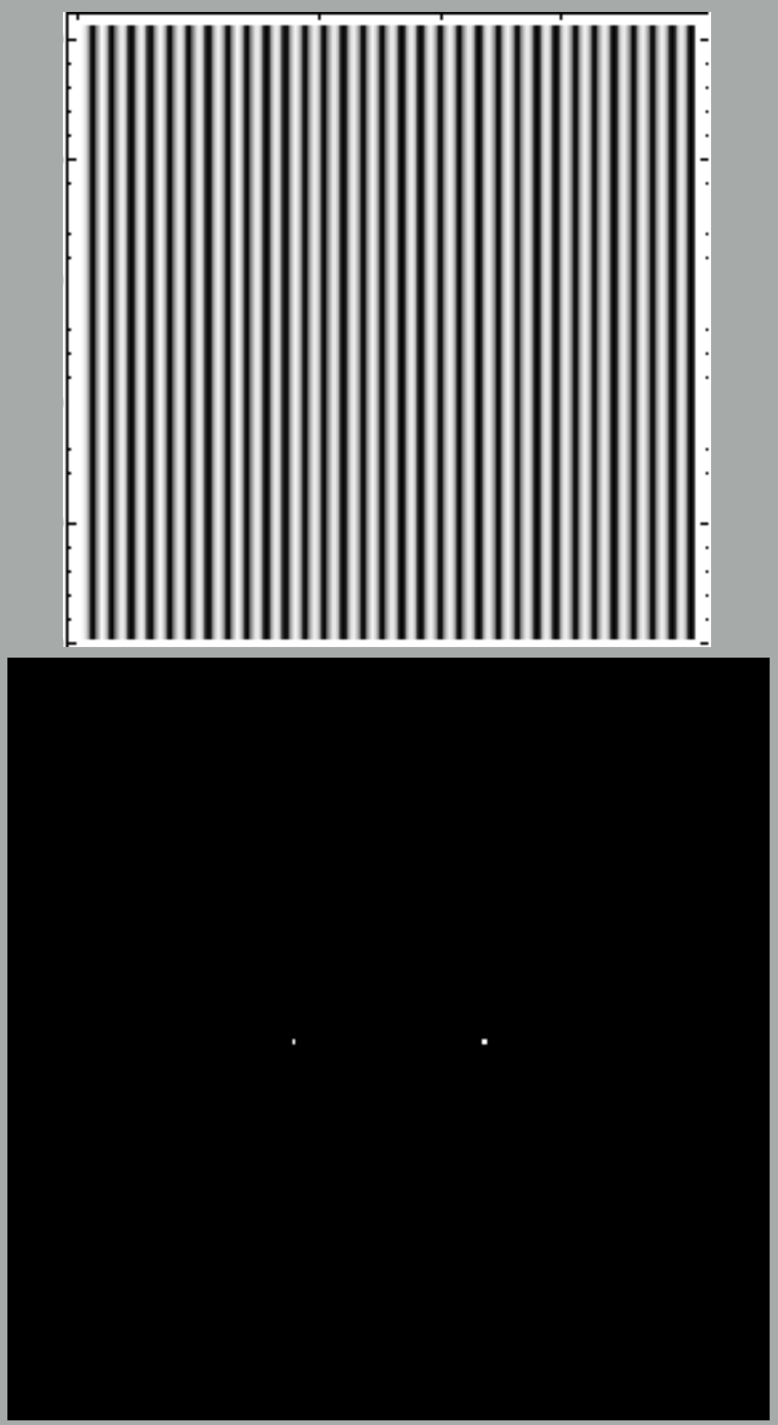
Example: A sinusoidal grating



$n = 4$

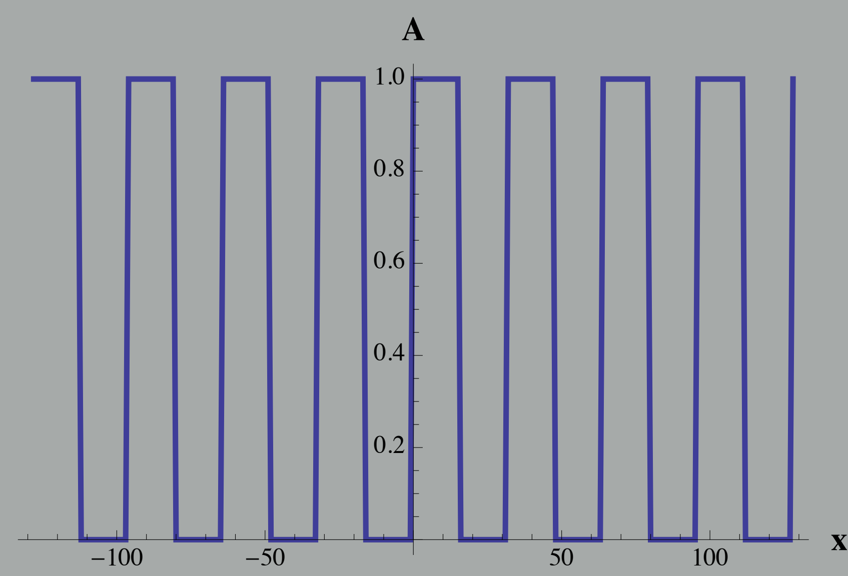


$n = 8$

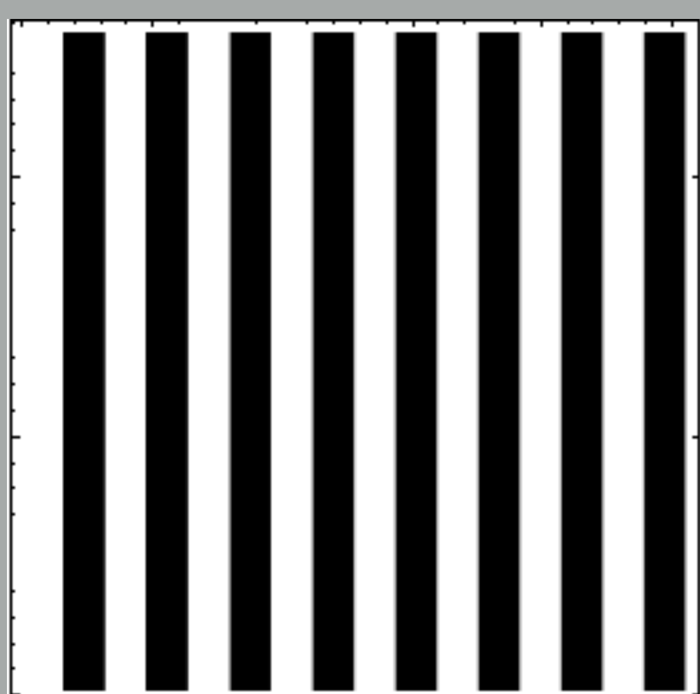


$n = 16$

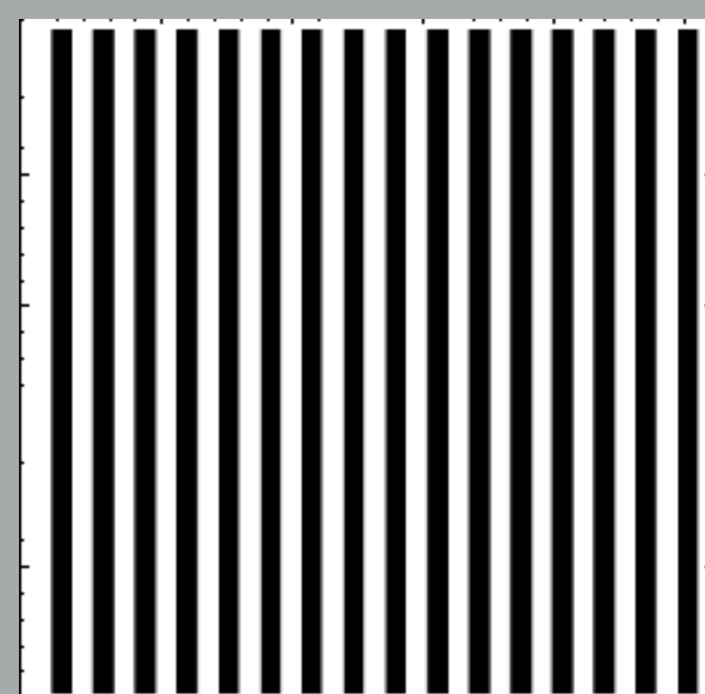
Example: A box function grating



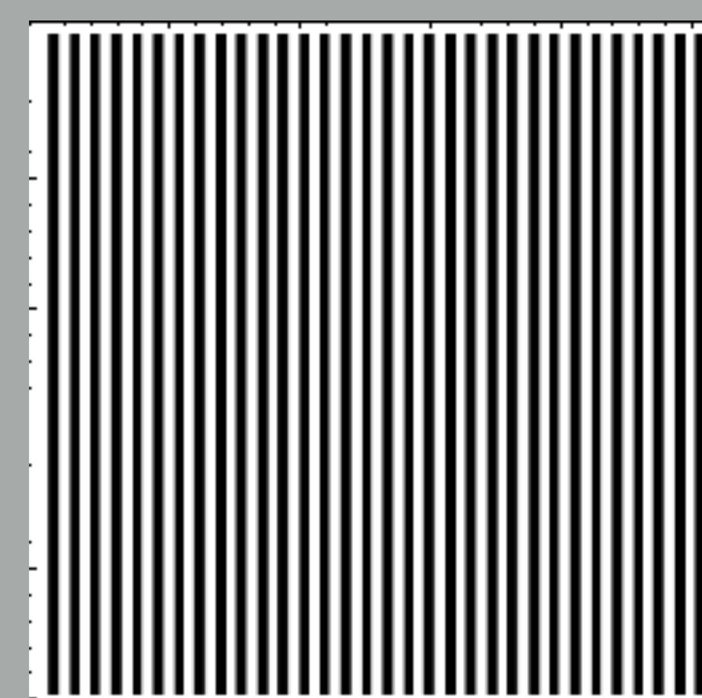
$$\mathrm{Box}(\mathrm{Mod}[x, L_x/n]/(L_x/n))$$



$n = 4$

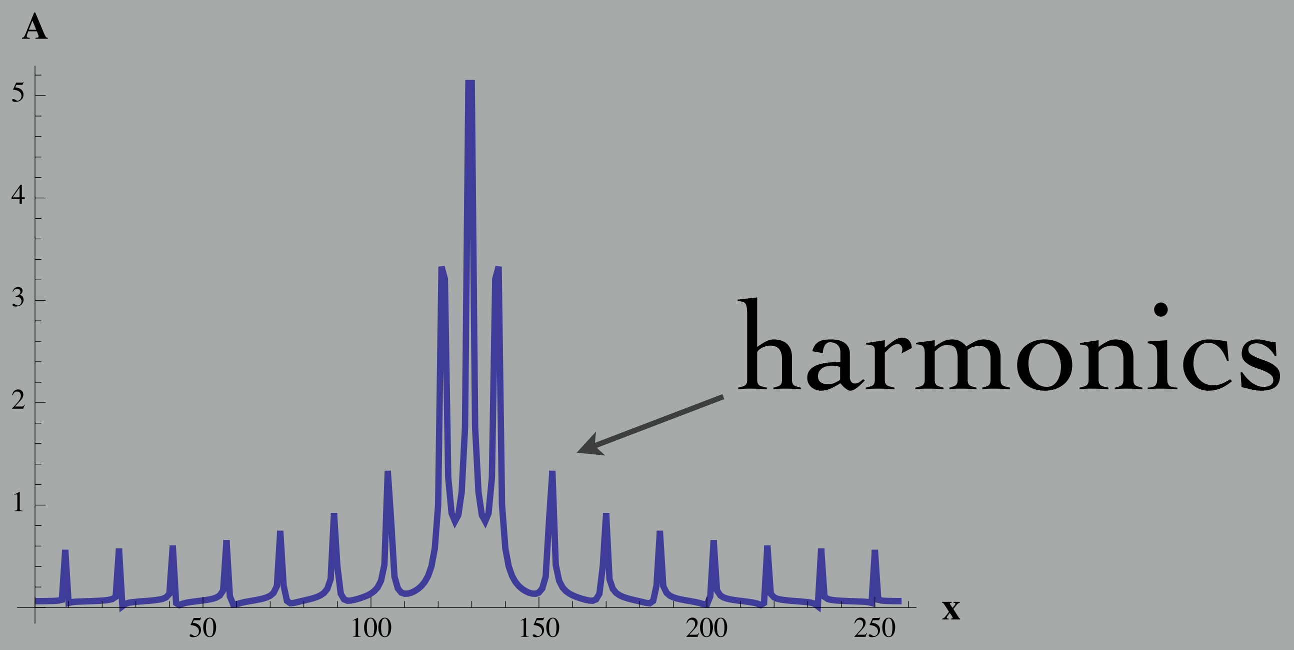


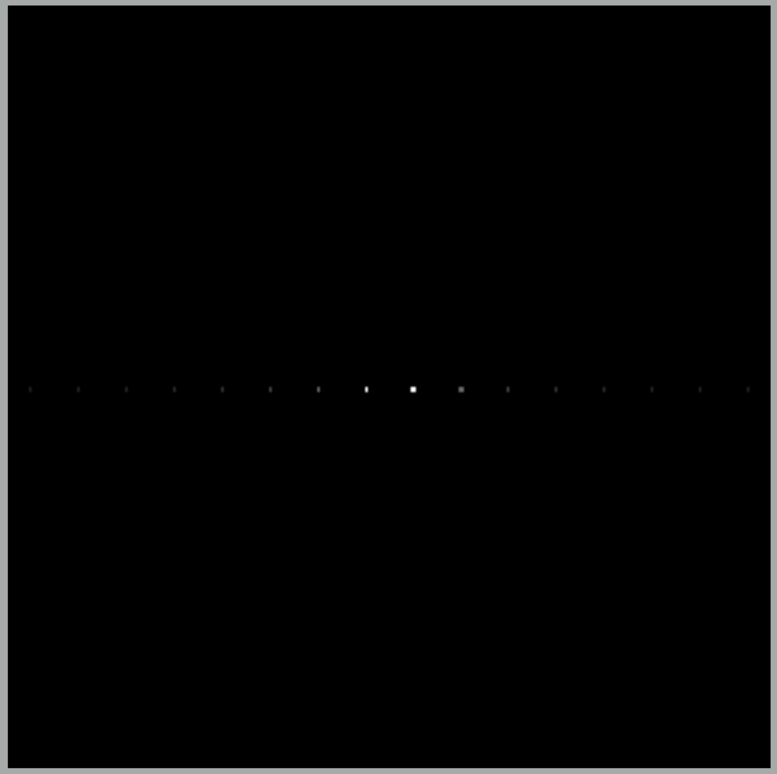
$n = 8$



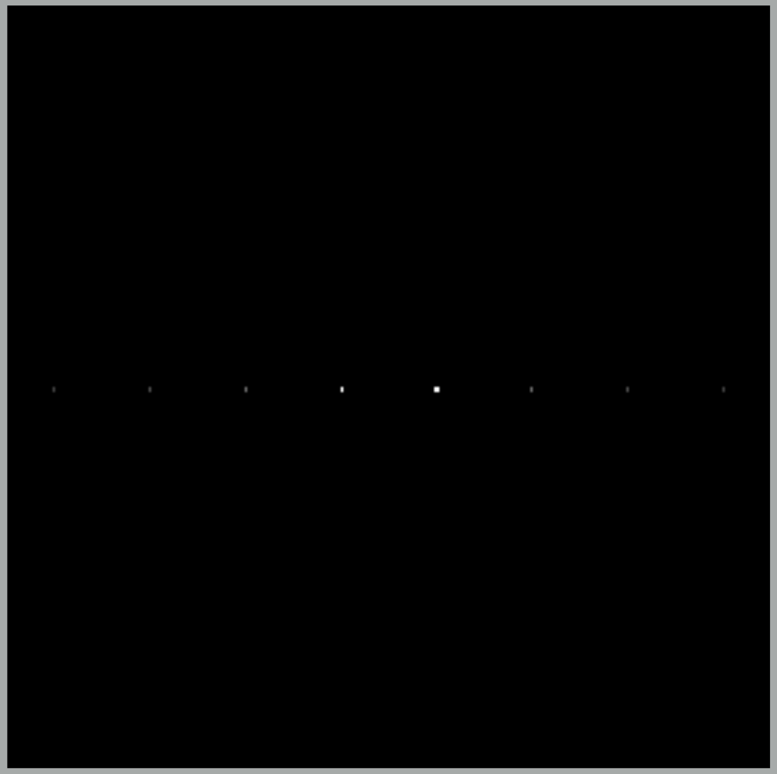
$n = 16$

Example: A box function grating

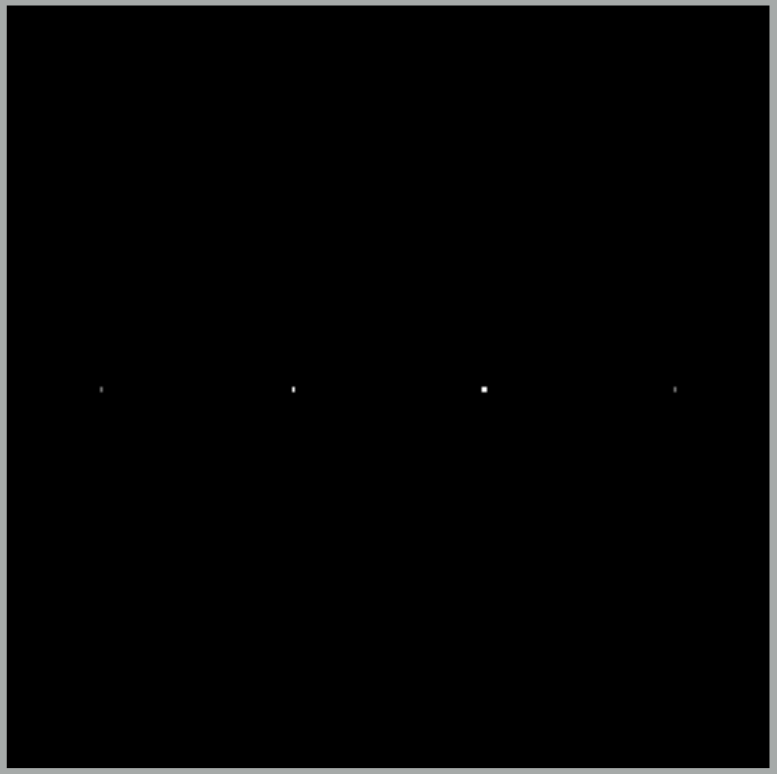




$n = 4$



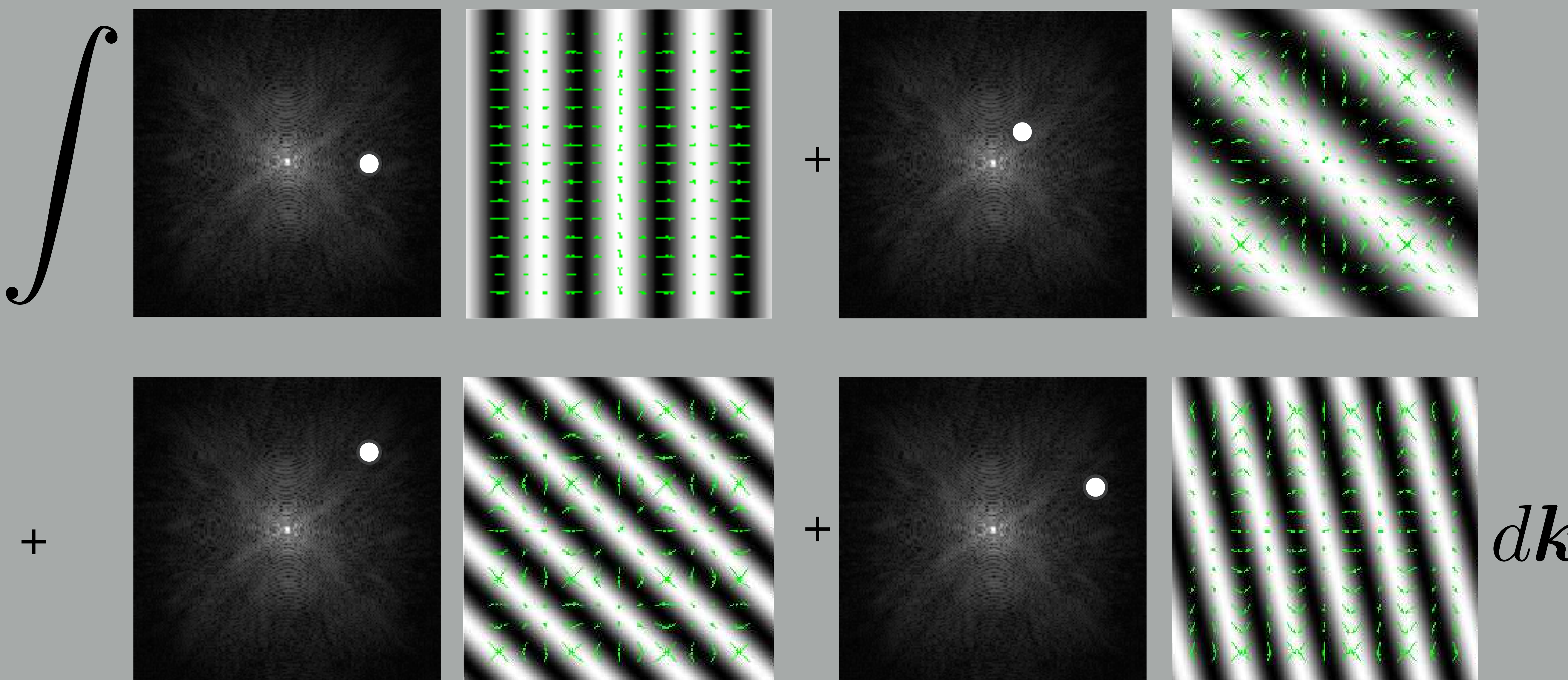
$n = 8$



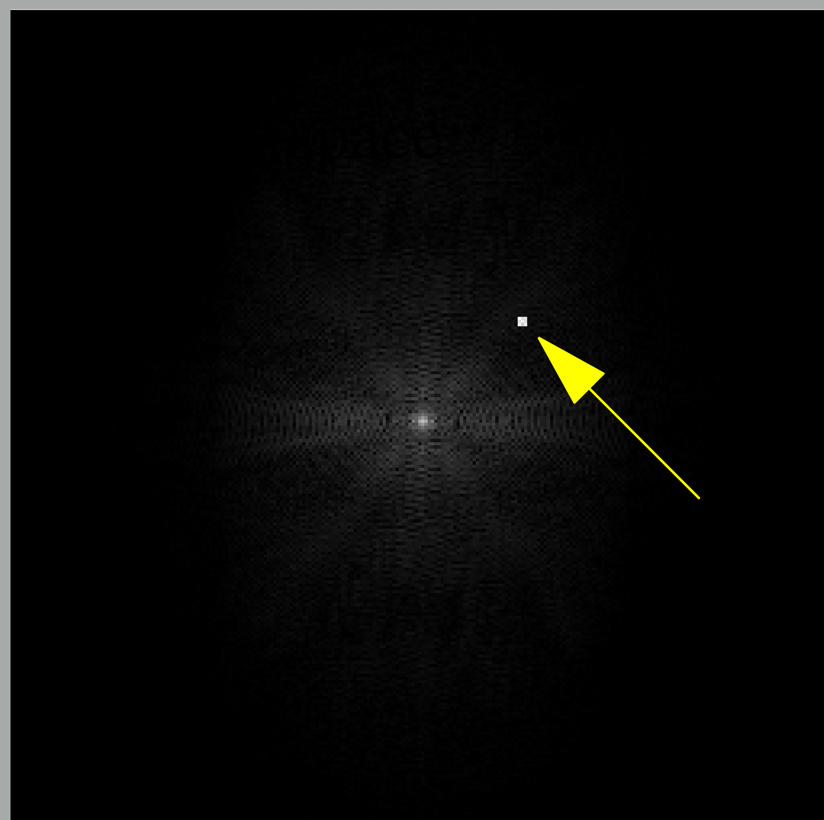
$n = 16$

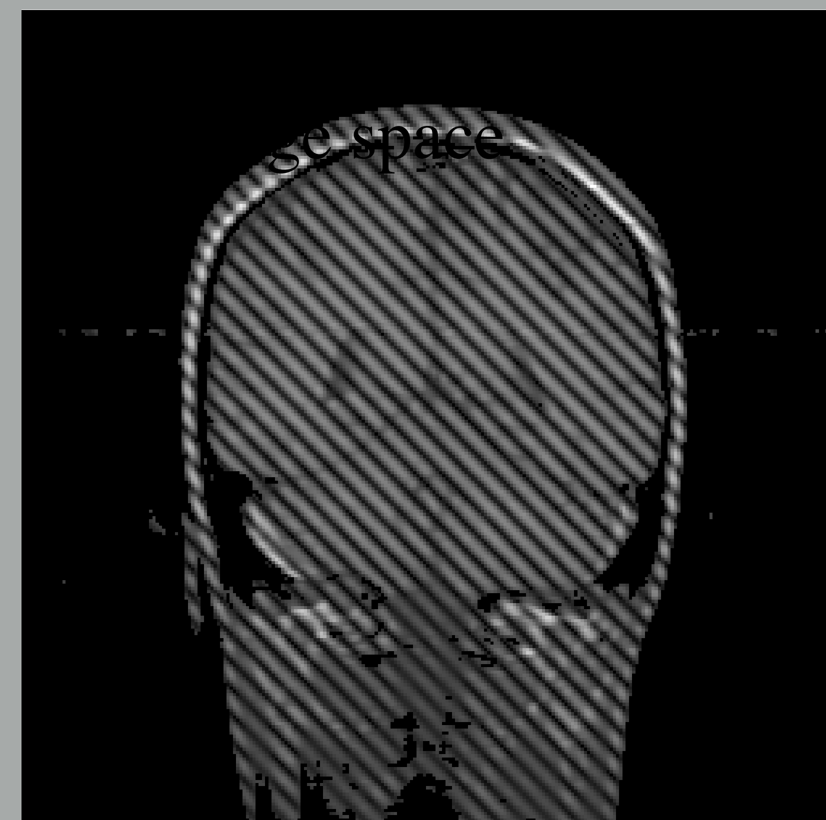
The image

$$m_\perp(\boldsymbol{x}) = \int s(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{k}$$

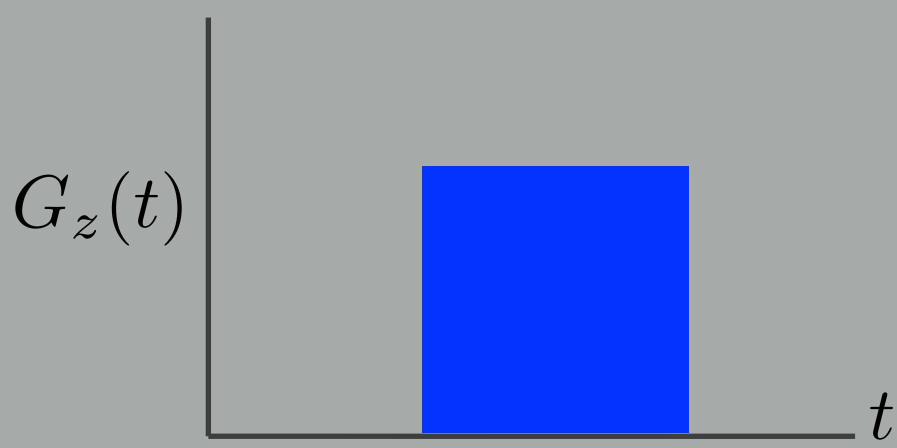


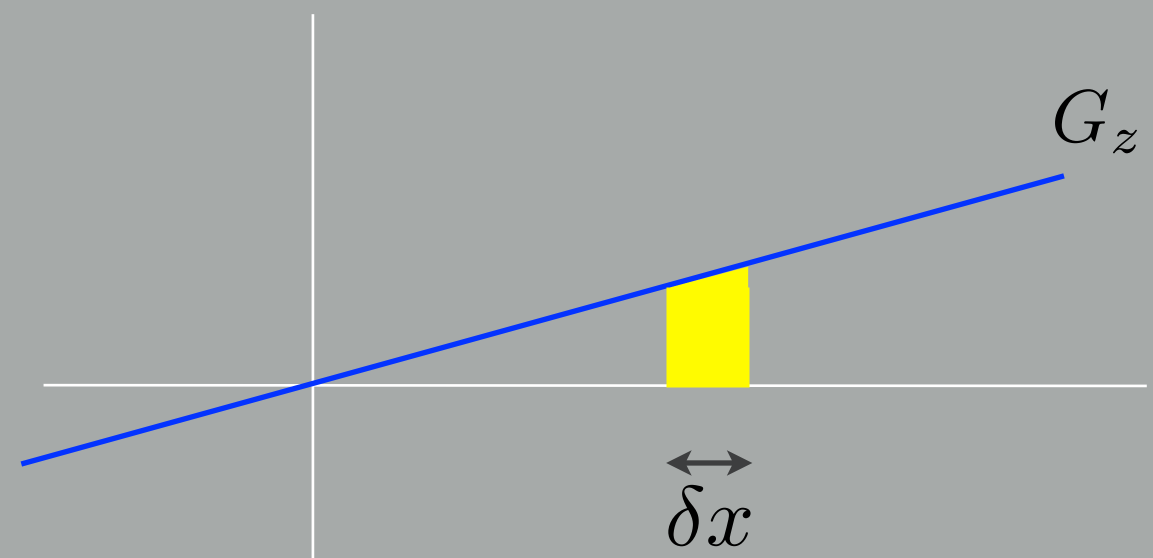
Spatial modulation of the phase



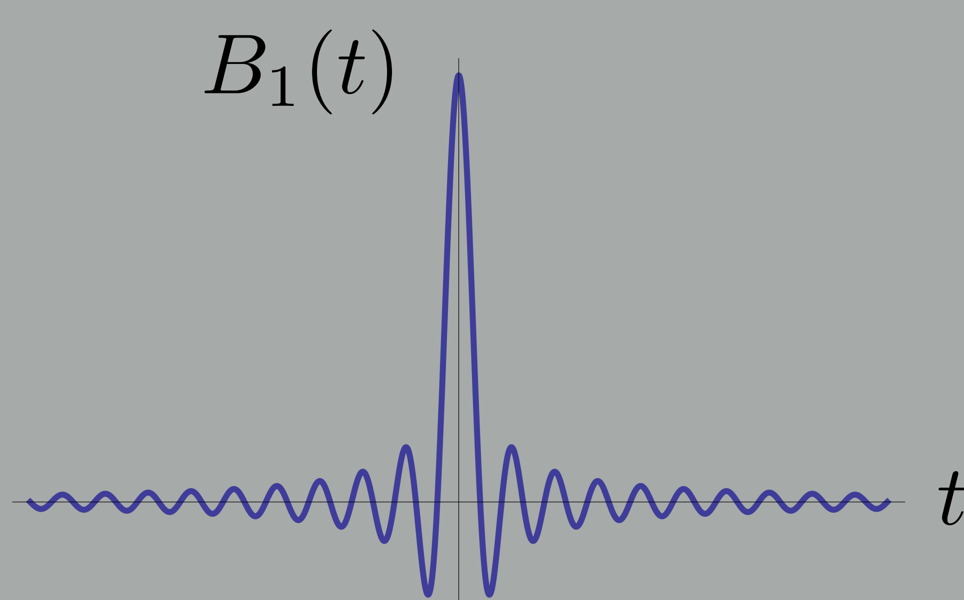


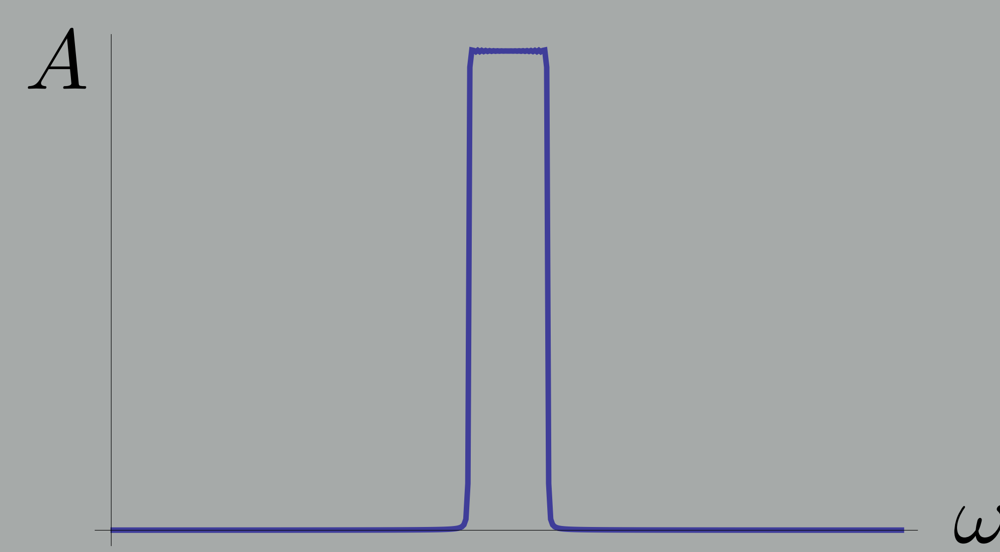
Slice selection



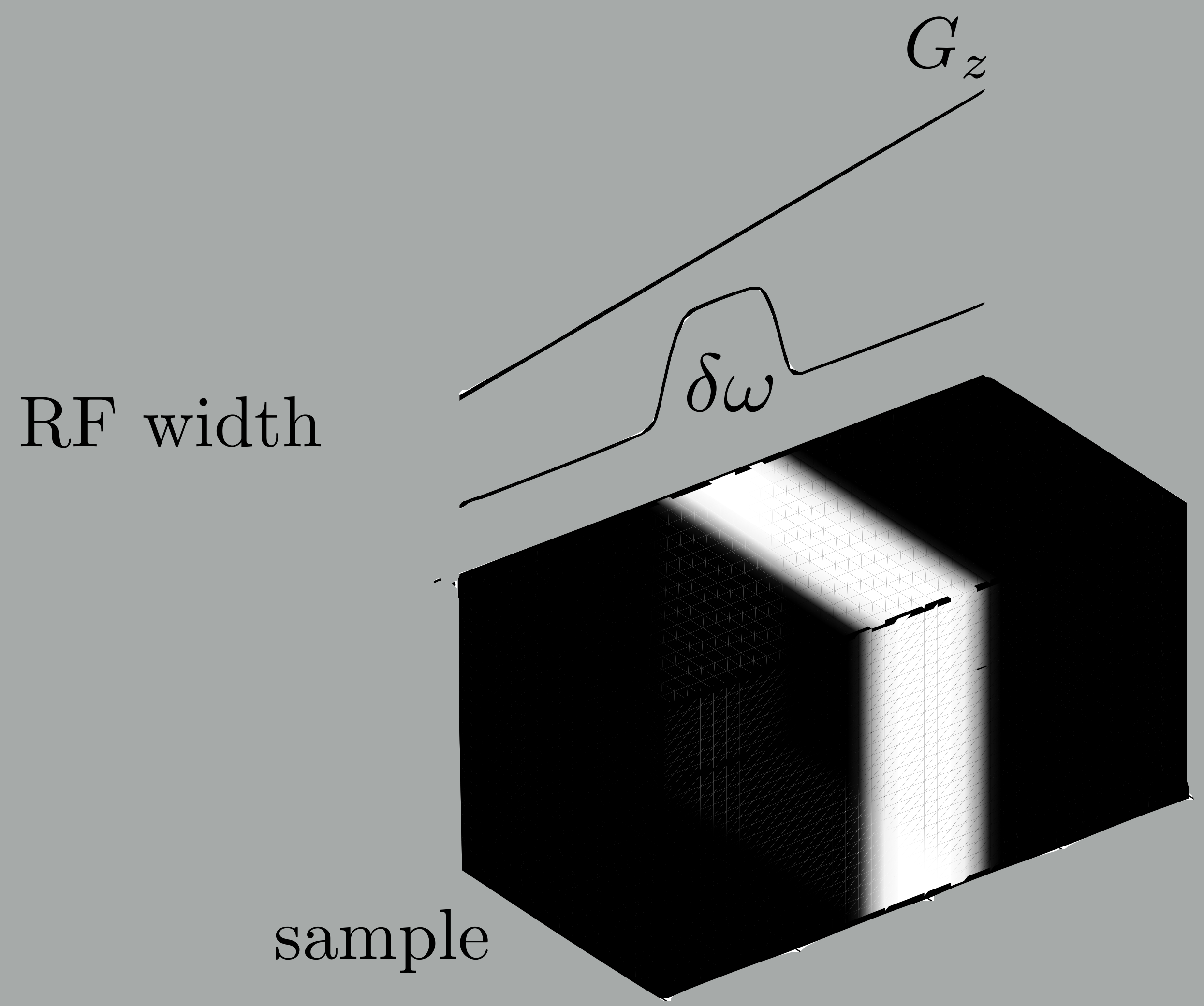


$$\delta\omega = \gamma G_x \delta x$$

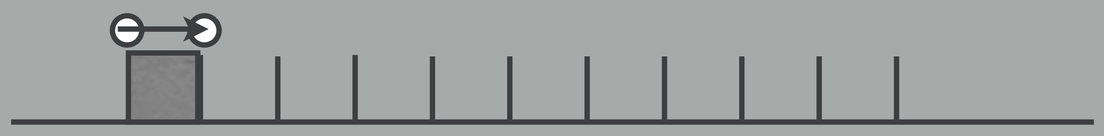




Slice selection



k-space trajectory



$G_x \Delta t$

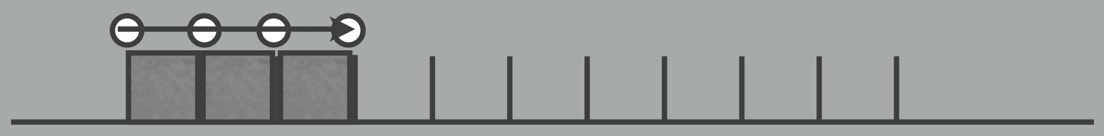
$$k_x = \gamma G_x \Delta t$$

k-space trajectory

$$2G_x \Delta t$$

$$k_x = \gamma 2 G_x \Delta t$$

k-space trajectory

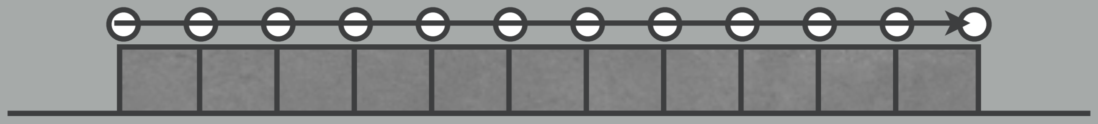


$3G_x\Delta t$

$$k_x = \gamma 3G_x\Delta t$$

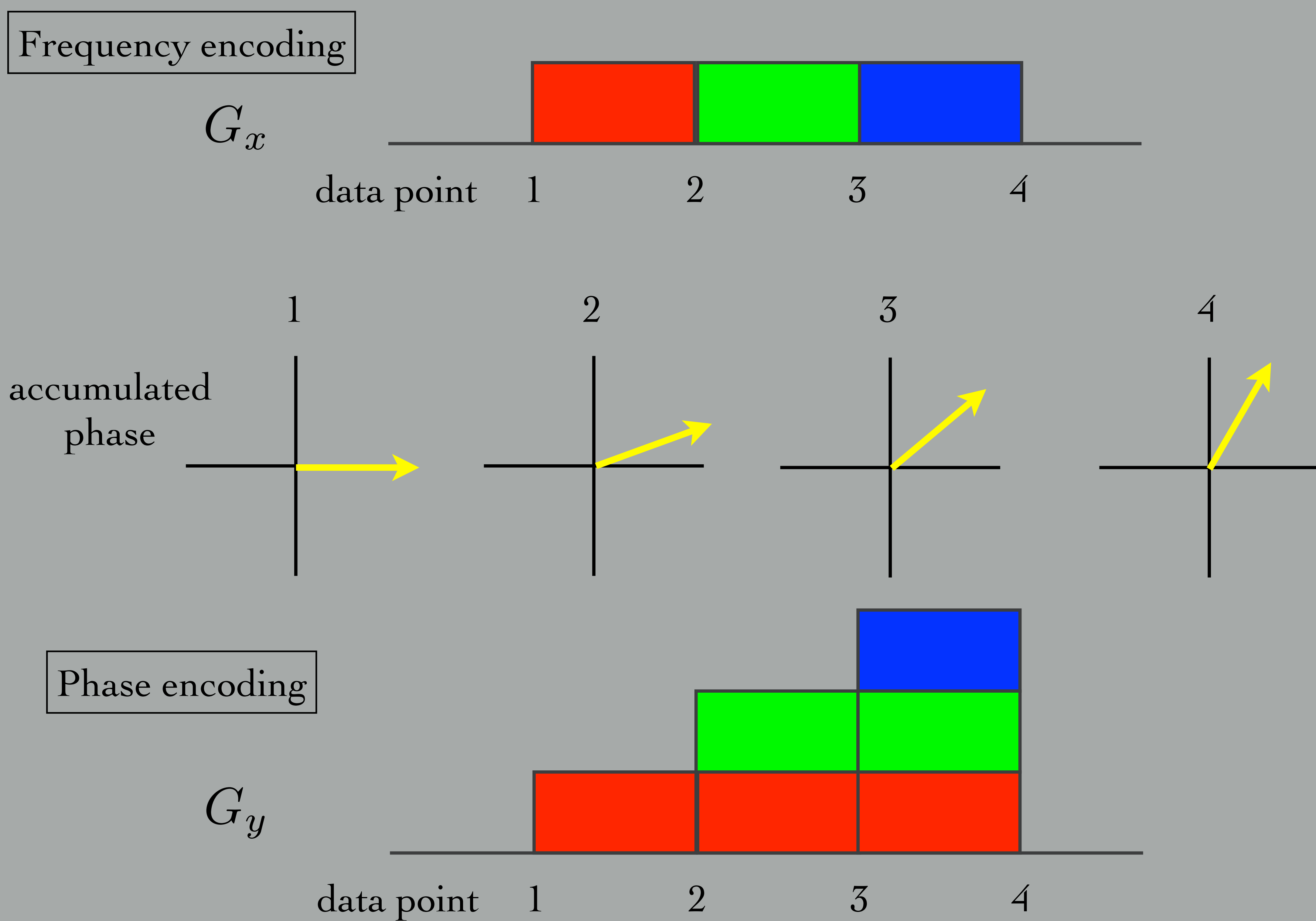
k-space trajectory

"Frequency encoding"

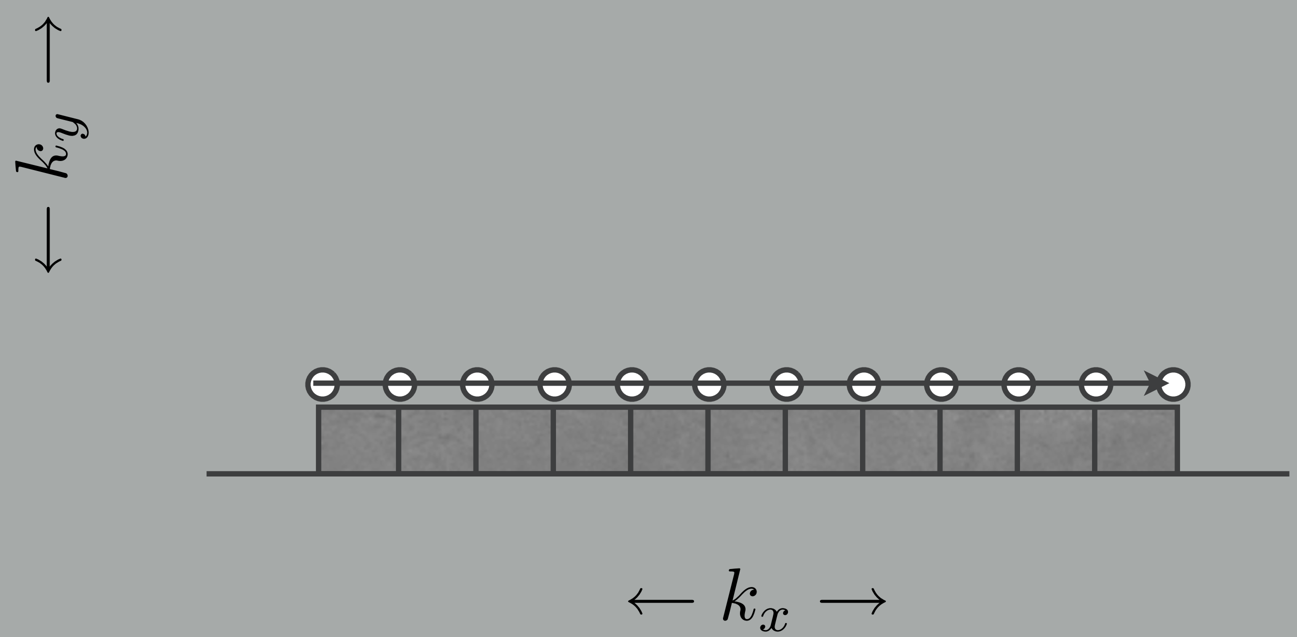


$$k_x = \gamma G_x t = \gamma G_x n \Delta t$$

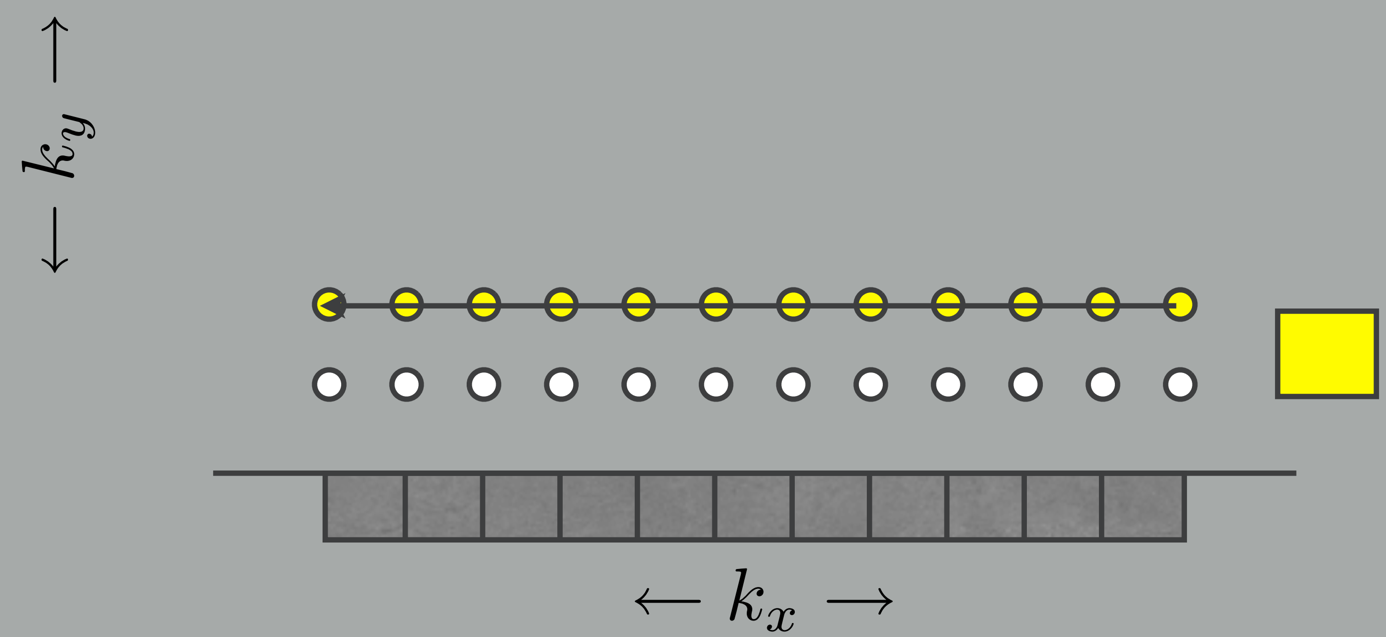
Frequency and phase encoding



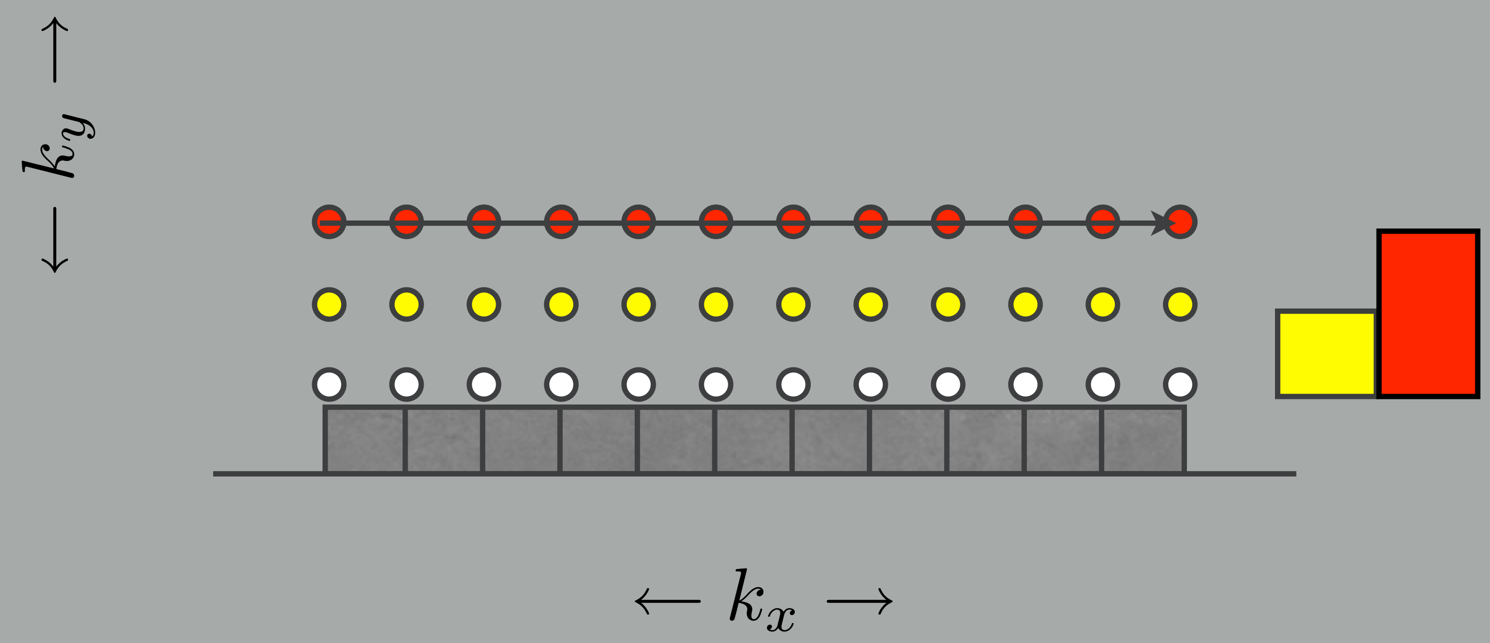
k-space trajectory



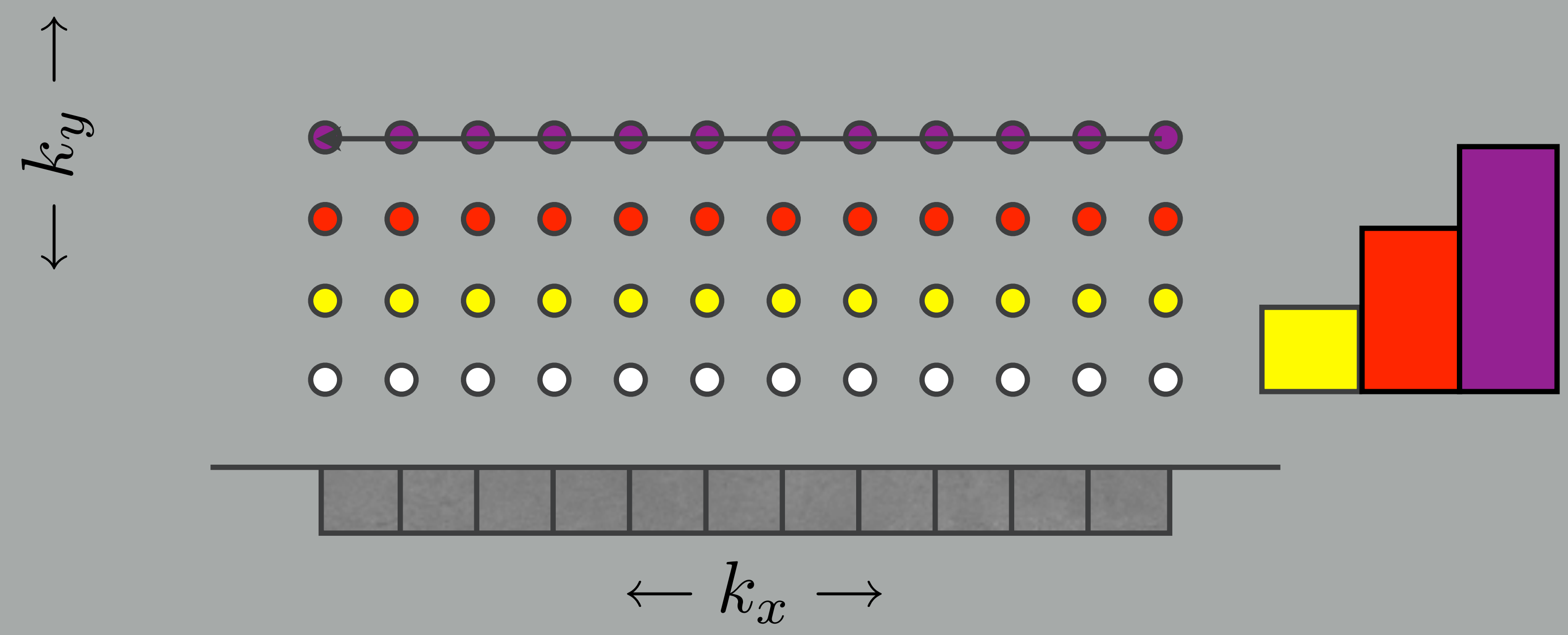
k-space trajectory



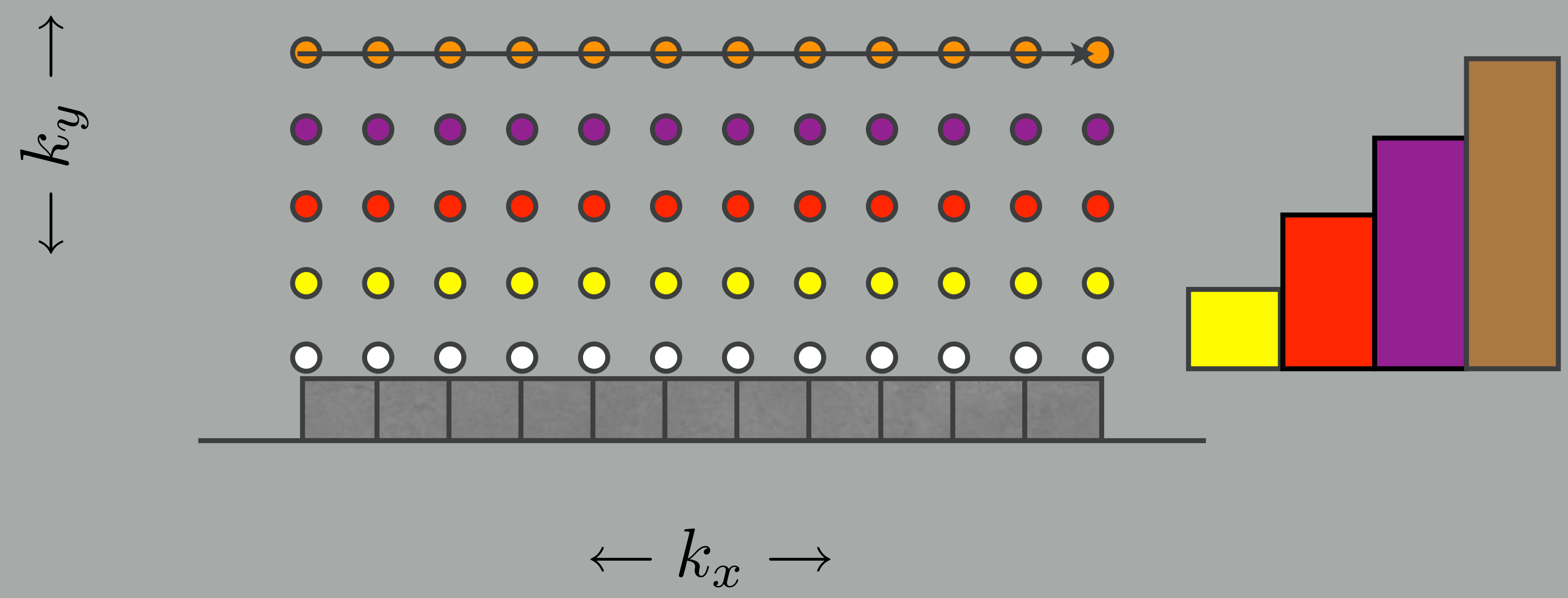
k-space trajectory



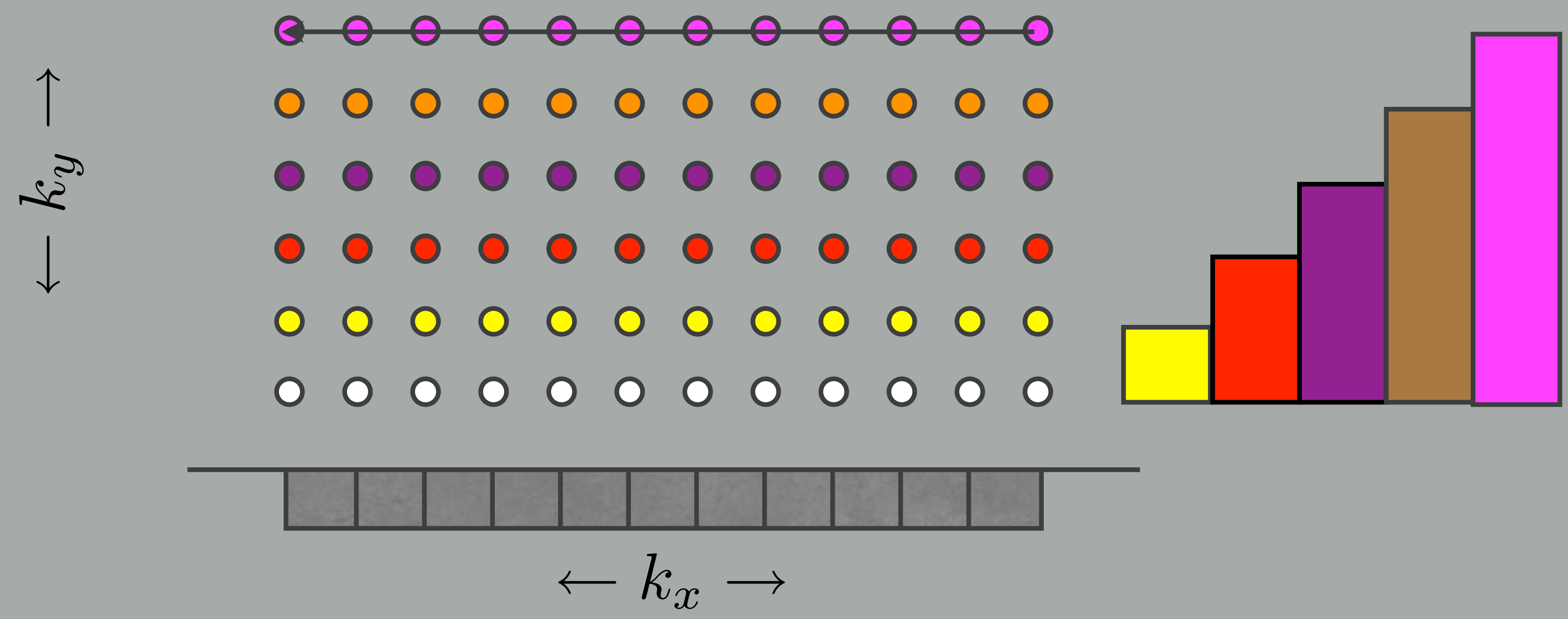
k-space trajectory



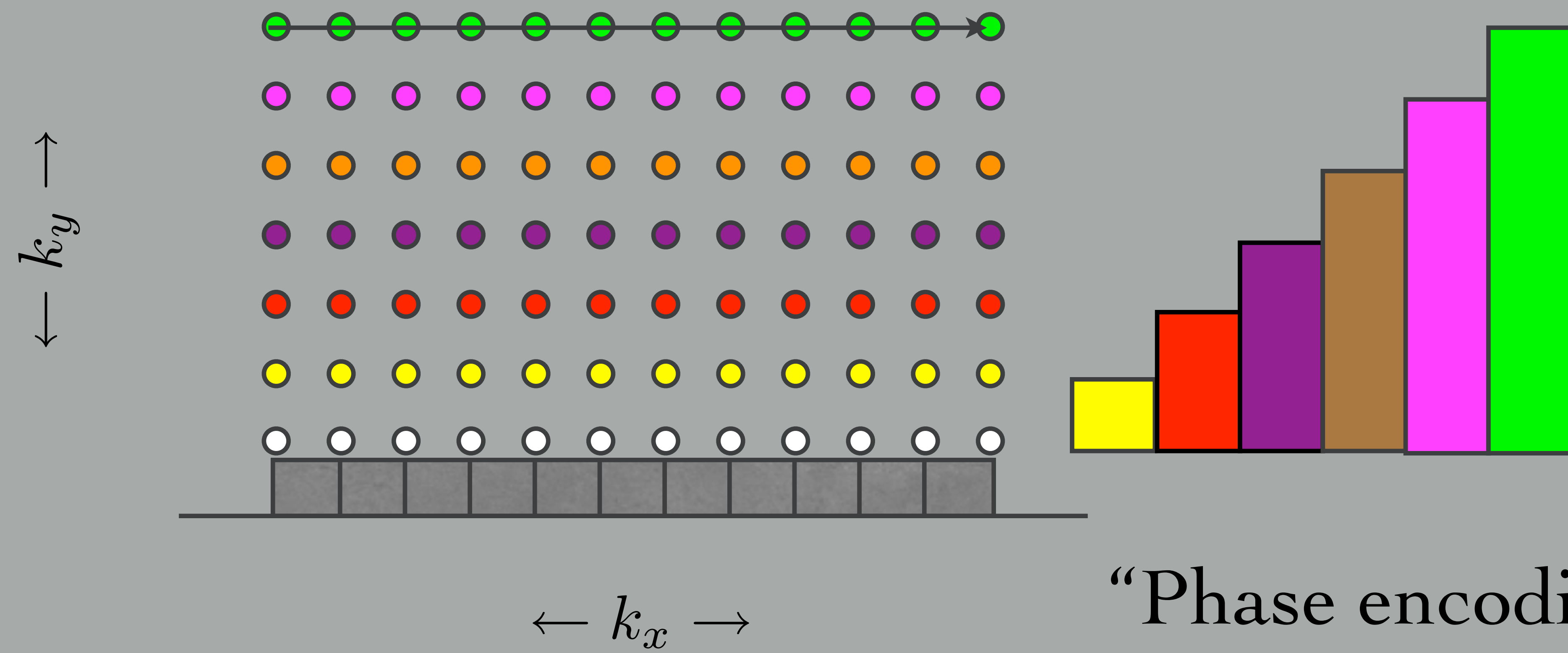
k-space trajectory



k-space trajectory



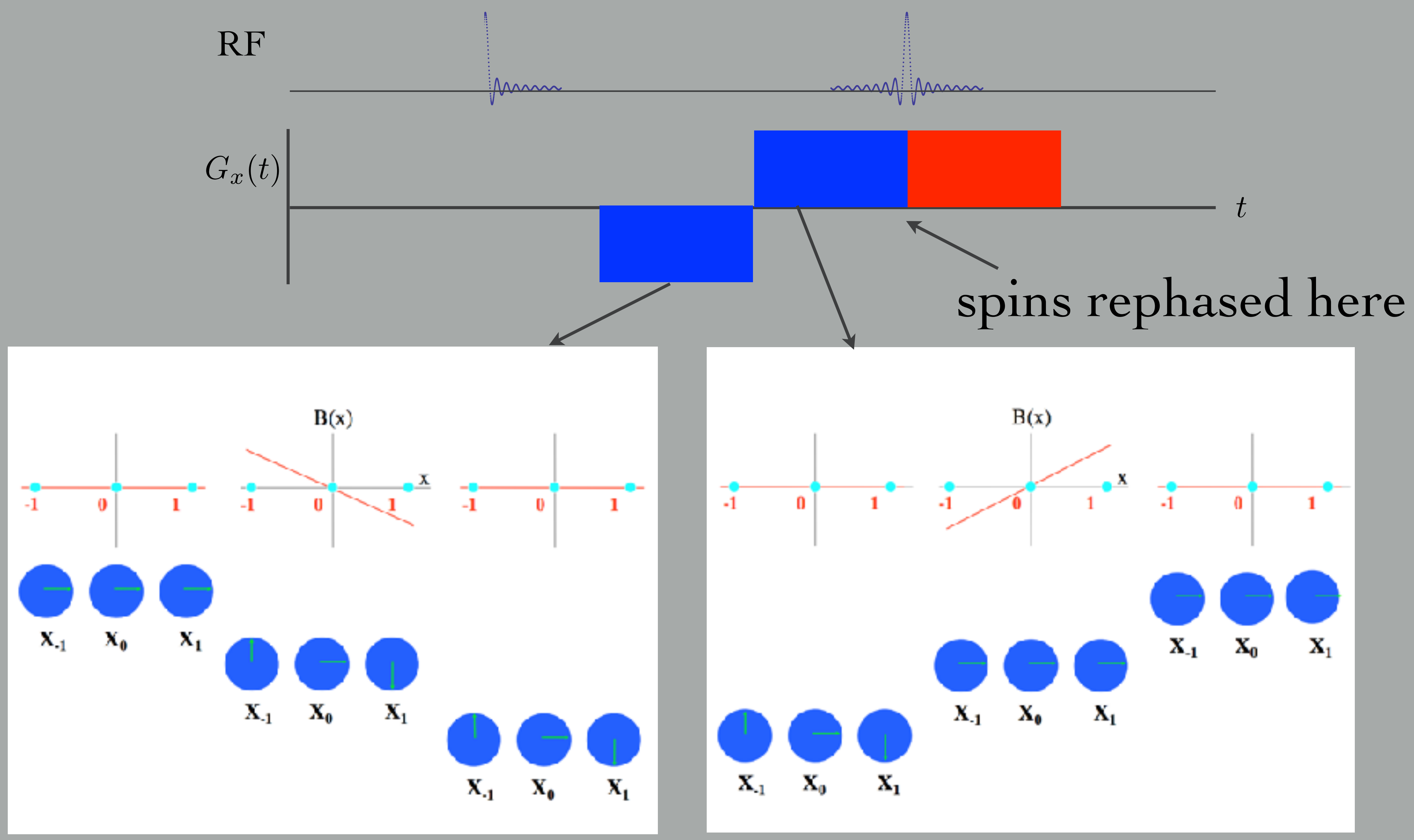
k-space trajectory



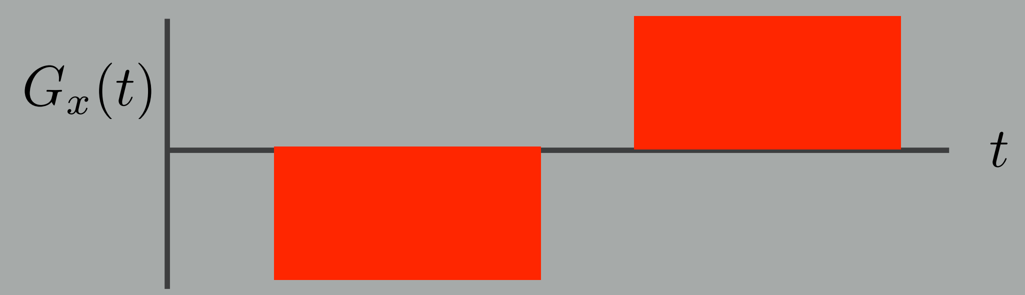
"Phase encoding"

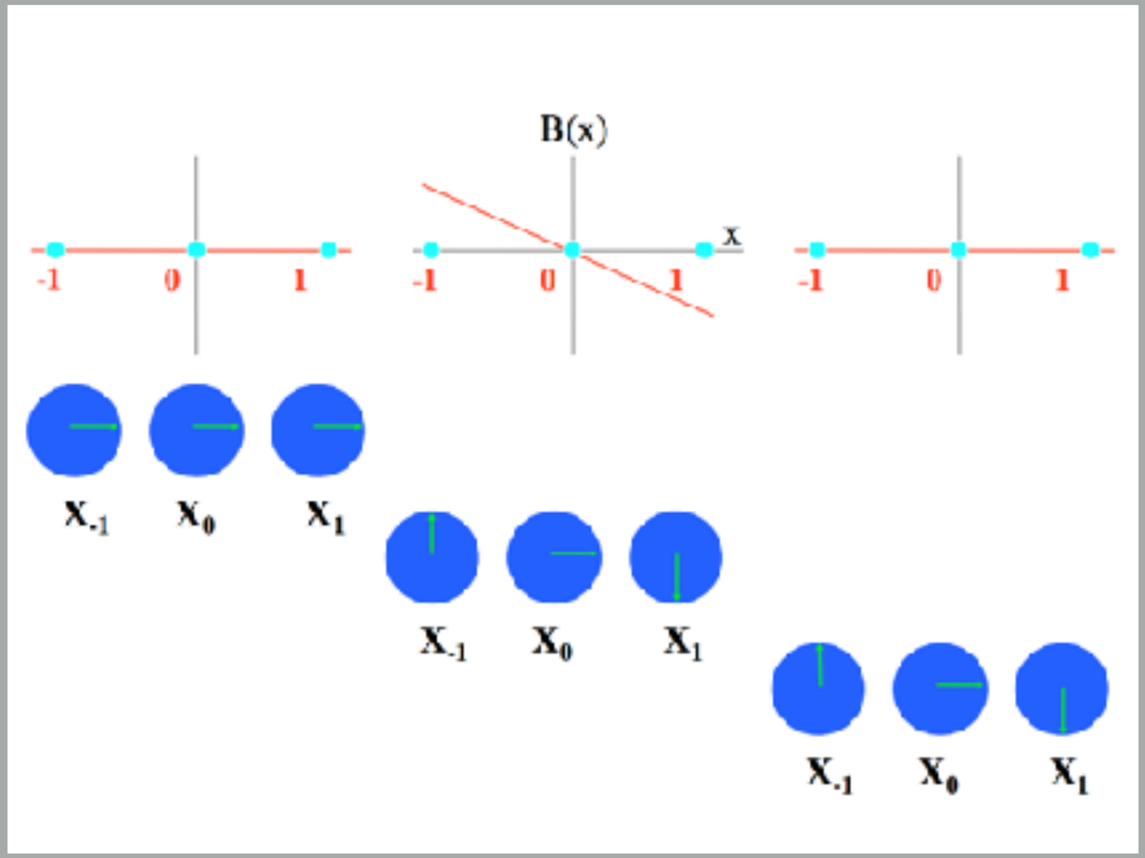
"Frequency encoding"

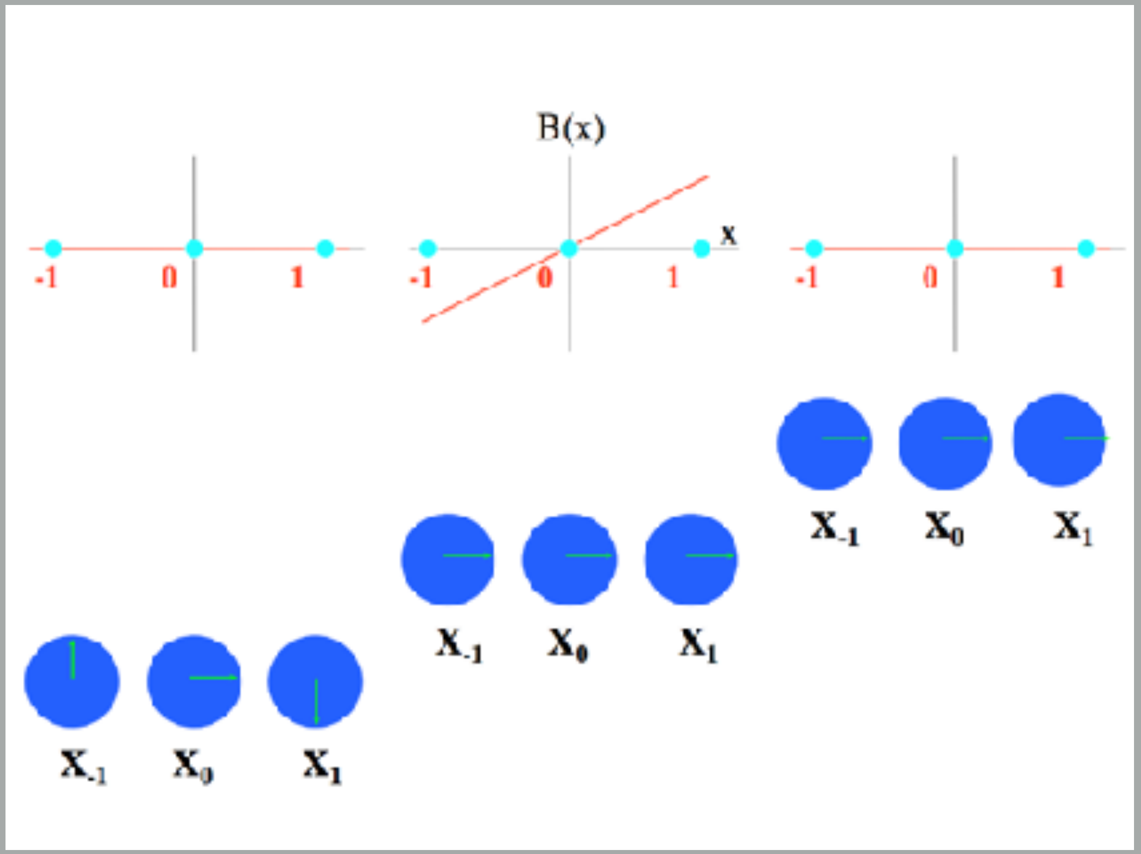
The gradient echo



Diffusion Preview: The Bipolar Gradient

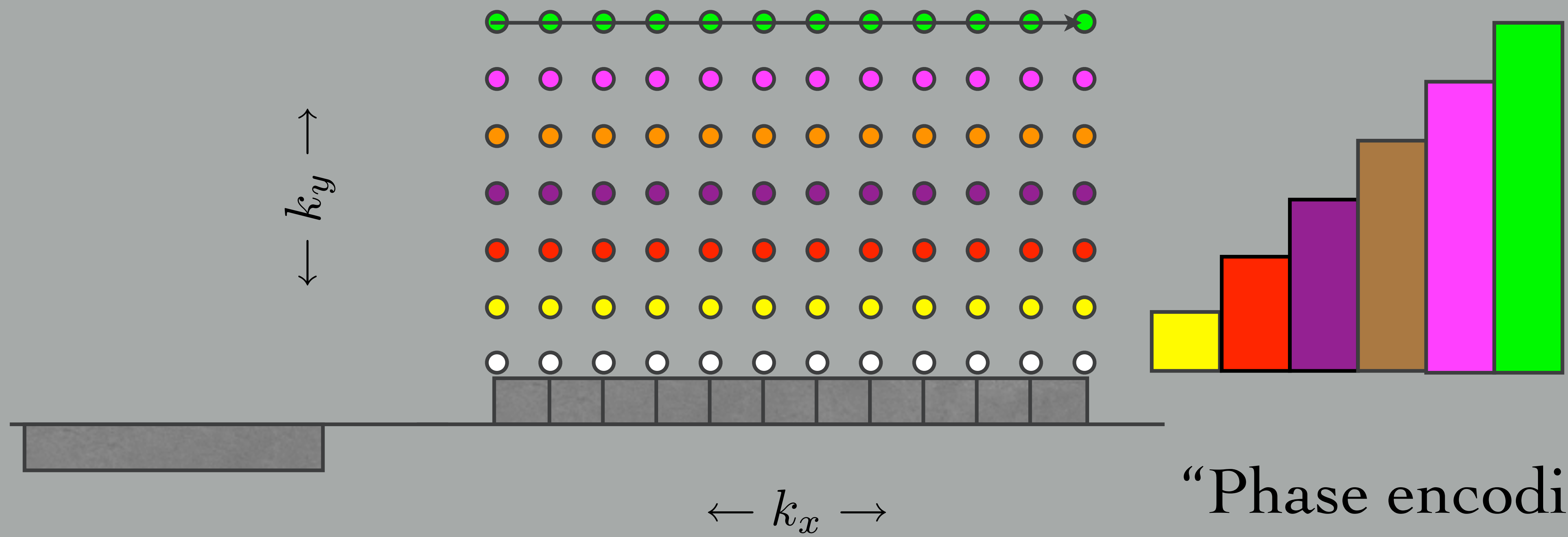






This is NOT true in diffusion!

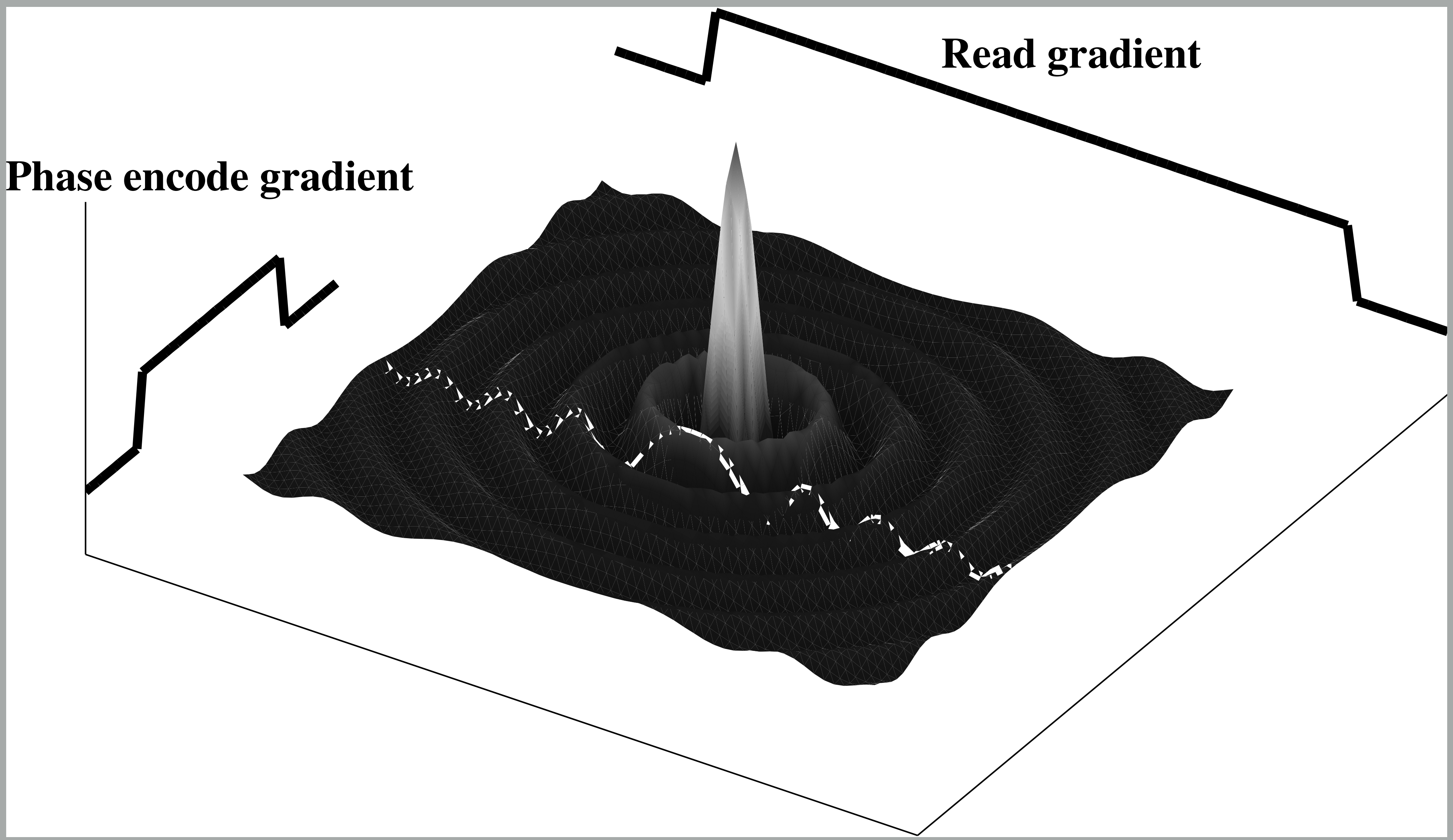
k-space trajectory



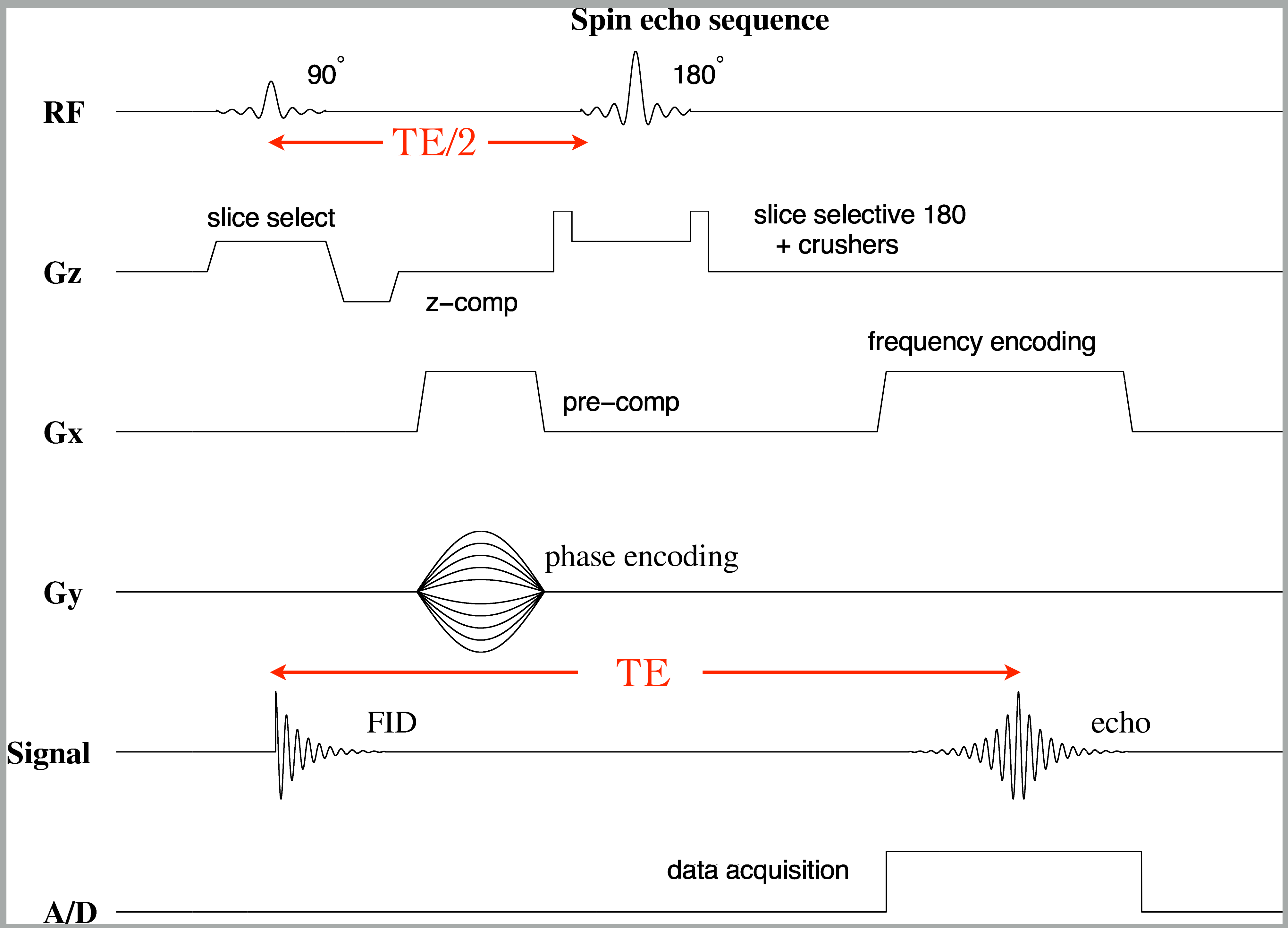
“Phase encoding”

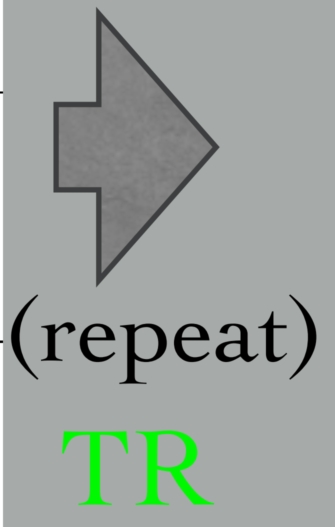
“Frequency encoding”

k-space trajectory



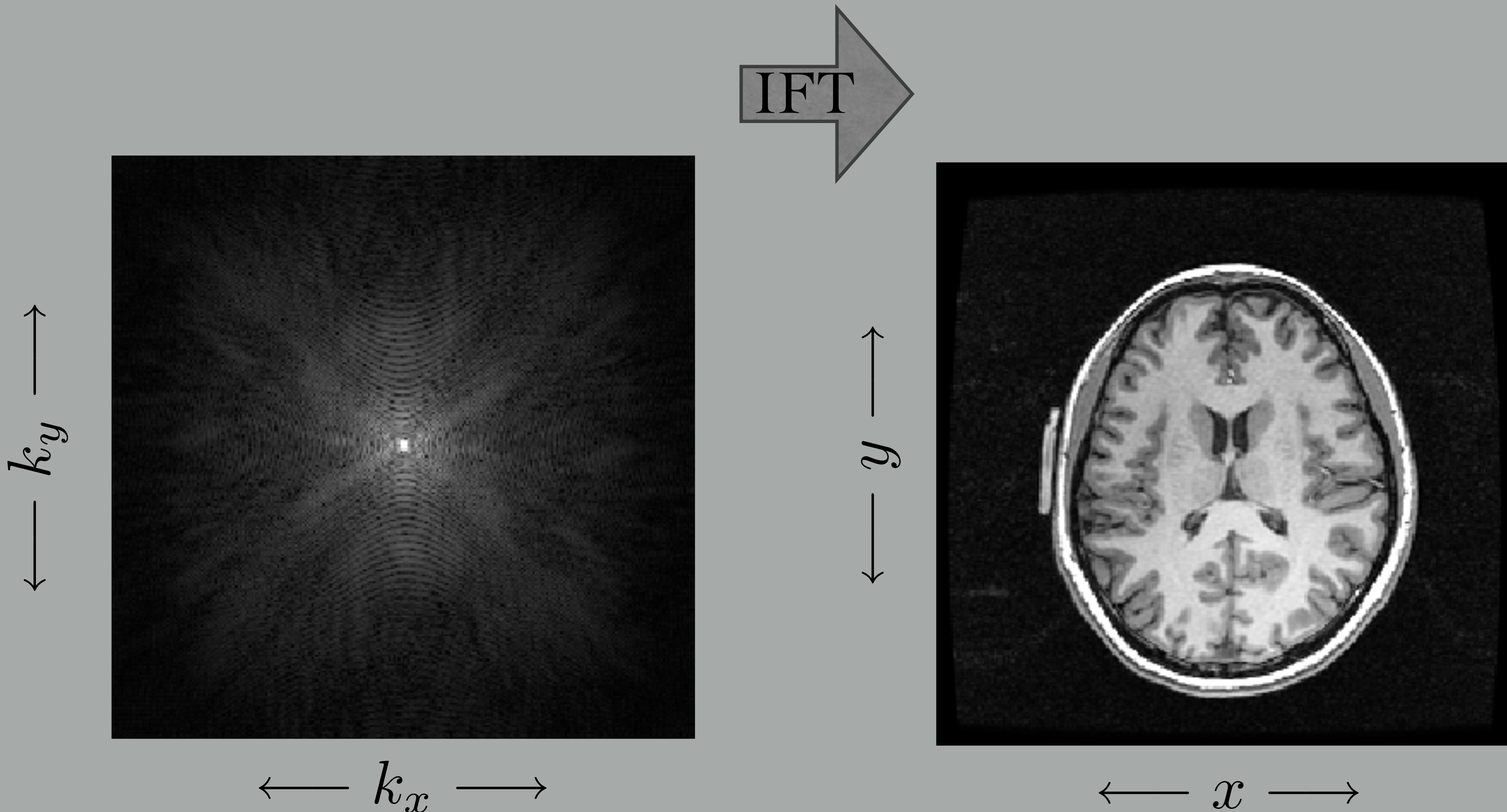
Spin echo pulse sequence





Centering the echo gives you great flexibility for contrast

MR image data



"Fourier" data

Image

The NMR signal

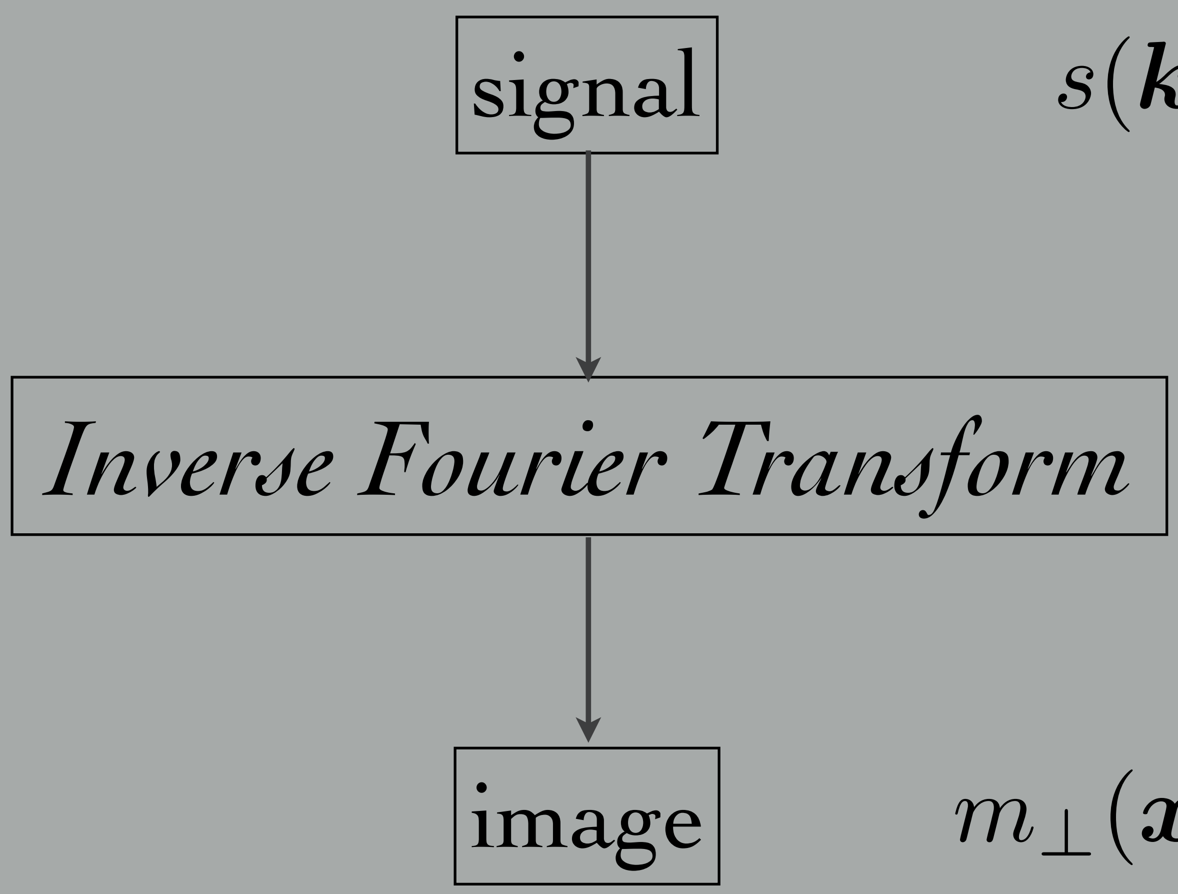
$$s(\boldsymbol{k}) = \int_{\Omega} m_{\perp}(\boldsymbol{x}, t) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \, d\boldsymbol{x}$$

The signal is the *Fourier Transform*
of the transverse magnetization

For static tissue (and perfect scanner)

$$m_{\perp}(\boldsymbol{x}, t) = m_{\perp}(\boldsymbol{x})$$

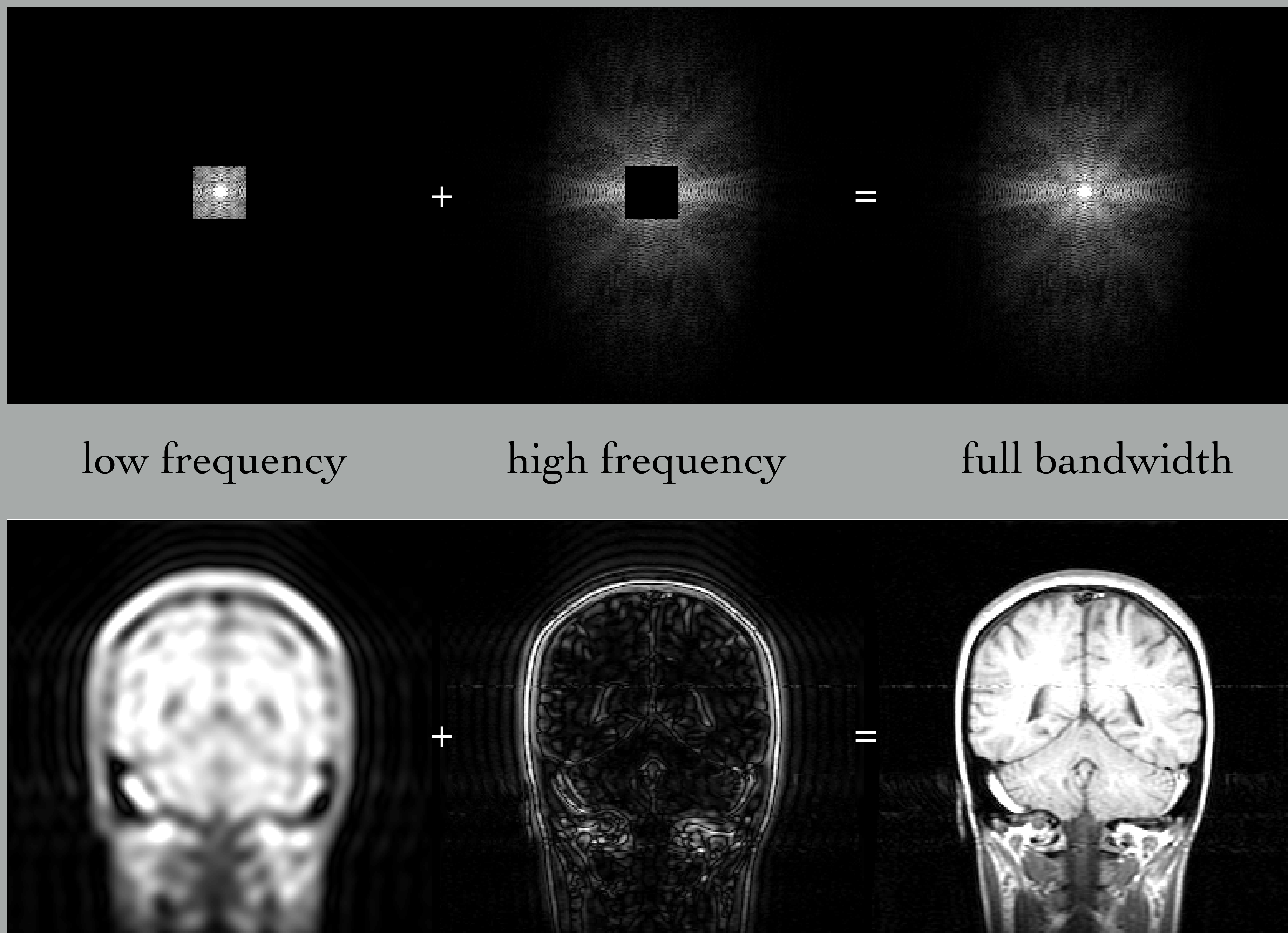
MRI data and image



$$s(\boldsymbol{k}) = \int_{\Omega} m_{\perp}(\boldsymbol{x}) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{x}$$

$$m_{\perp}(\boldsymbol{x}) = \int s(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{k}$$

Anatomy of k-space



Resolution

$$\delta x = \frac{\pi}{k_{max}}$$

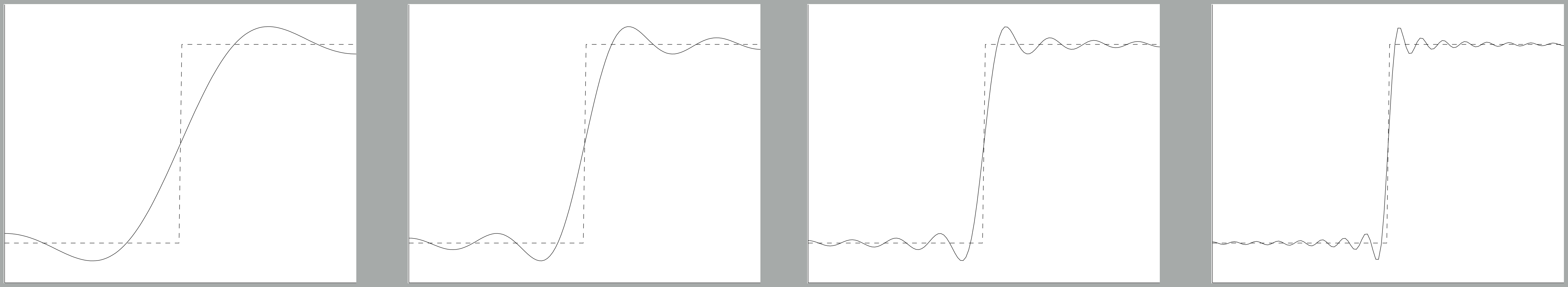
highest spatial frequency
determines how well objects can be resolved

Field of view

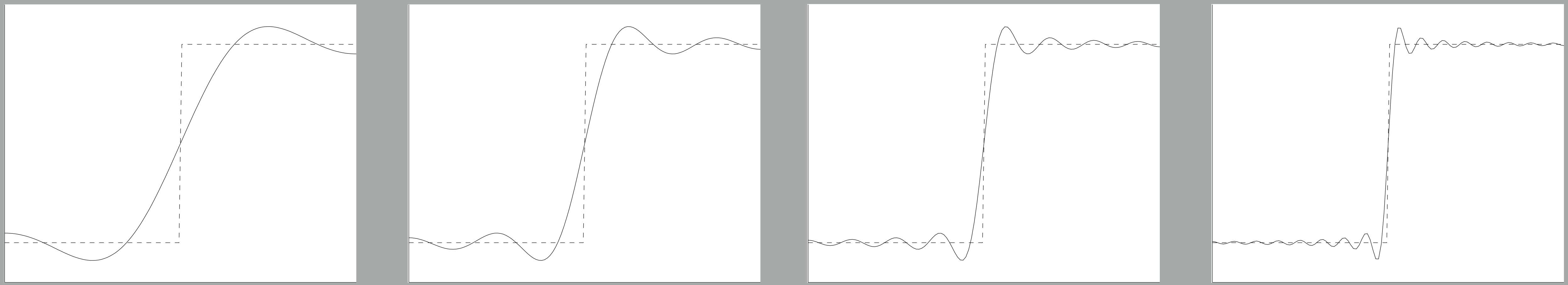
$$FOV = \frac{2\pi}{\Delta k}$$

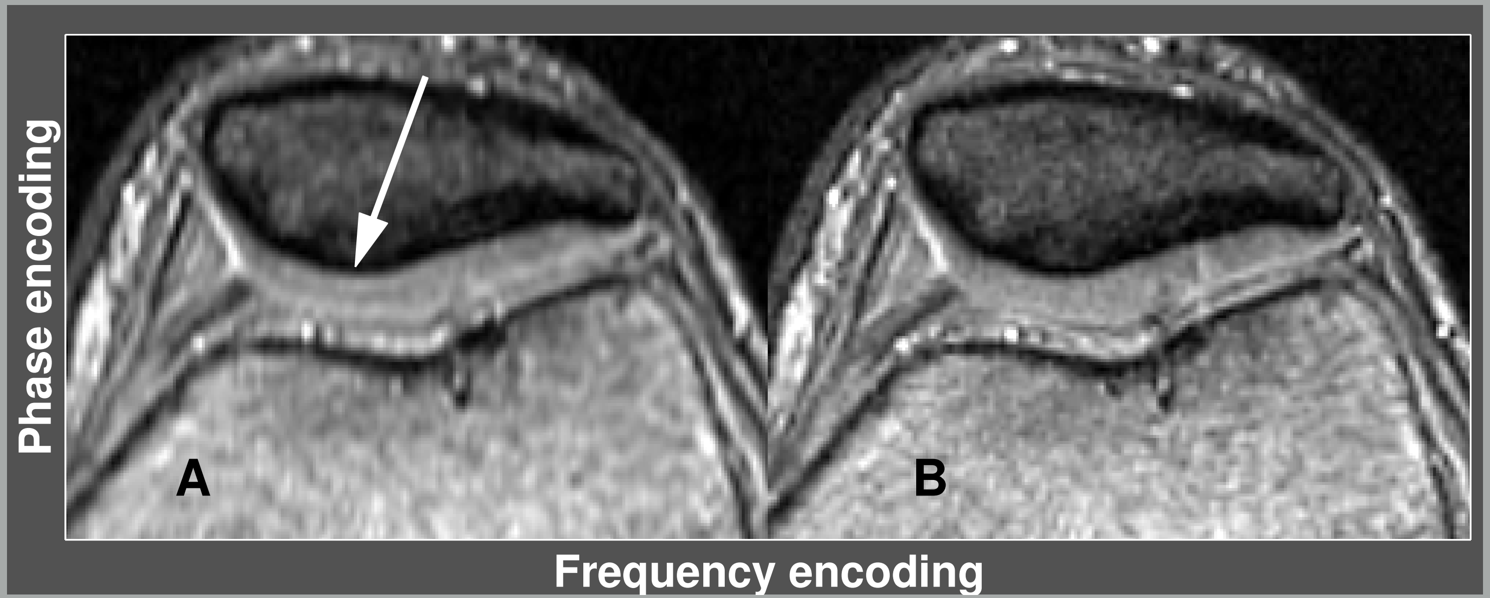
Lowest spatial frequency determines extent of image

Fourier's Theorem



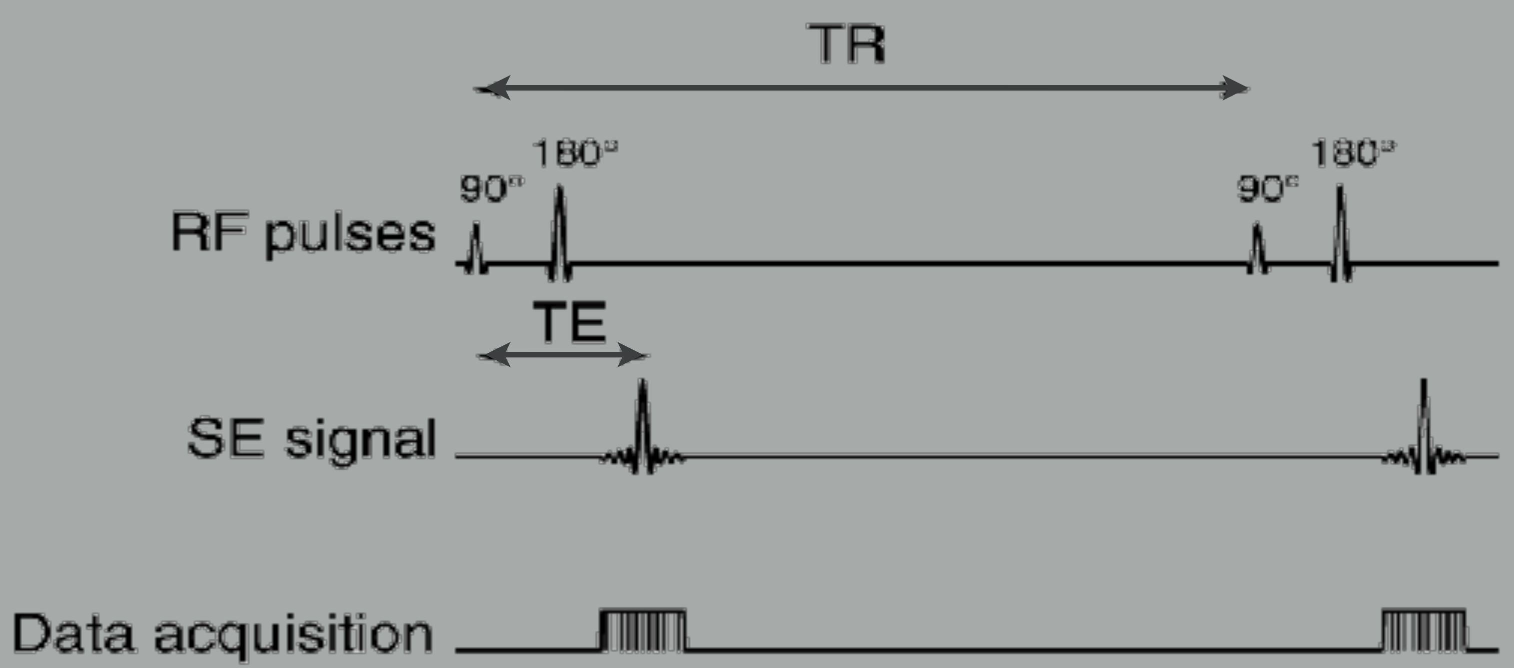
Truncation Artifact

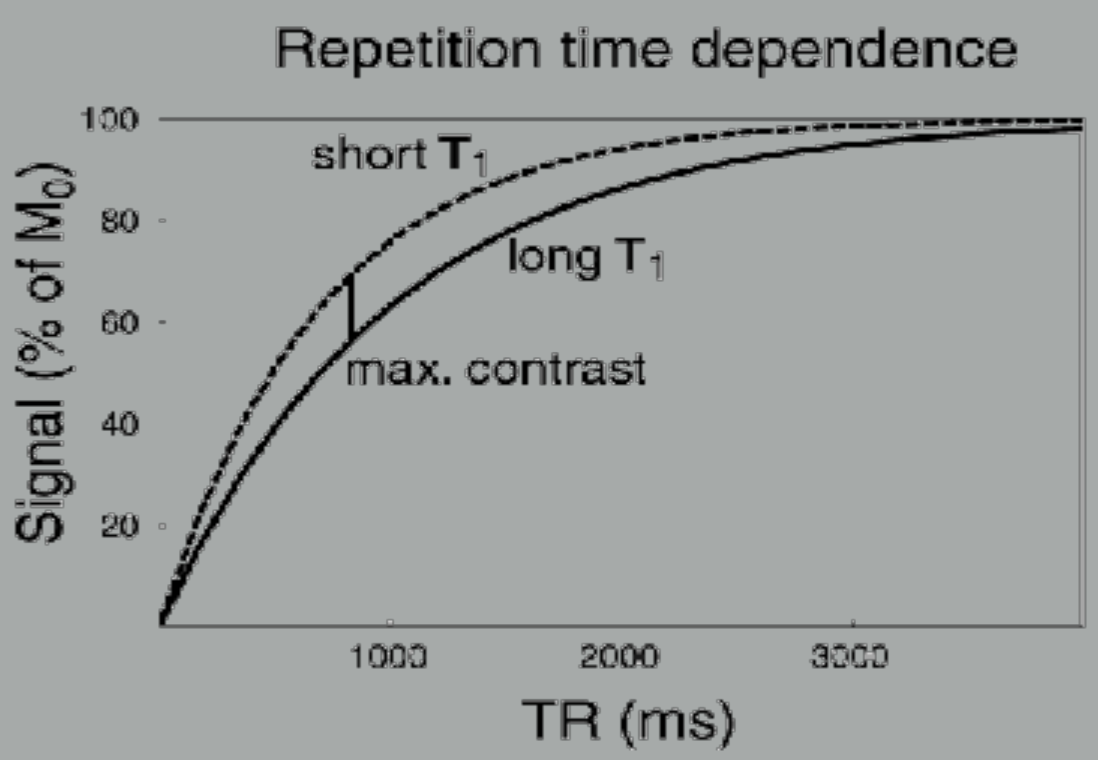


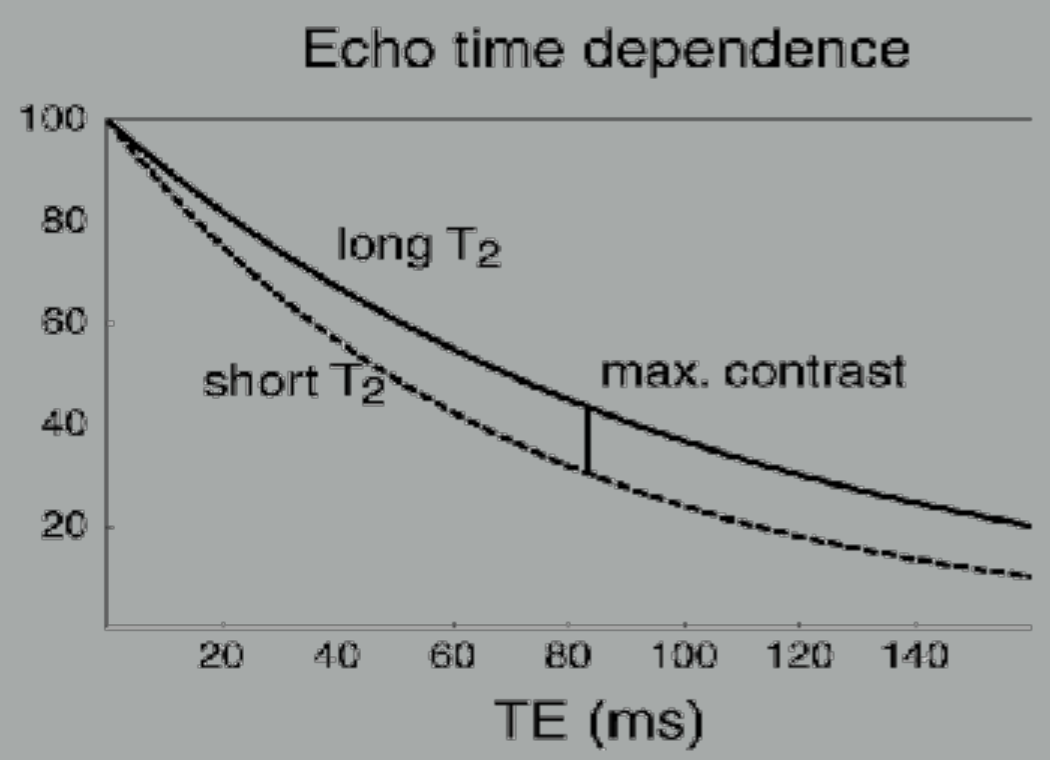


(A) 256 x 128 (B) 256 x 256

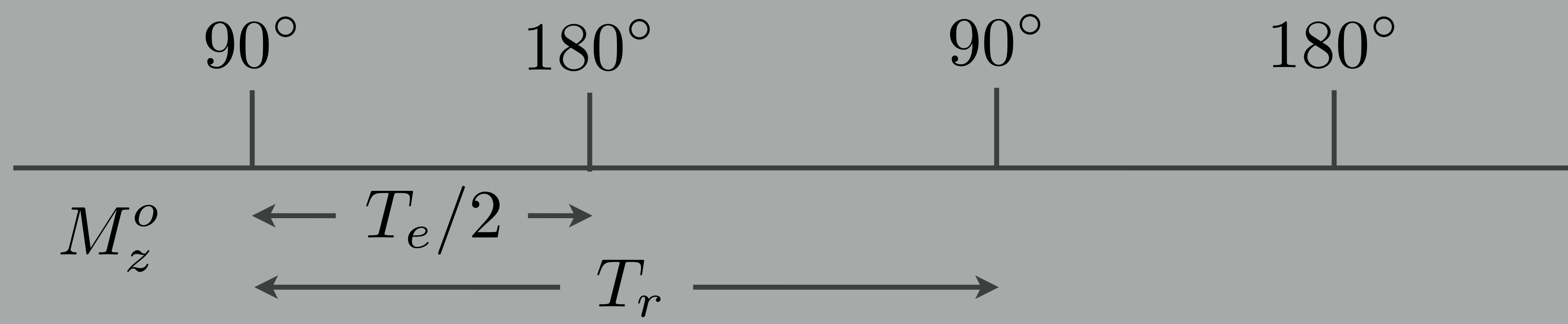
Spin Echo Contrast



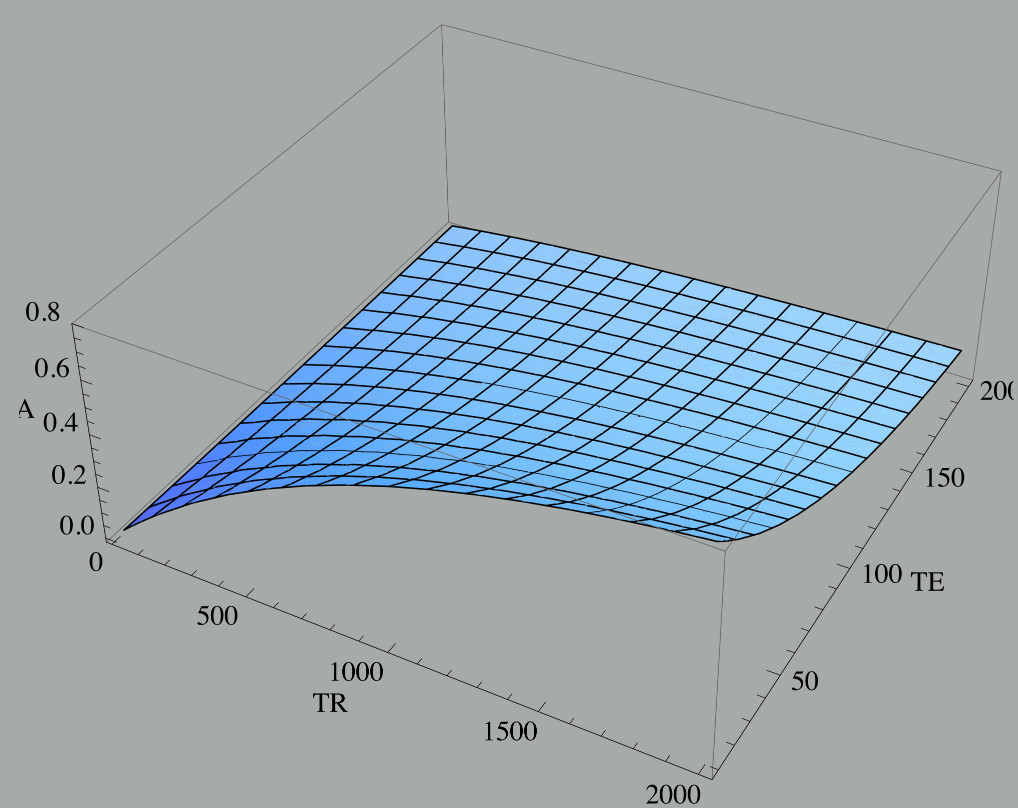


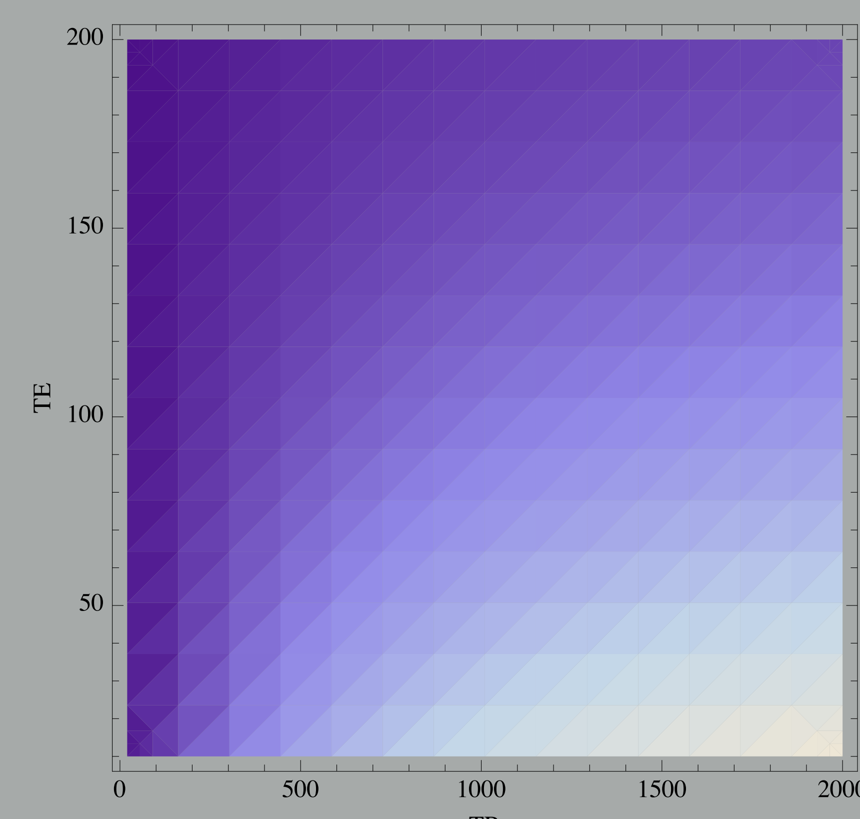


Spin Echo Contrast

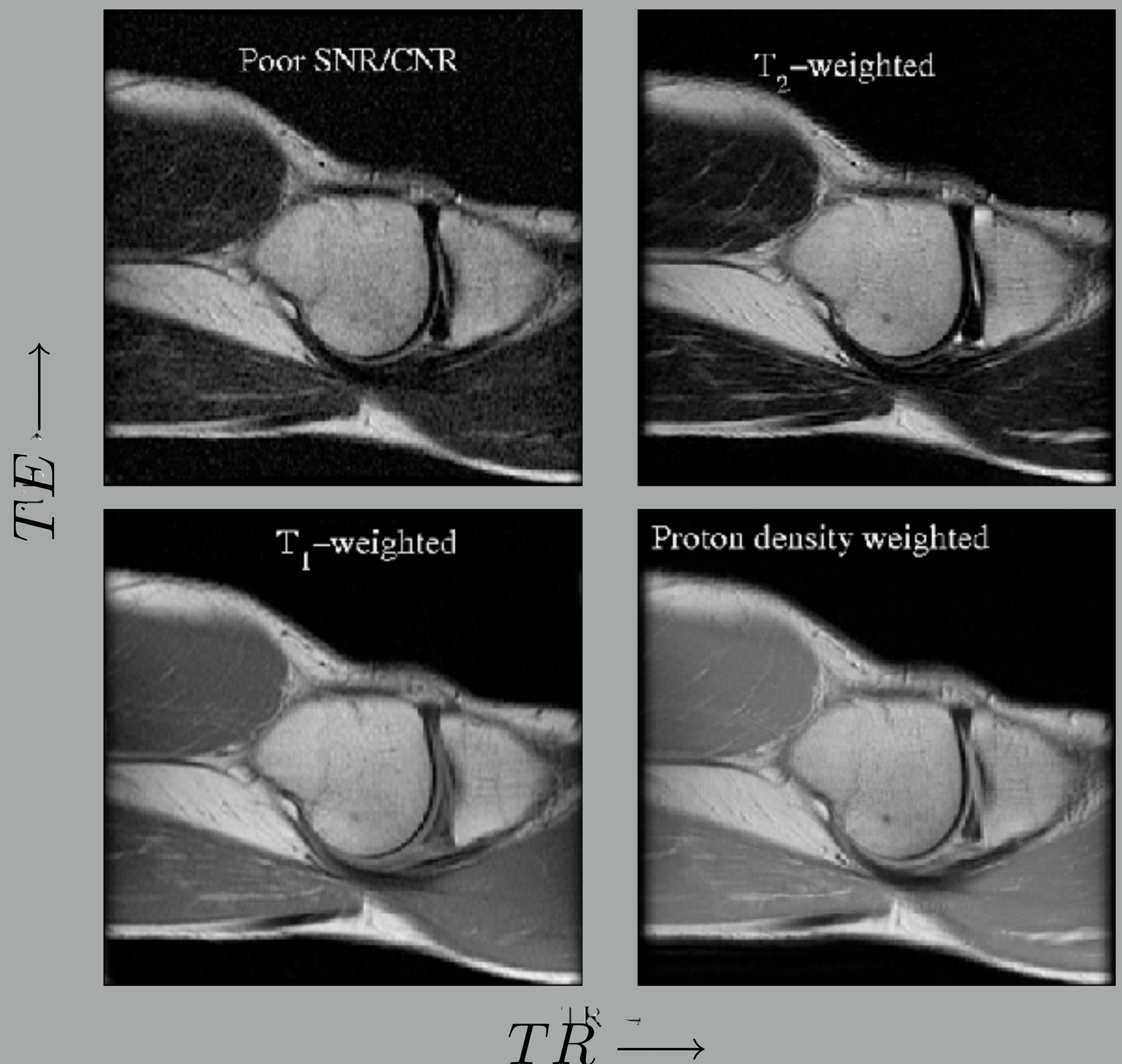


$$A \approx M_z^o(1 - e^{-T_r/T_1})e^{-T_e/T_2}$$





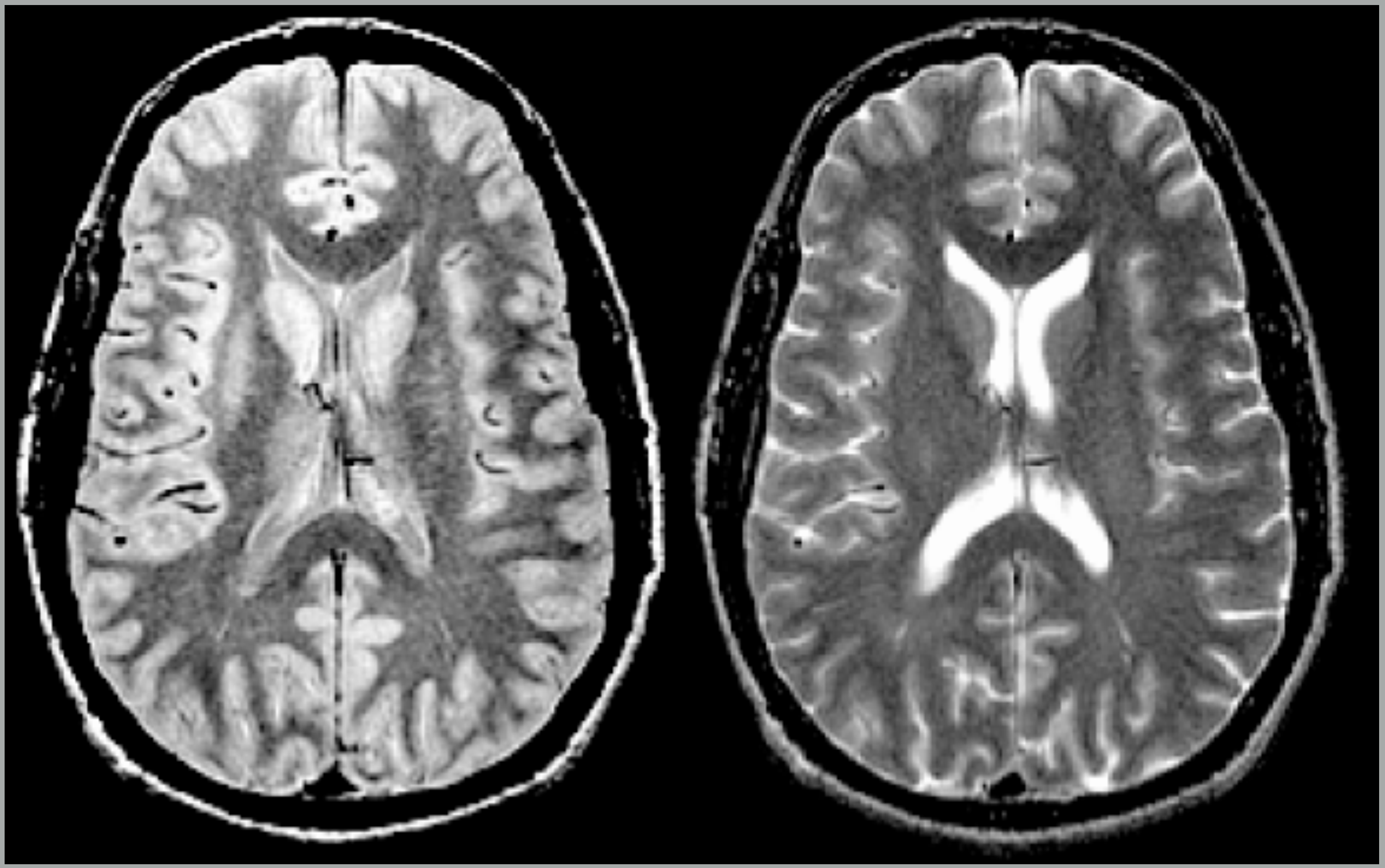
Spin Echo Contrast



Spin Echo Contrast

T1-weighted density weighted T2-weighted

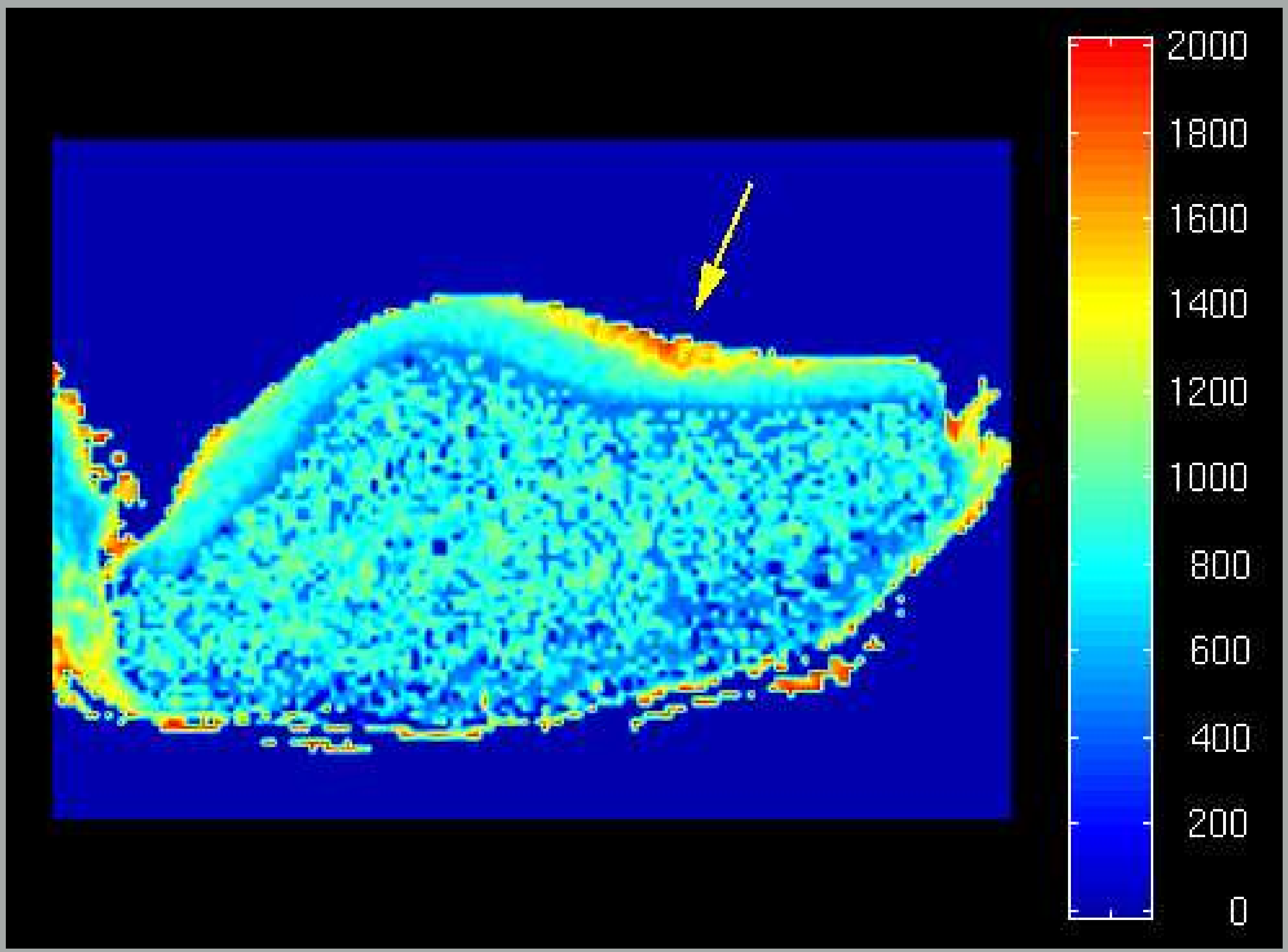
Relaxation Contrast



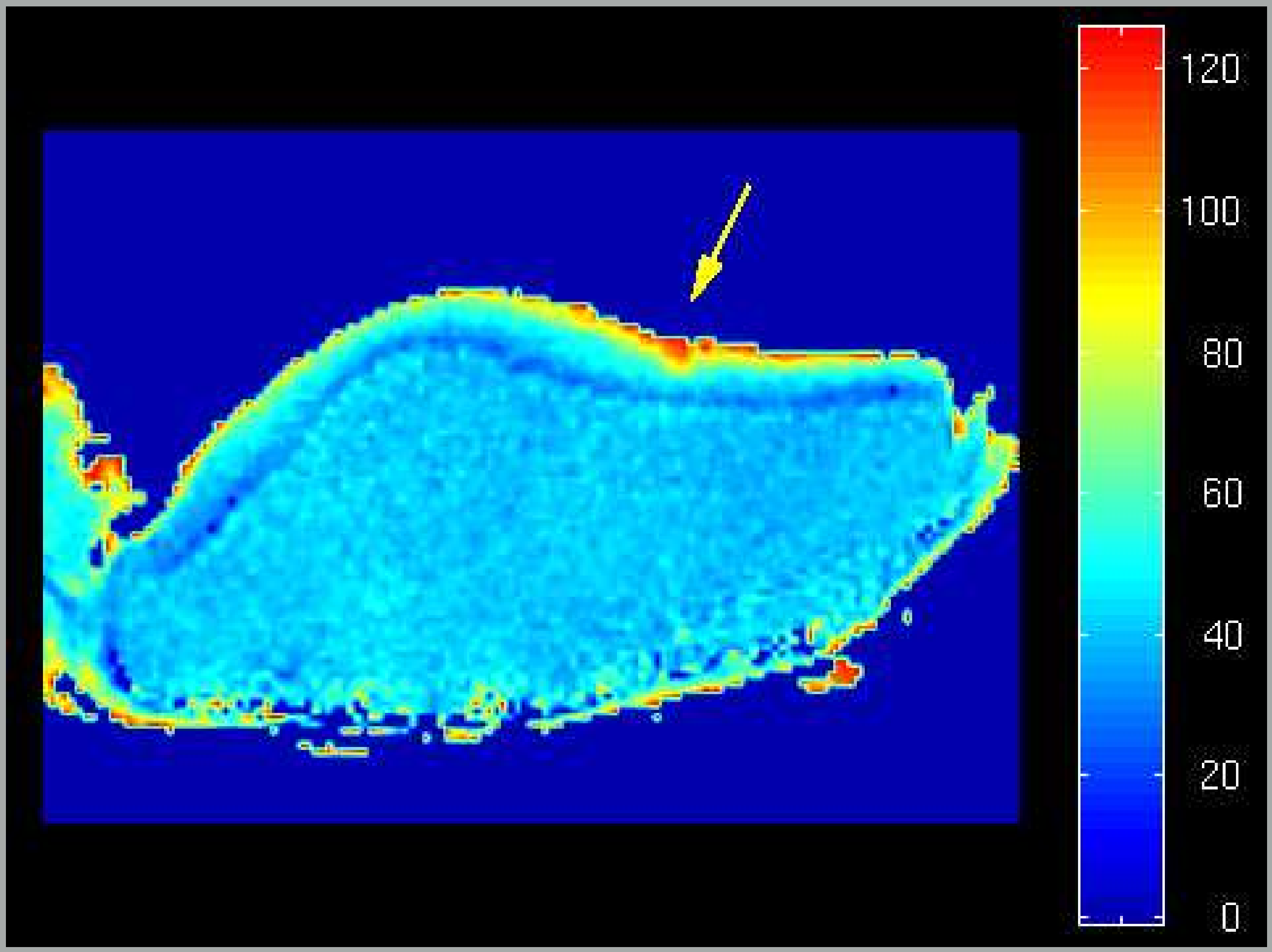
short TE long TE

The MR signal depends on the local relaxation time (T2) and the delay (TE) between excitation and data collection

Mapping relaxation rates directly

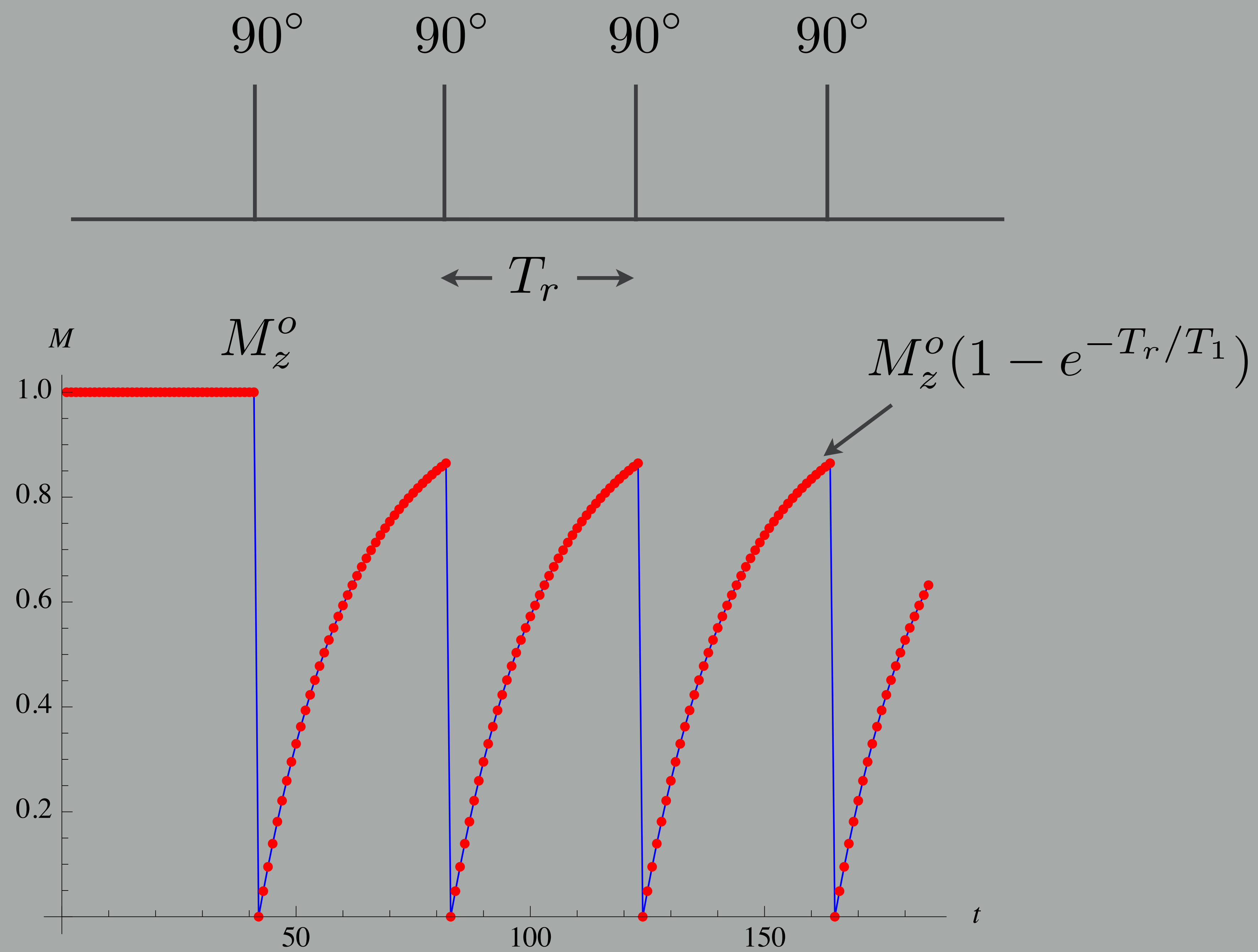


T1 map



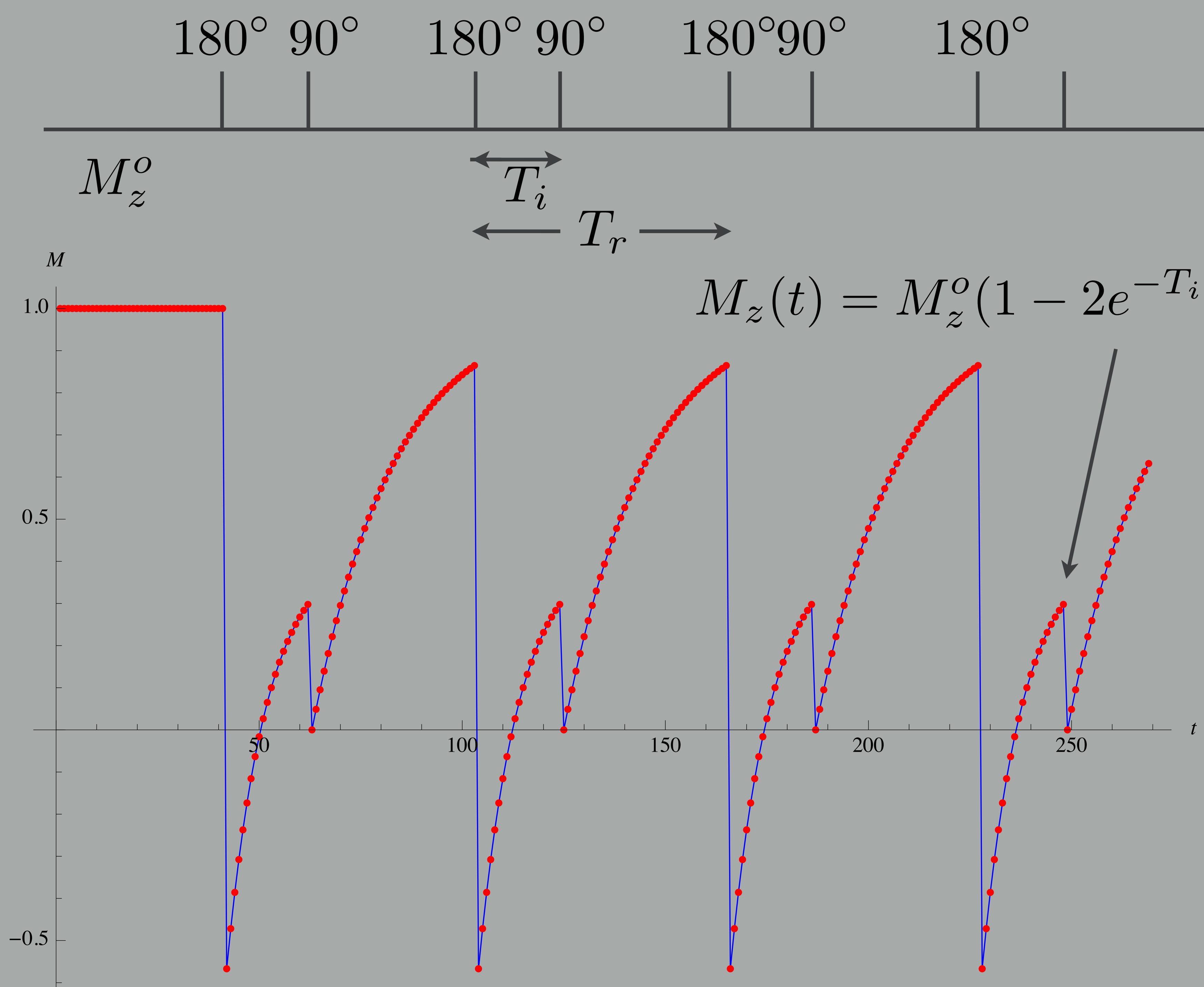
T2 map

Saturation Recovery



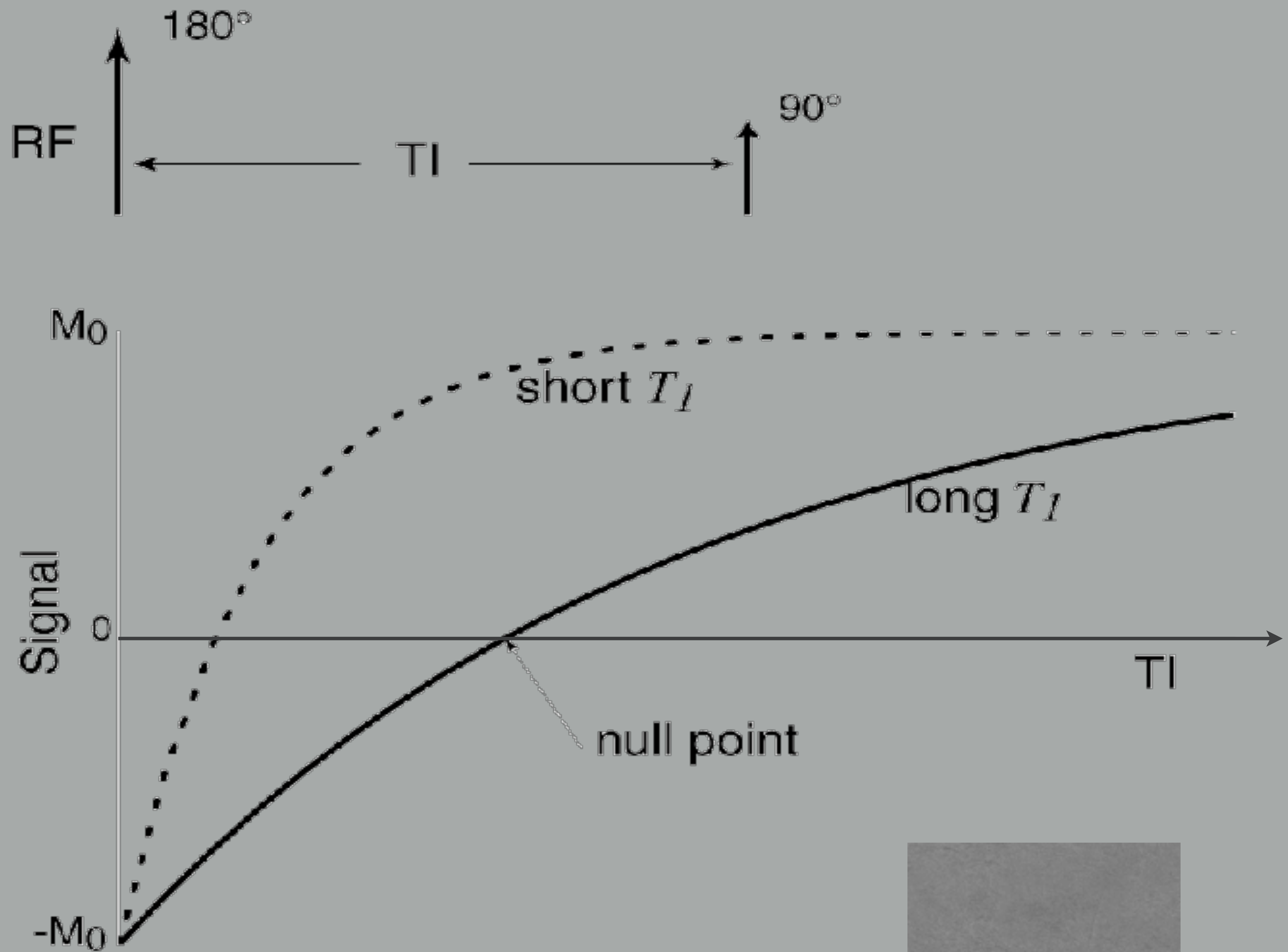
recovery curve for longitudinal magnetization

Inversion Recovery Sequence



$$M_z(t) = M_z^o(1 - 2e^{-T_i/T_1} + e^{-T_r/T_1})$$

Inversion Recovery Sequence



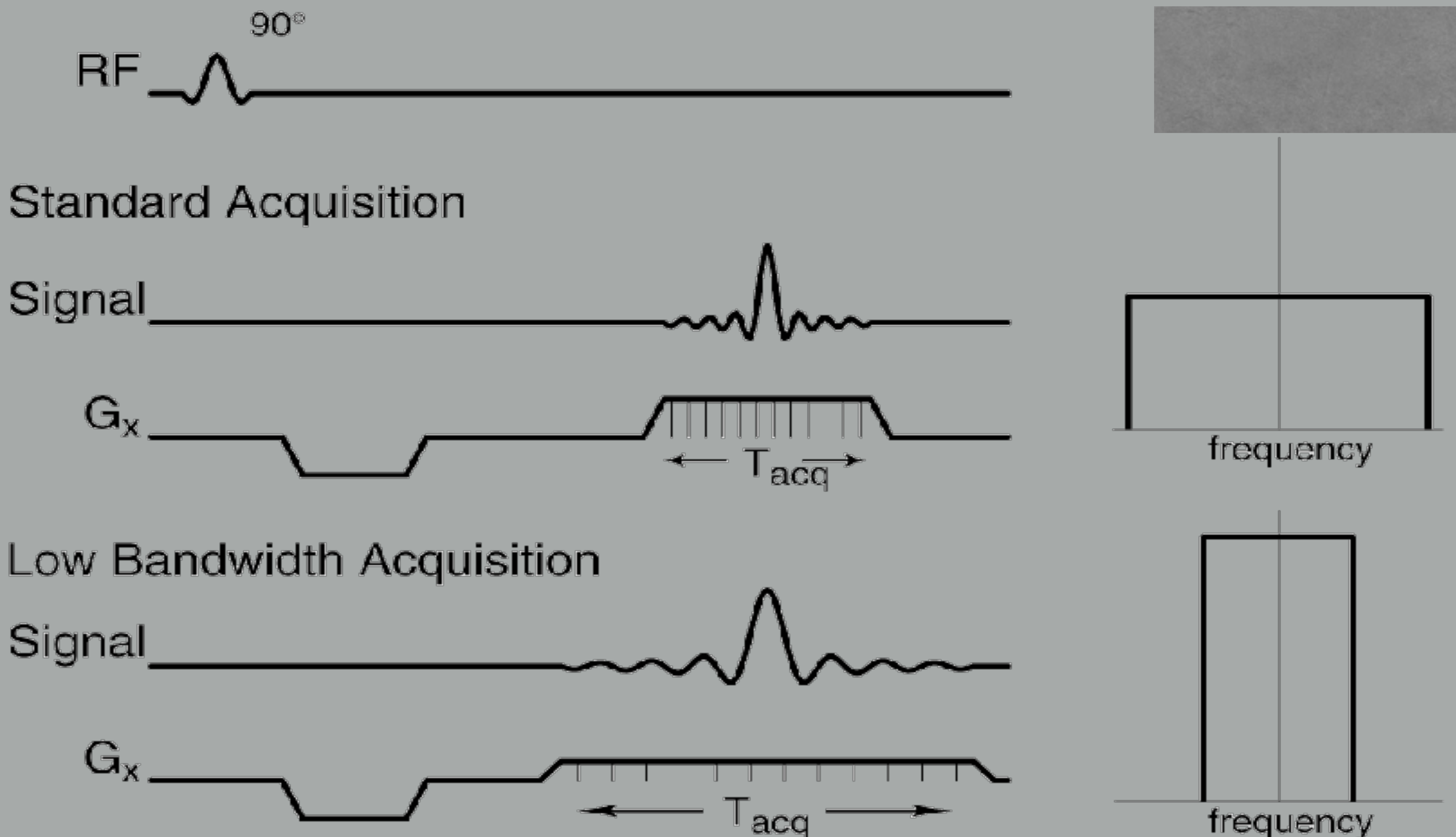
Signal-to-Noise (SNR)

$$\mathrm{SNR} \propto V\sqrt{T}$$

V = voxel volume
T = total acquisition time

T = (#averages) x (#phase encoding steps) x (data acquisition time)

Bandwidth and SNR



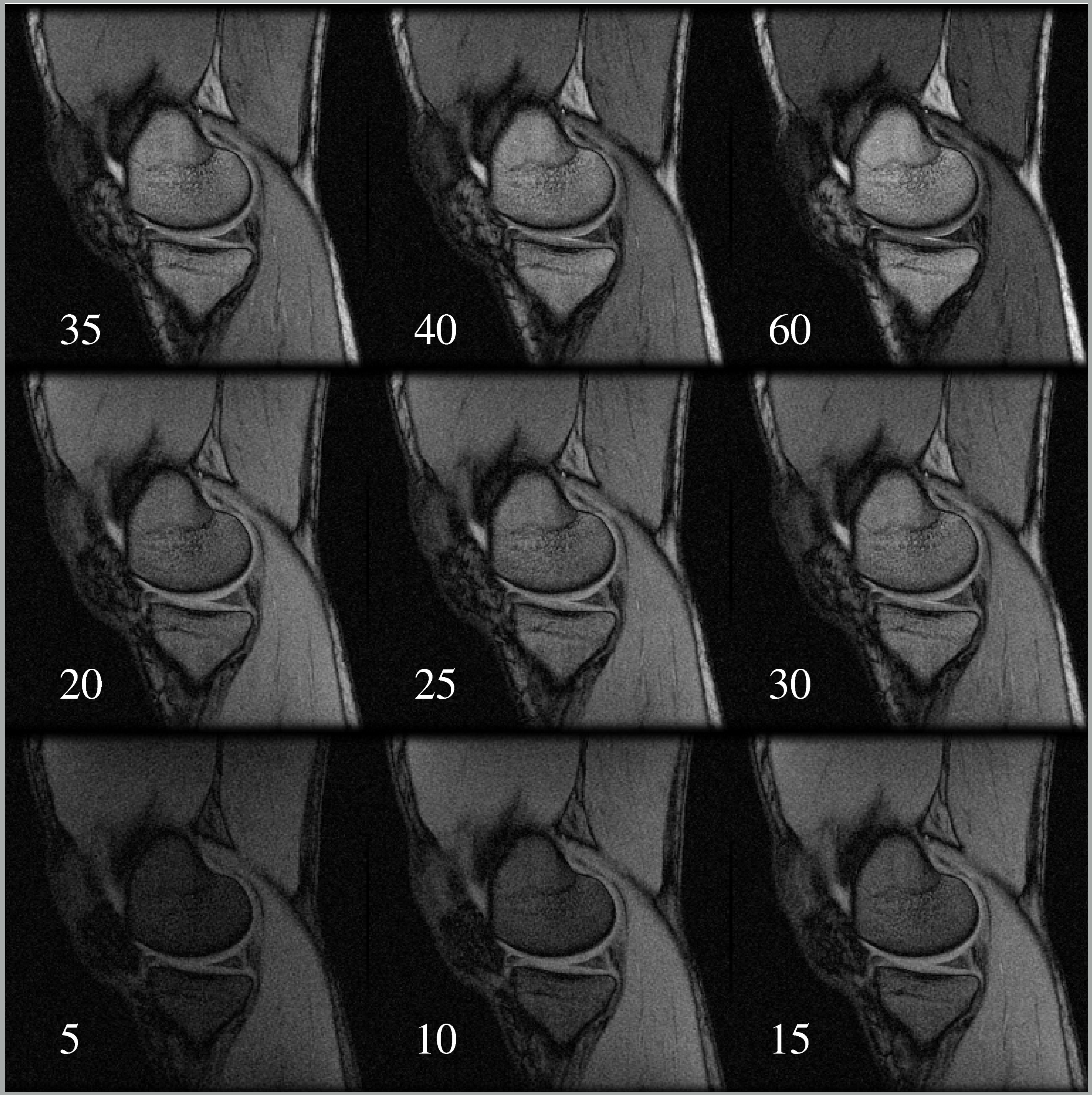
Gradient echo imaging

max signal at Ernst angle:

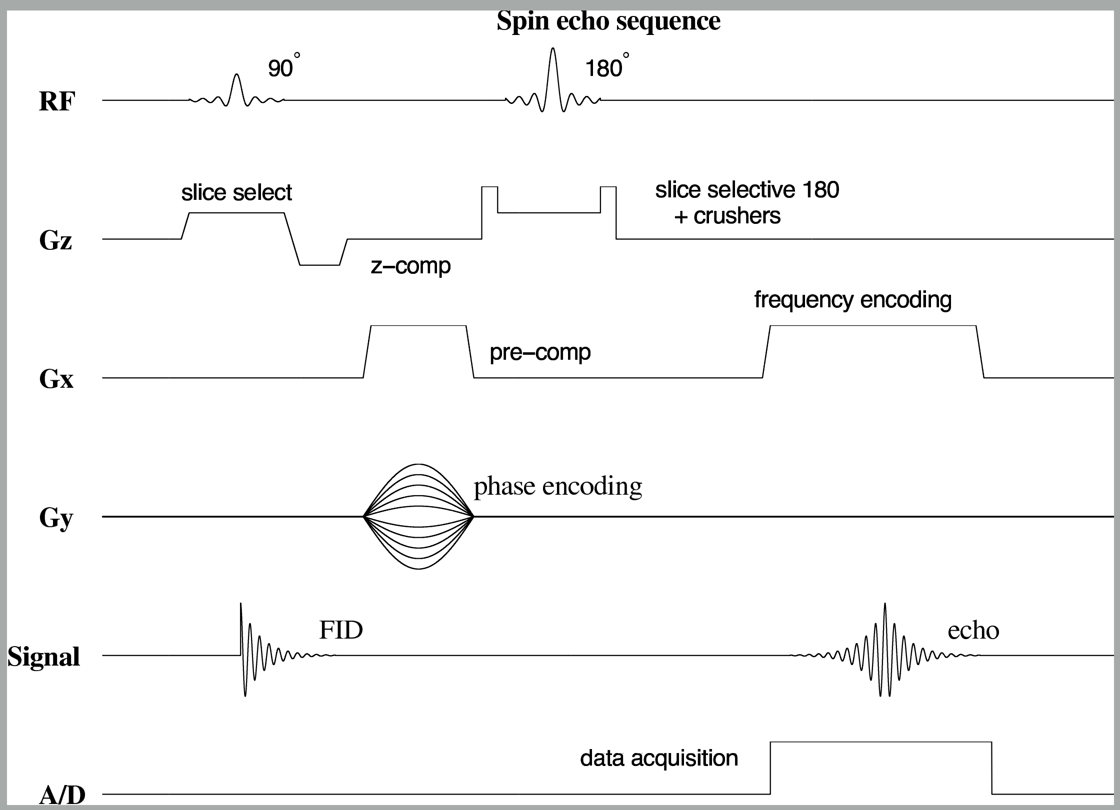
$$\cos\alpha = e^{-T_r/T_1}$$

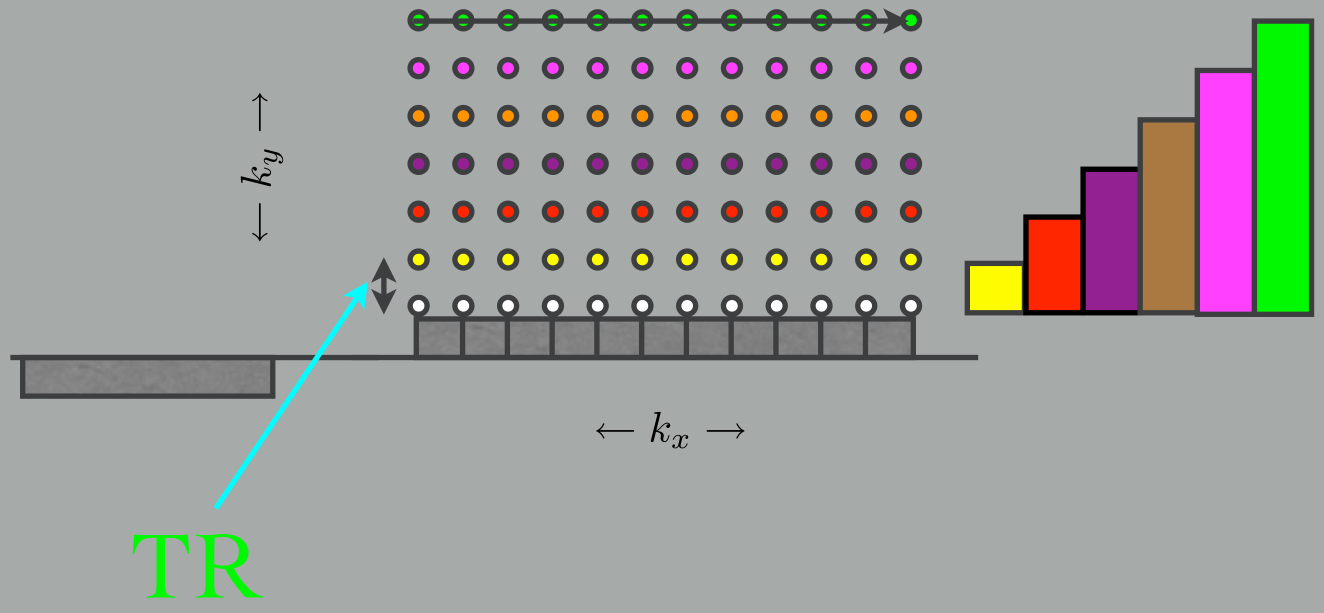
1. Maximum signal at Ernst angle
2. Contrast mediated by flip angle
3. Ernst angle is where spoiled and non-spoiled signal curves intersect
4. Ernst angle near point of maximum *contrast*
5. Below Ernst angle, both spoiled and non-spoiled sequences are relatively insensitive to T1, making them proton density weighted

Gradient echo flip angle dependence

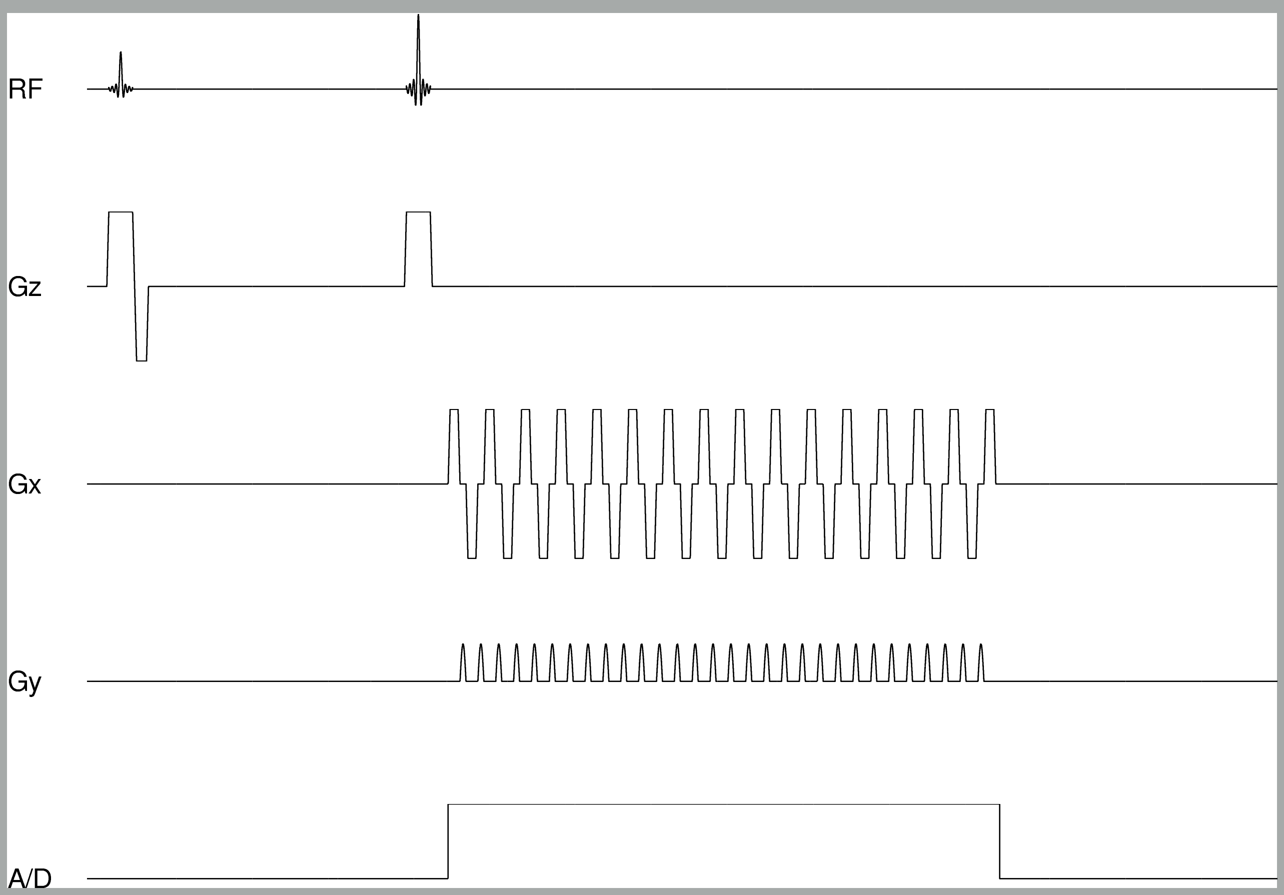


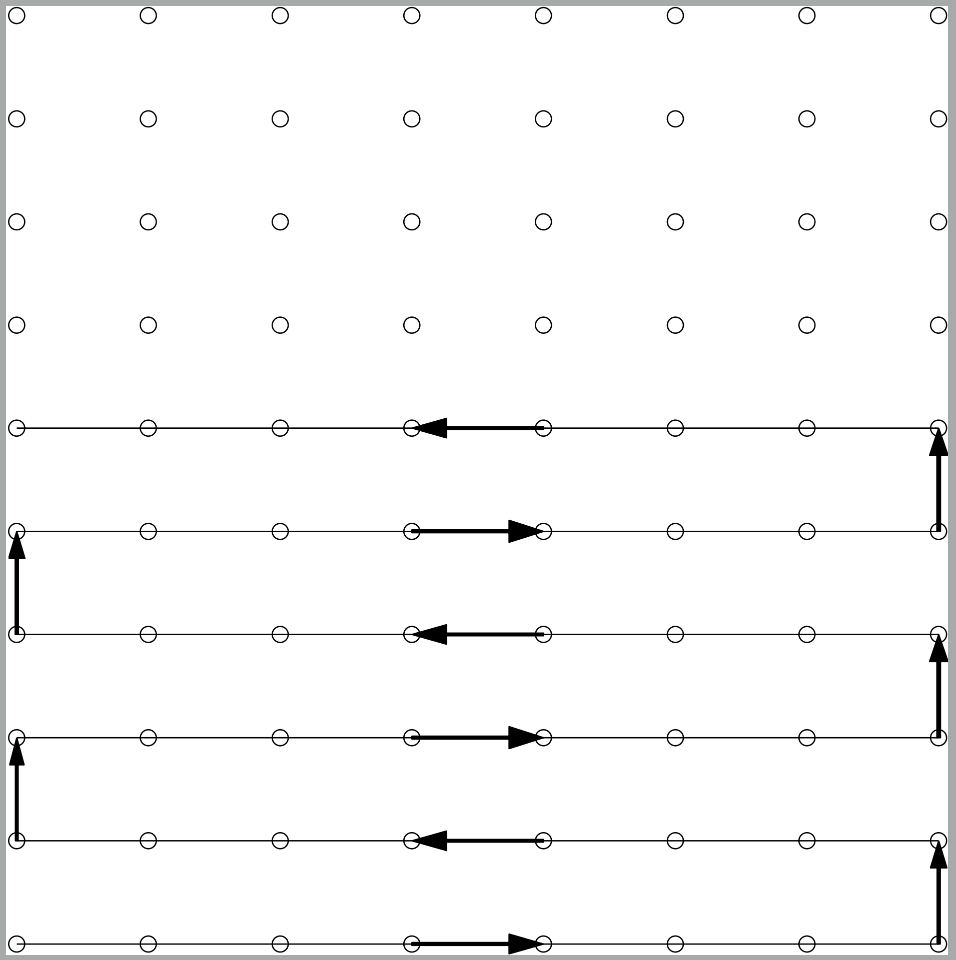
How do you mitigate the effects of motion?



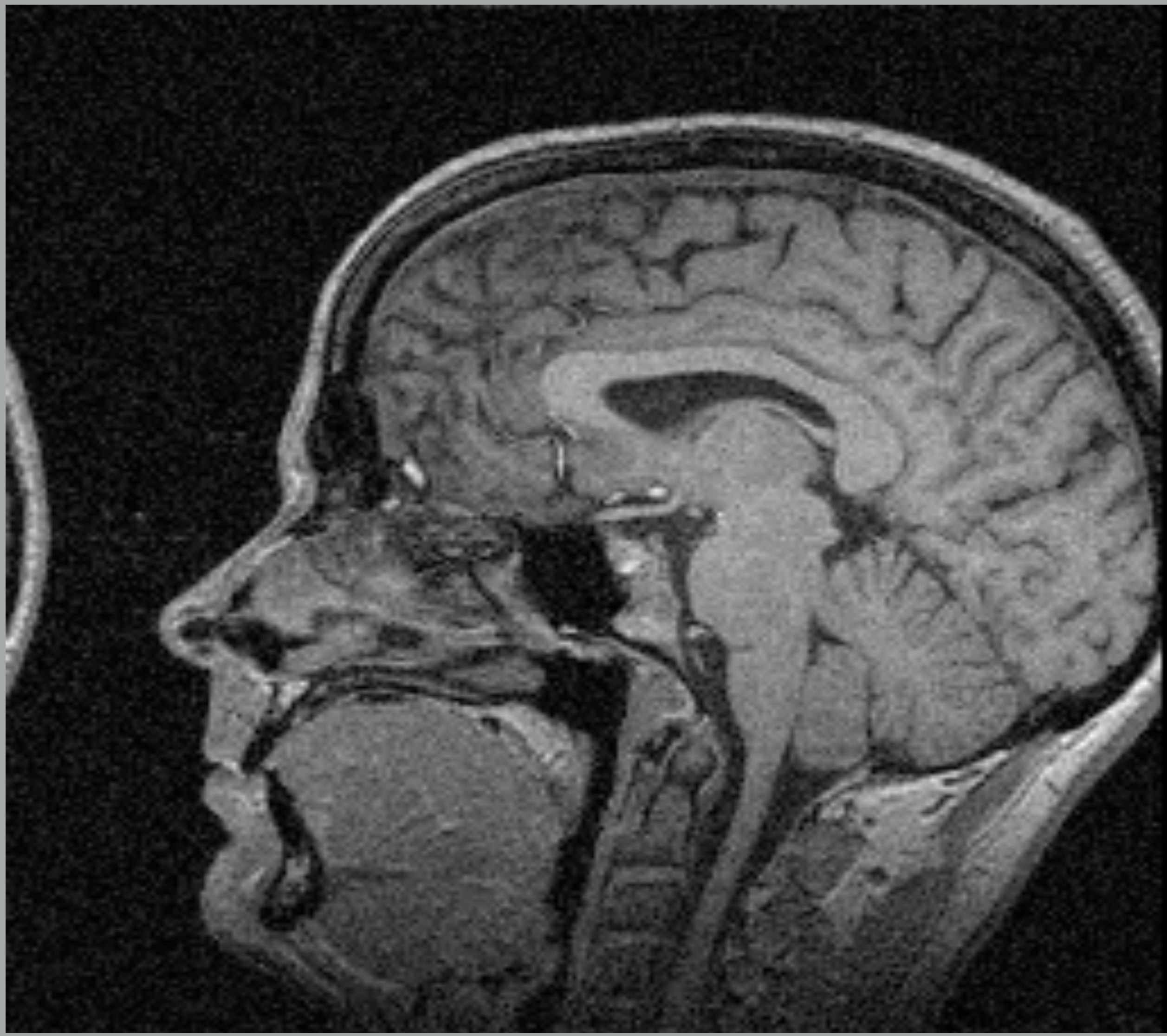


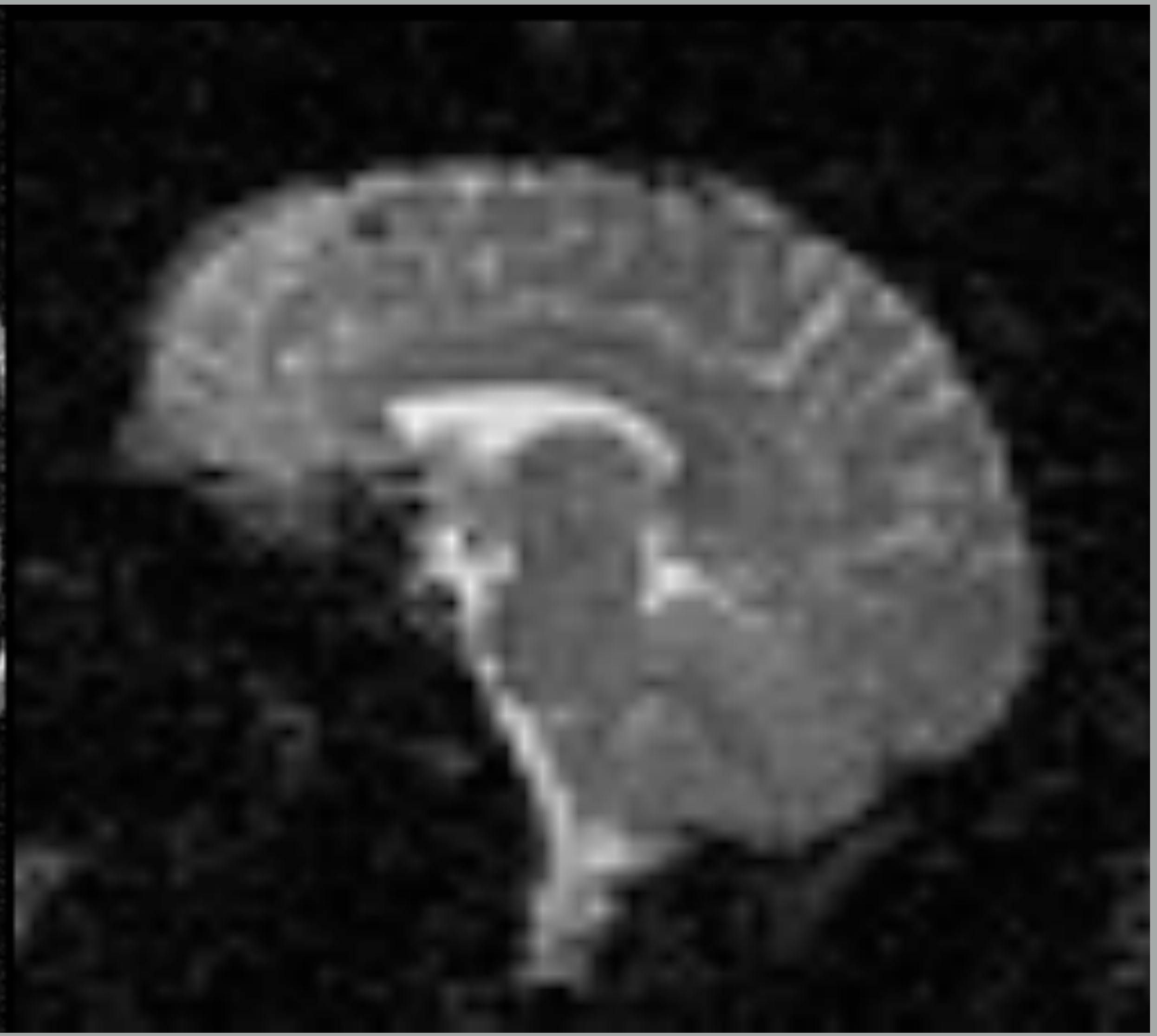
Echo Planar Imaging (EPI)





Echo Planar Imaging (EPI)





MP-RAGE
Voxel volume: 1 mm^3
Imaging time: 6 min

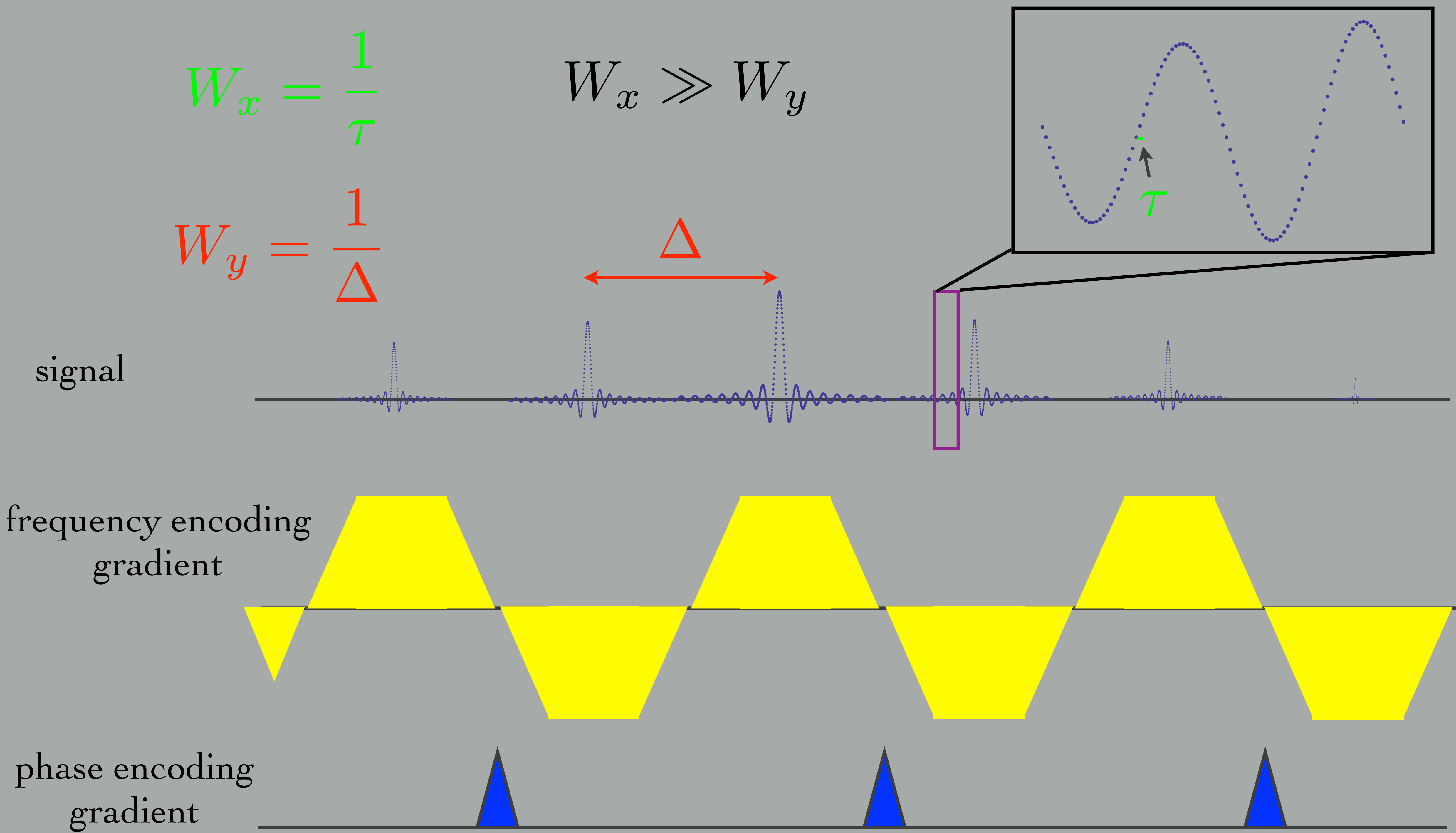
EPI
Voxel volume: 45 mm^3
Imaging time: 60 msec

EPI Bandwidths

$$W_x = \frac{1}{\tau}$$

$$W_x \gg W_y$$

$$W_y = \frac{1}{\Delta}$$



EPI Bandwidths

Pixel shift in *r*

$$\Delta r = \left(\frac{\delta B_o}{W_r} \right) F_r$$

ratio of frequency offset to bandwith

$$\begin{aligned} \delta B_o &= \text{field offset} \\ W_r &= \text{bandwidth in r} \\ F_r &= \text{Field-of-view in r} \end{aligned}$$

EPI Bandwidths

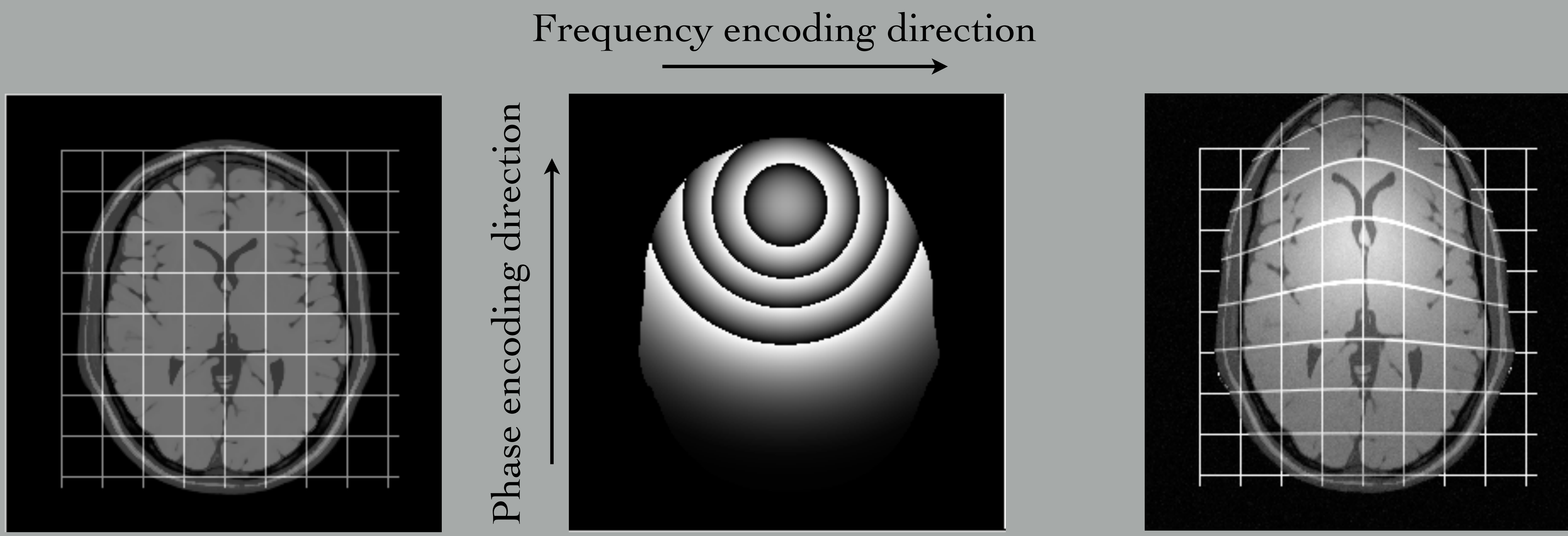
$$\Delta x = \left(\frac{\delta B_o}{W_x} \right) F_x = \tau \delta B_o F_x$$

$$\Delta \gg \tau \rightarrow \Delta y \gg \Delta x$$

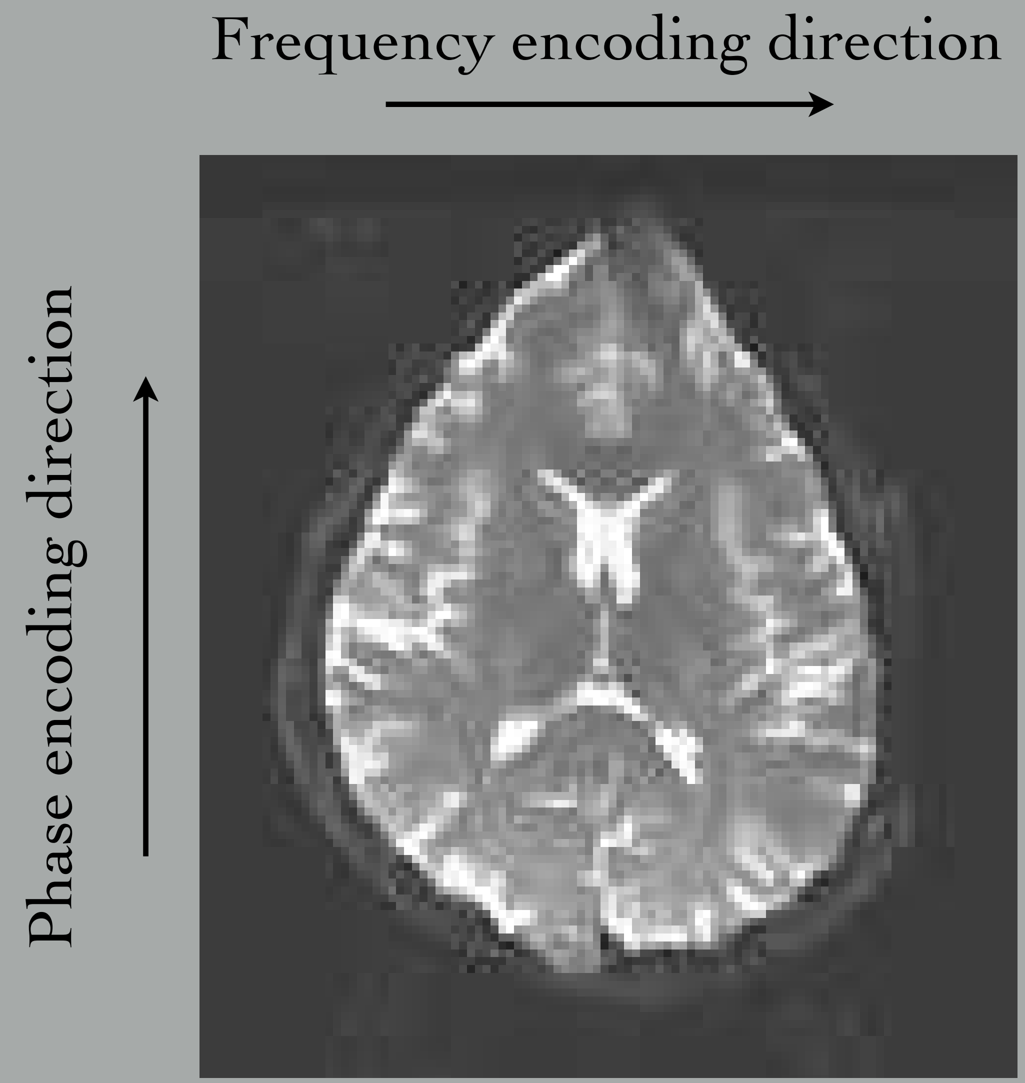
$$\Delta y = \left(\frac{\delta B_o}{W_y} \right) F_y = \Delta \delta B_o F_y$$

(assume $F_x = F_y$)

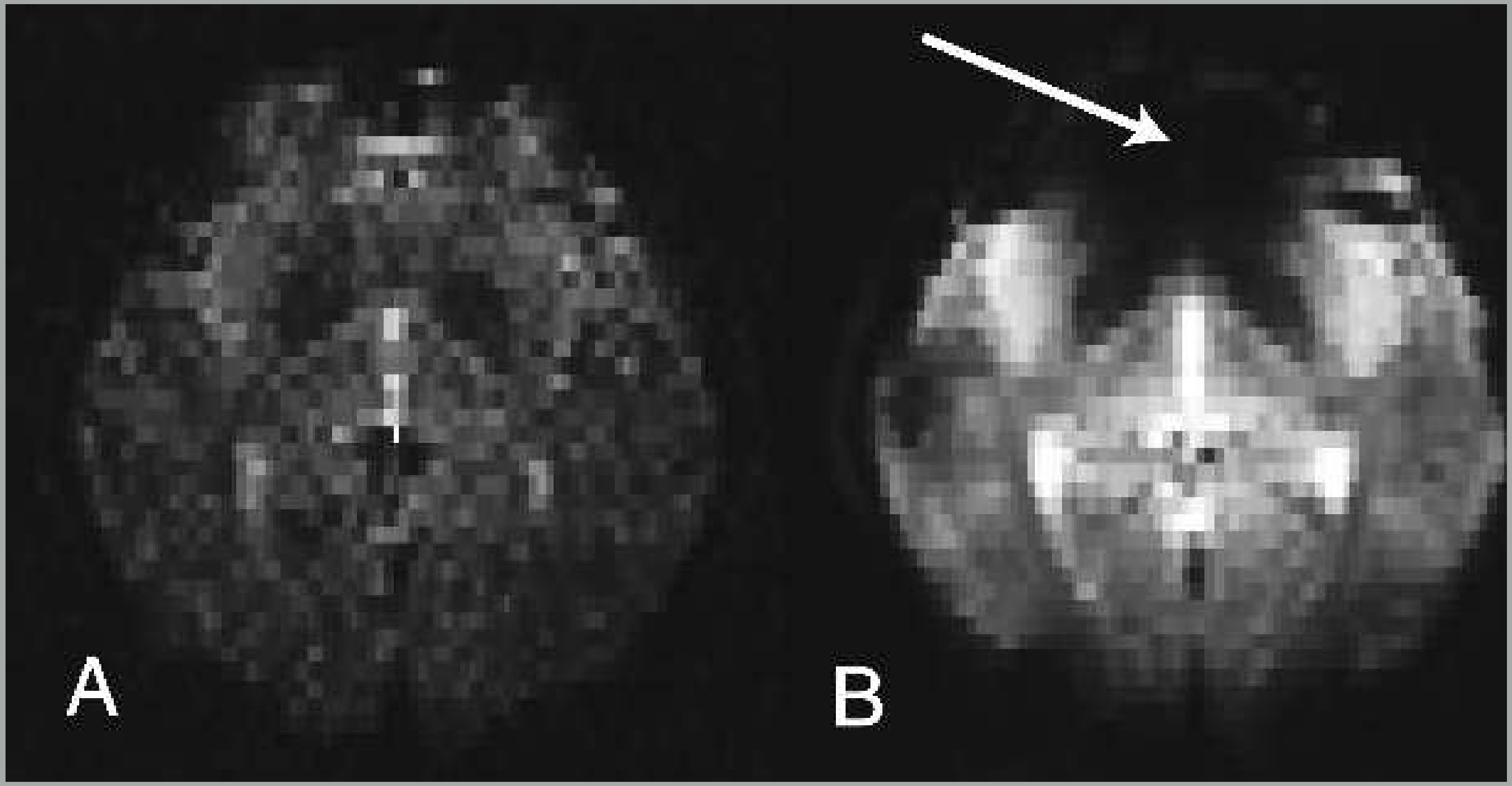
Static Field Inhomogeneity and EPI



Static Field Inhomogeneity and EPI



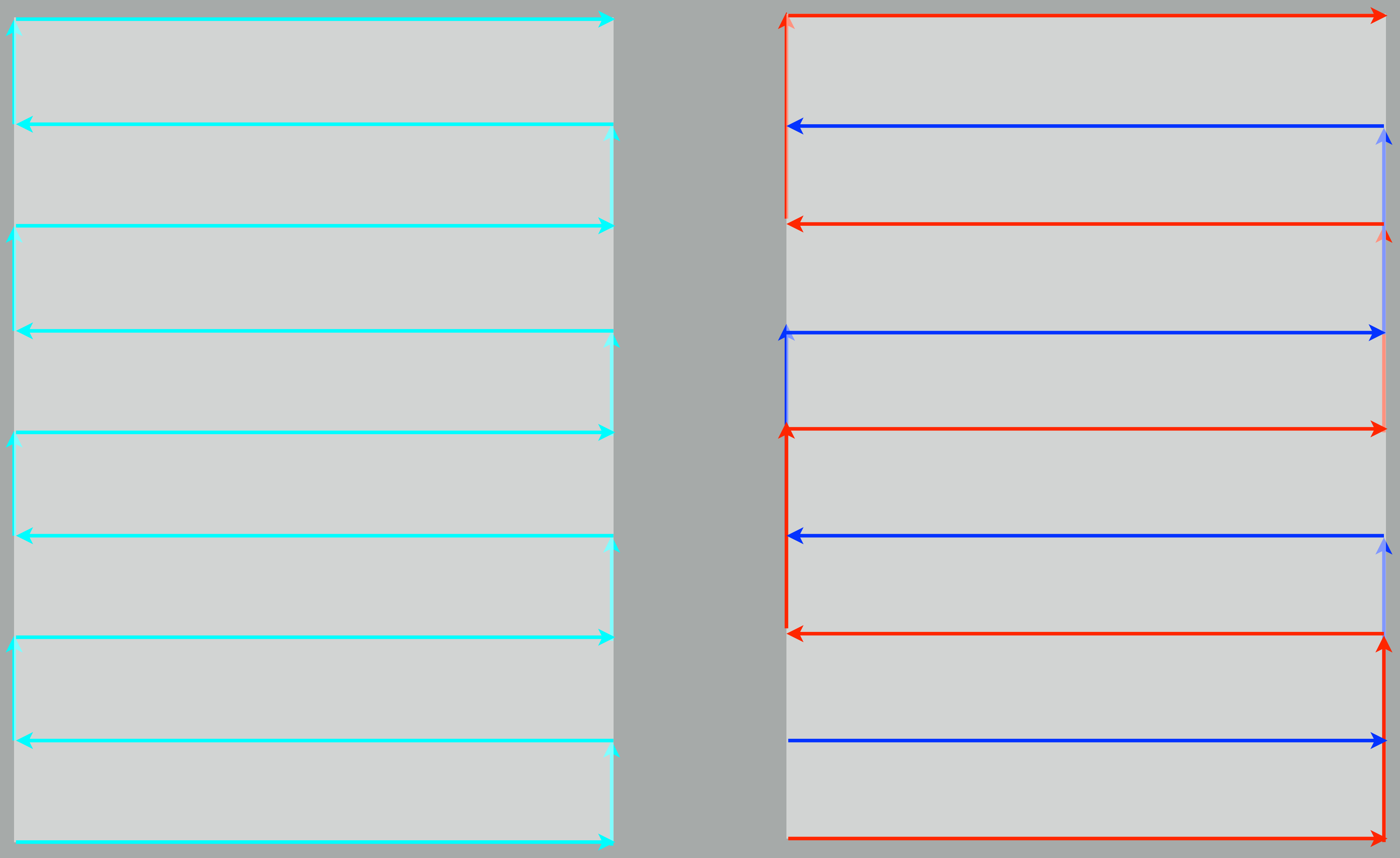
Signal dropout



volume acquisition

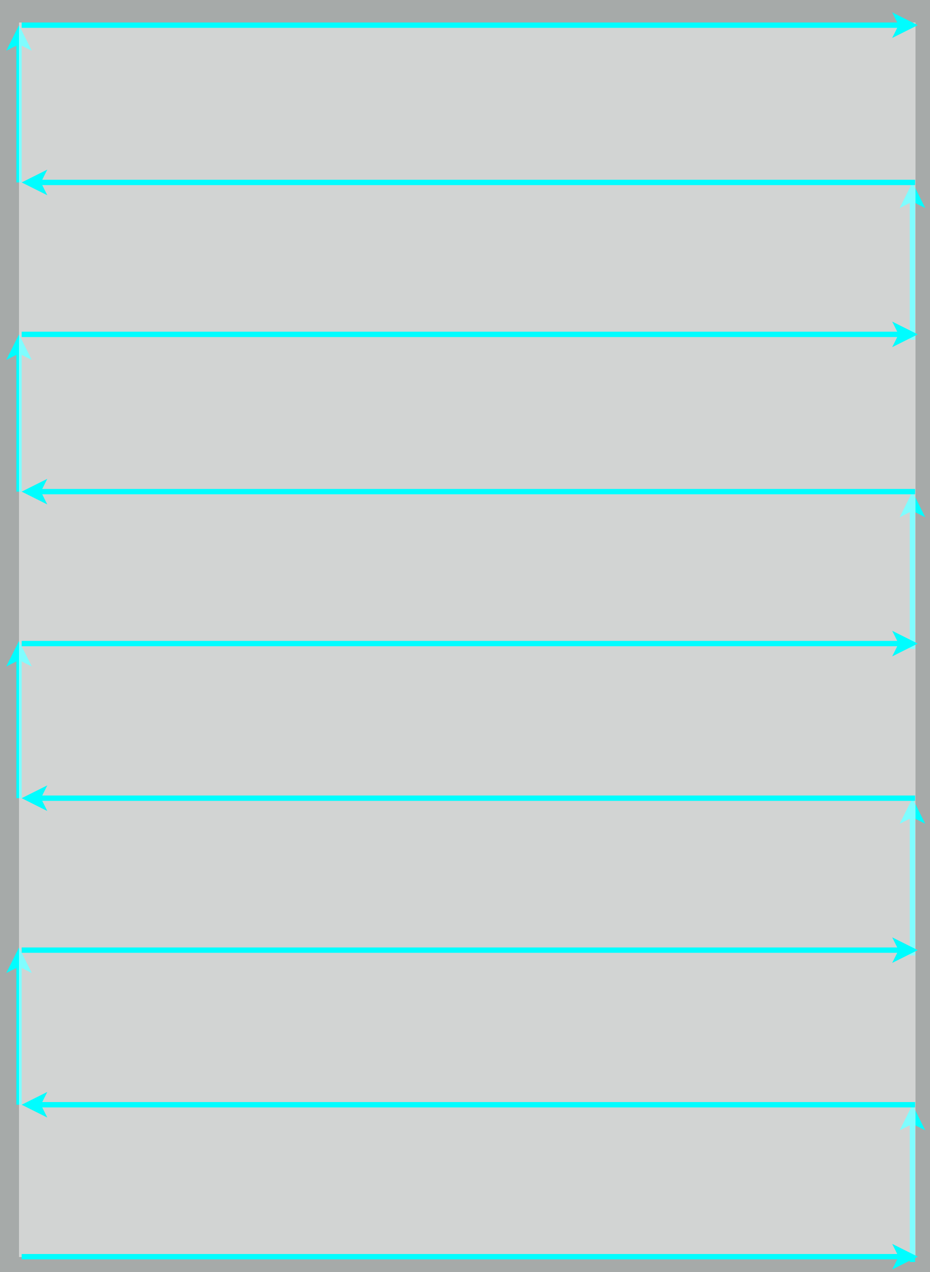
EPI acquisition

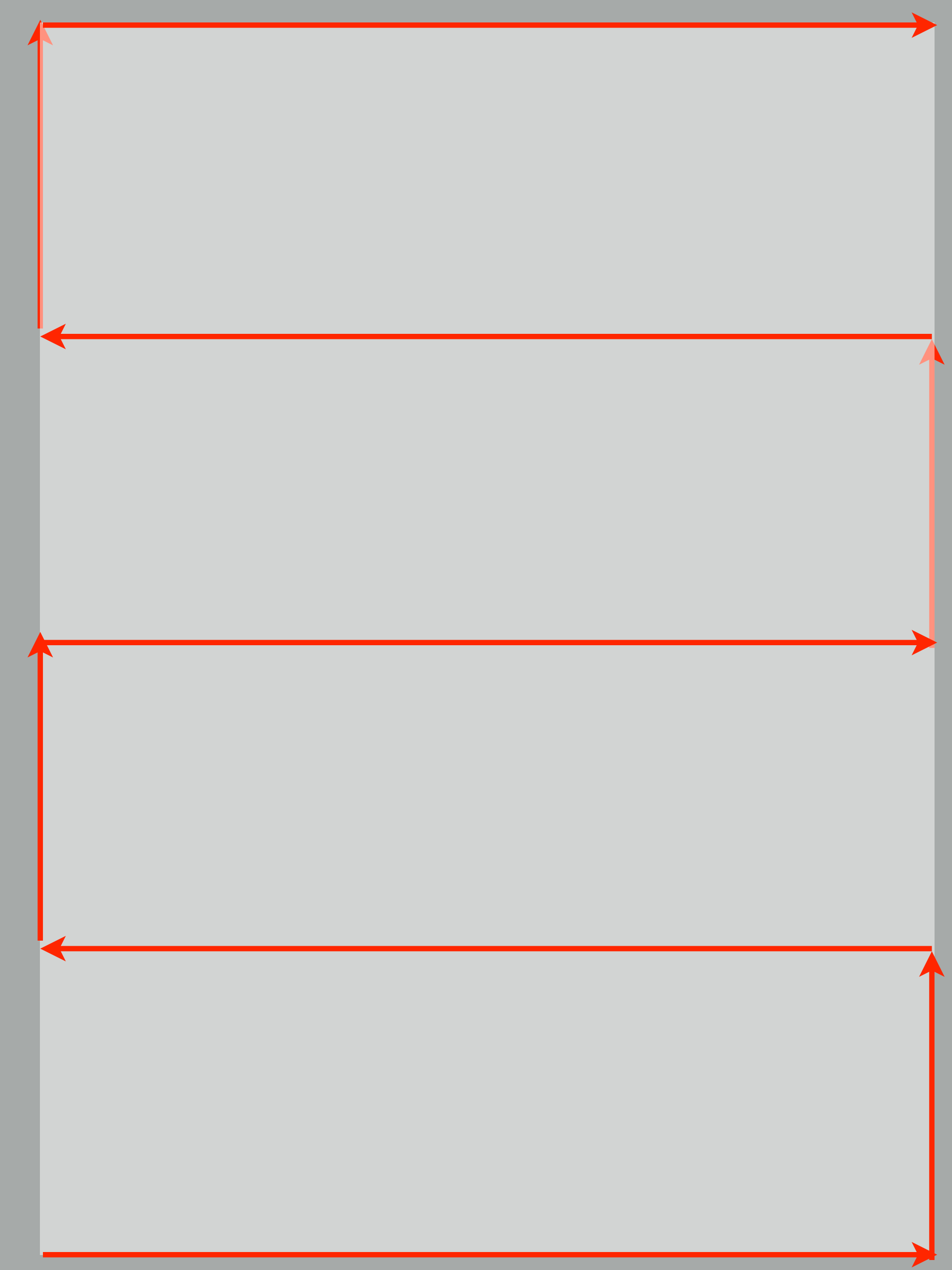
Multi-shot acquisition



Multi-shot acquisition

F_y $\qquad$ $F_y/2$



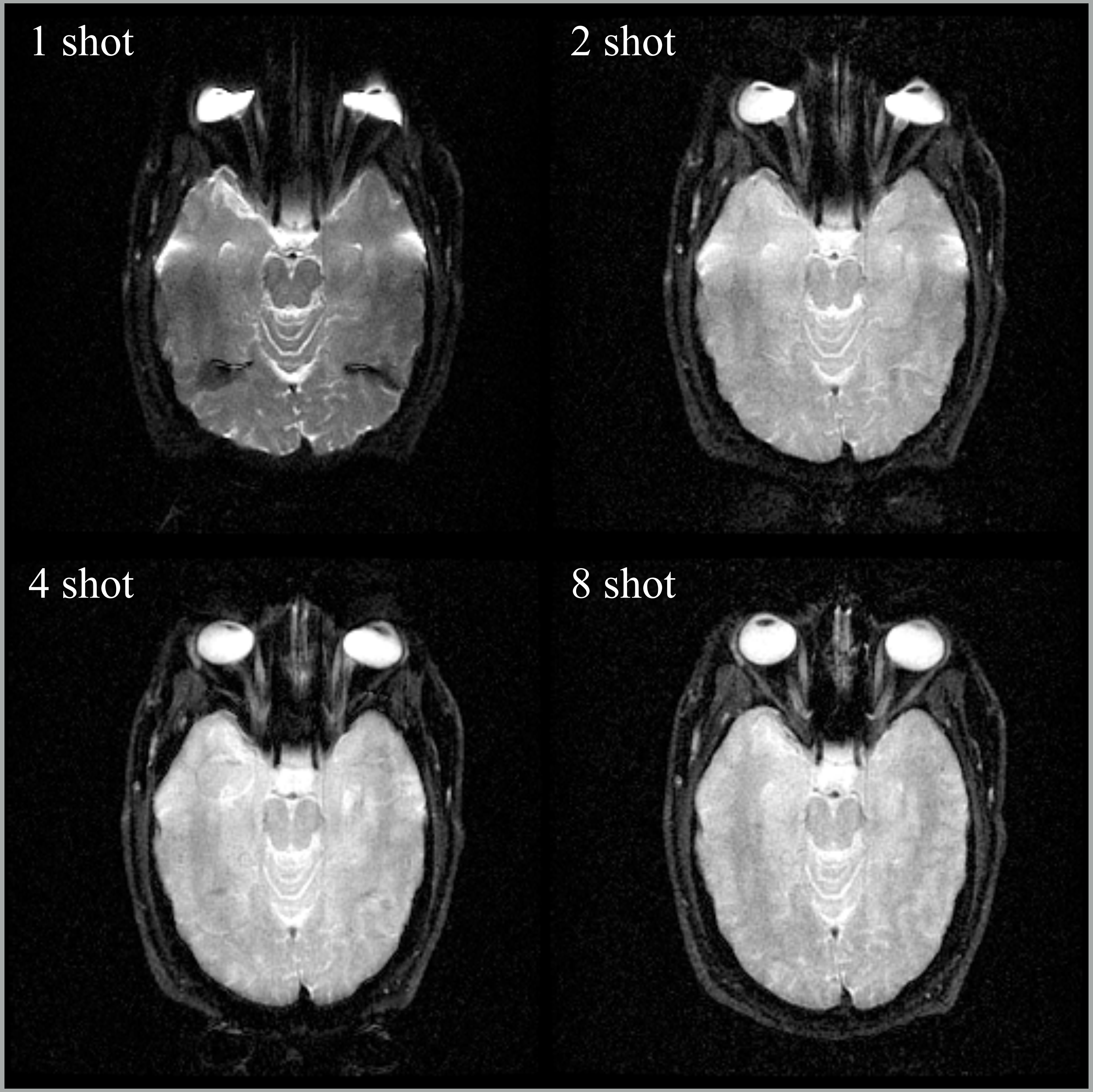


Δy

$$\Delta r = \left(\frac{\delta B_o}{W_r}\right) F_r$$

$\Delta y/2$

Multi-shot acquisition

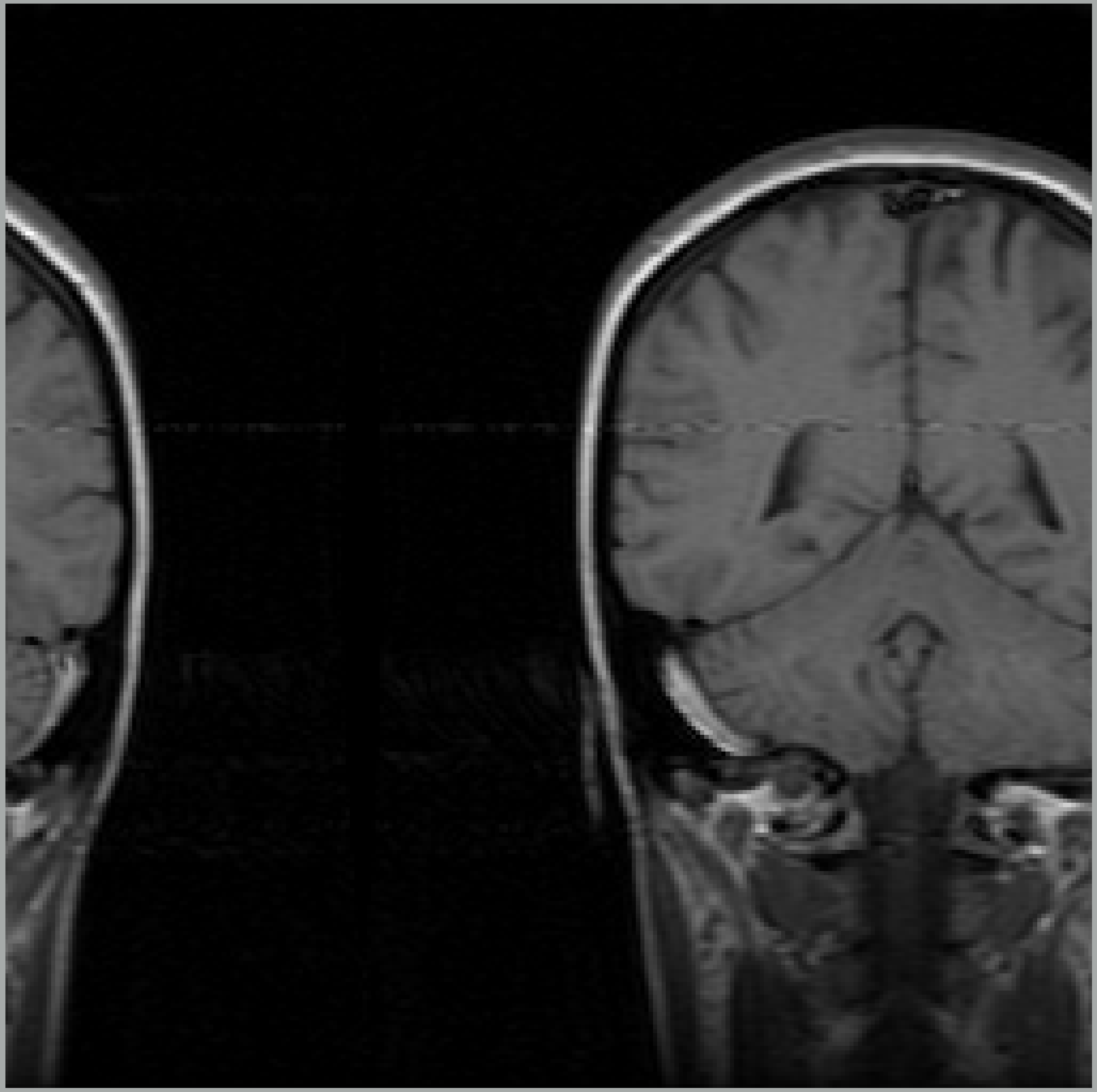


Increasing the number of shots per image decreases the EPI echo-train length per shot.

<u>SE-EPI</u>

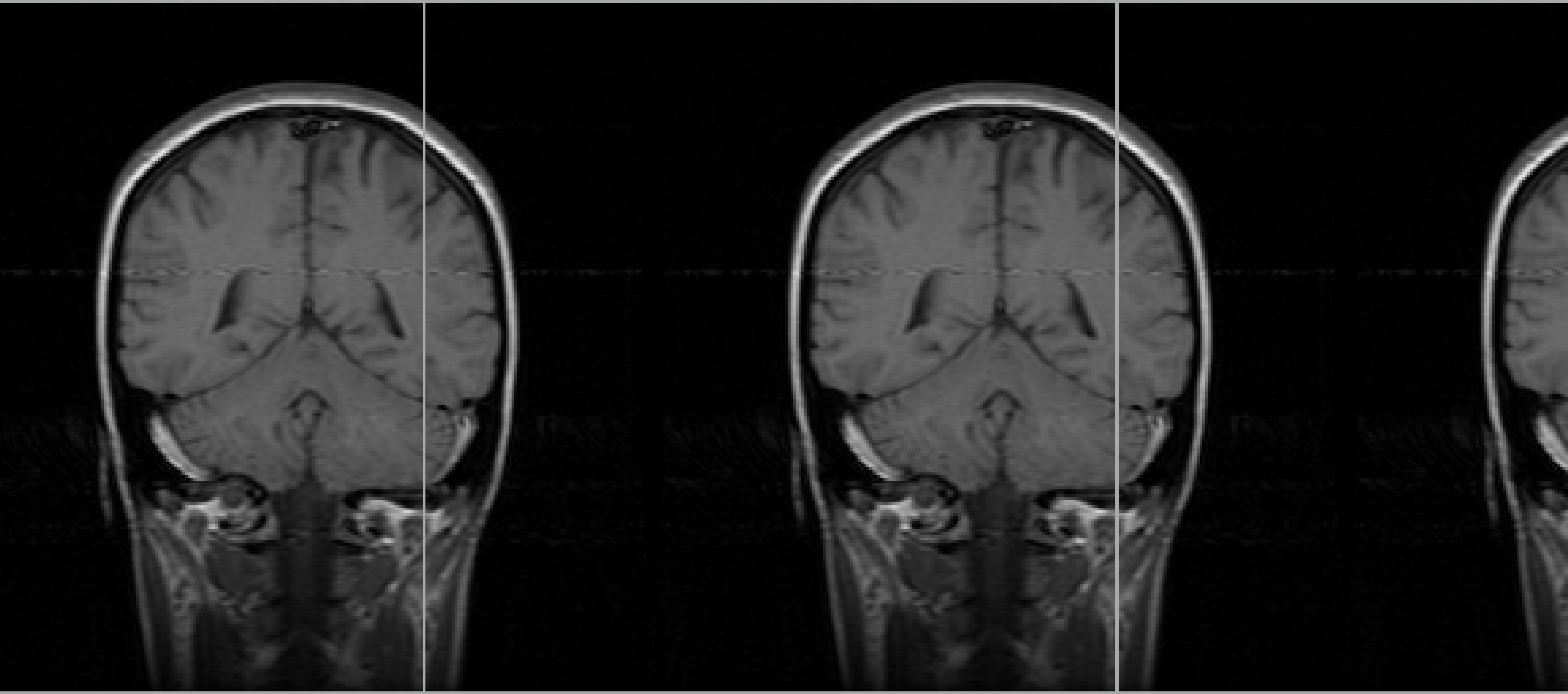
Shots	: 1-8
TR	: 3000ms
TE	: 60ms
Slice	: 5mm/2.5mm (18)
Matrix	: 256 x 256
FOV	: 24cm x 24cm
Time	: 12s-27s
NEX	: 1

Aliasing



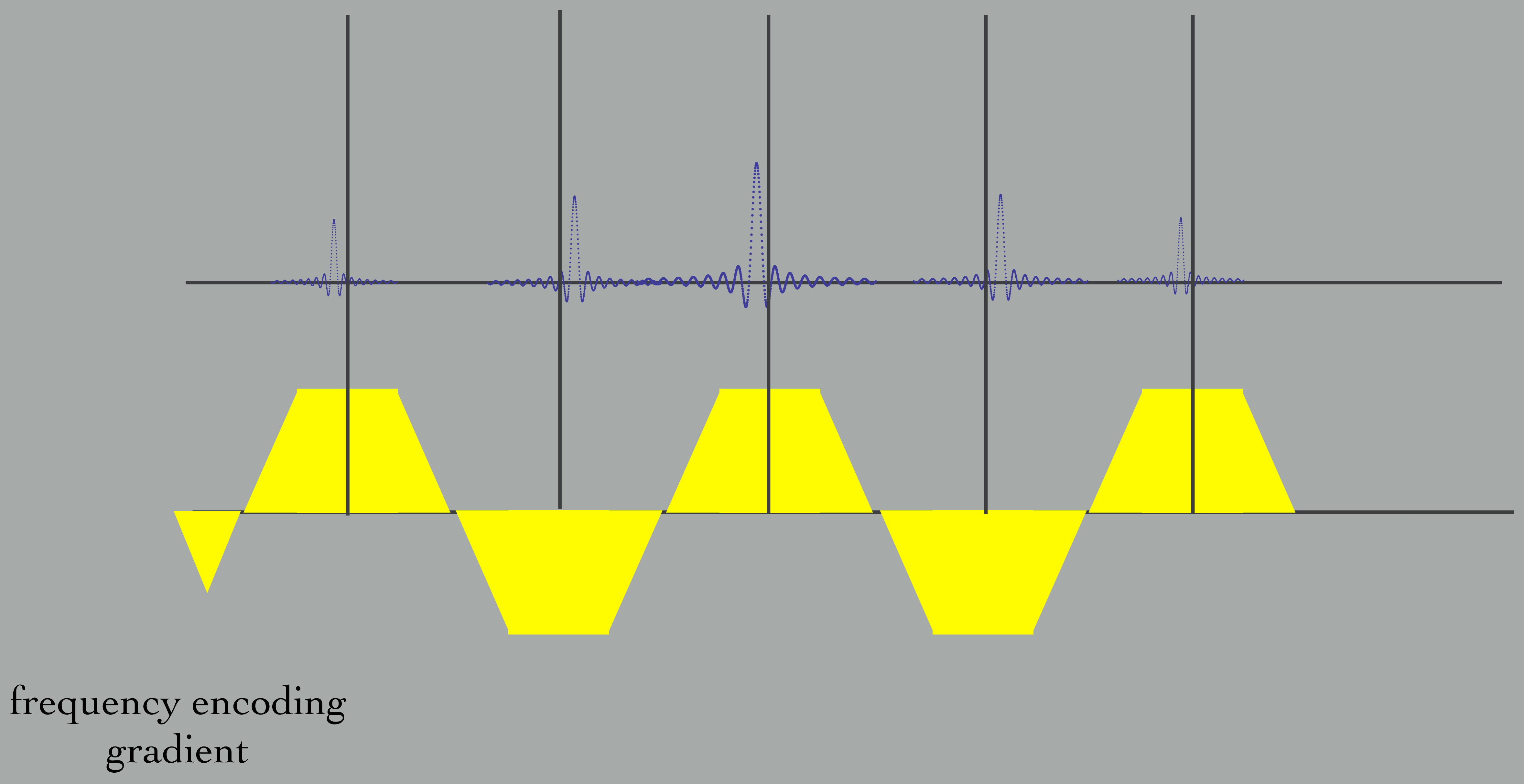
bandwidth of object > receiver bandwidth

Aliasing

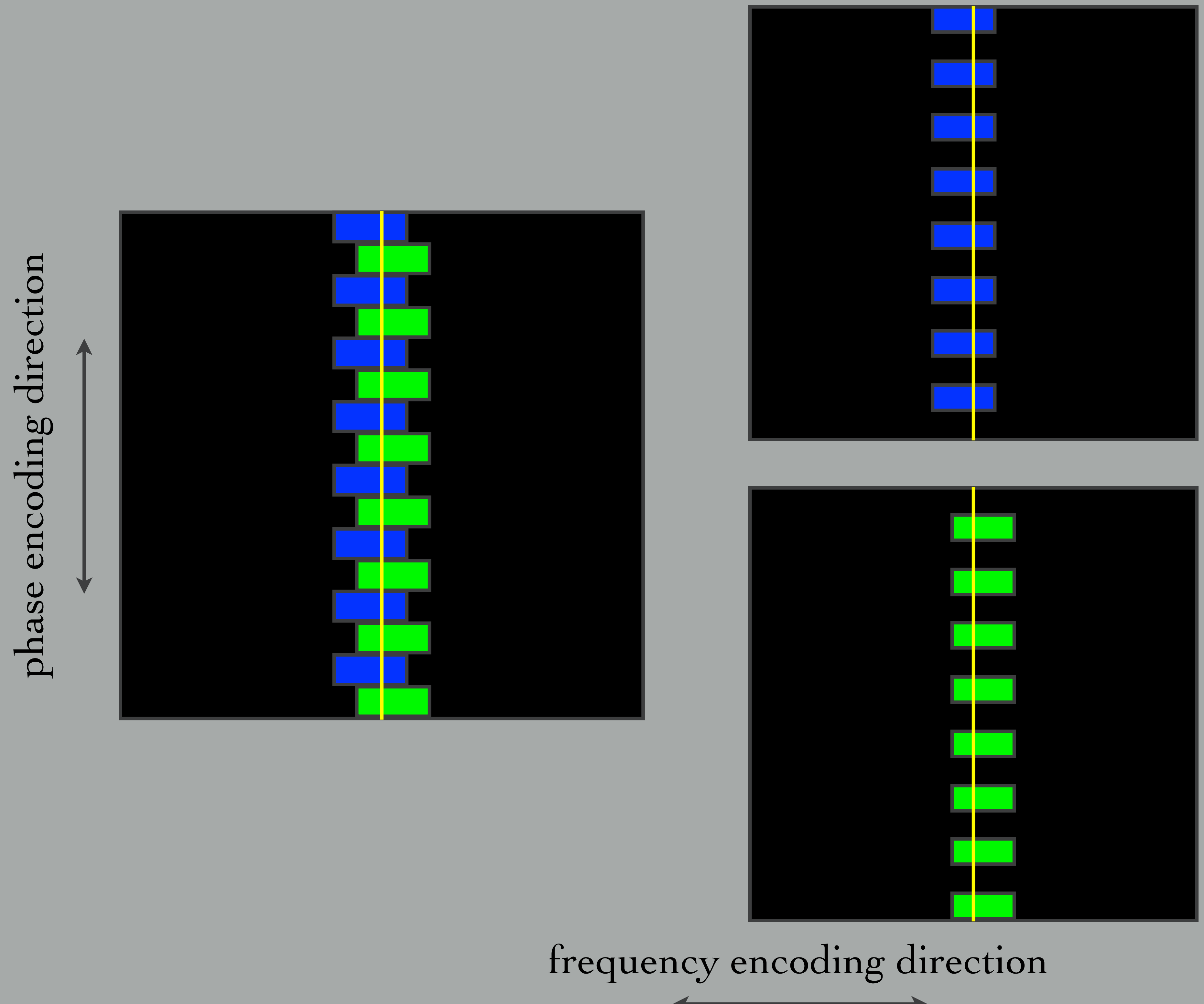


Fourier representation: periodically repeating

Nyquist ghosts



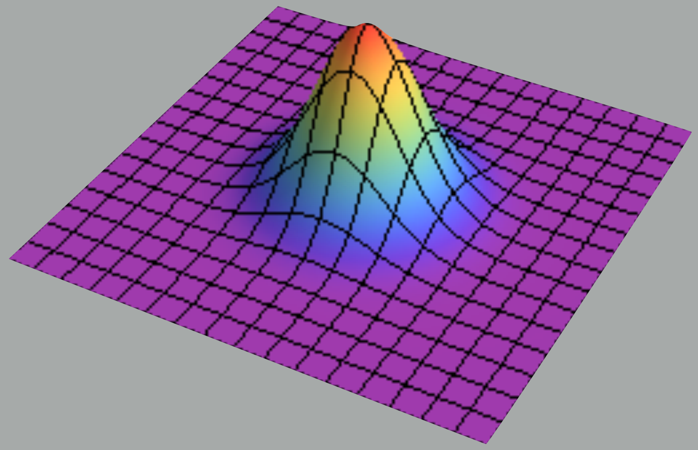
Nyquist ghosts

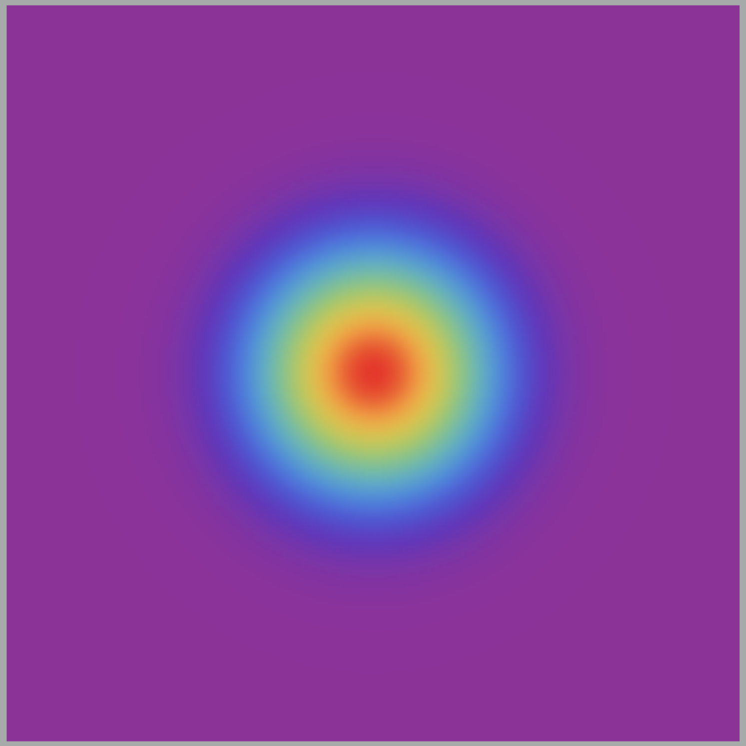


$$FOV_{even} = \frac{2\pi}{2\Delta k}$$

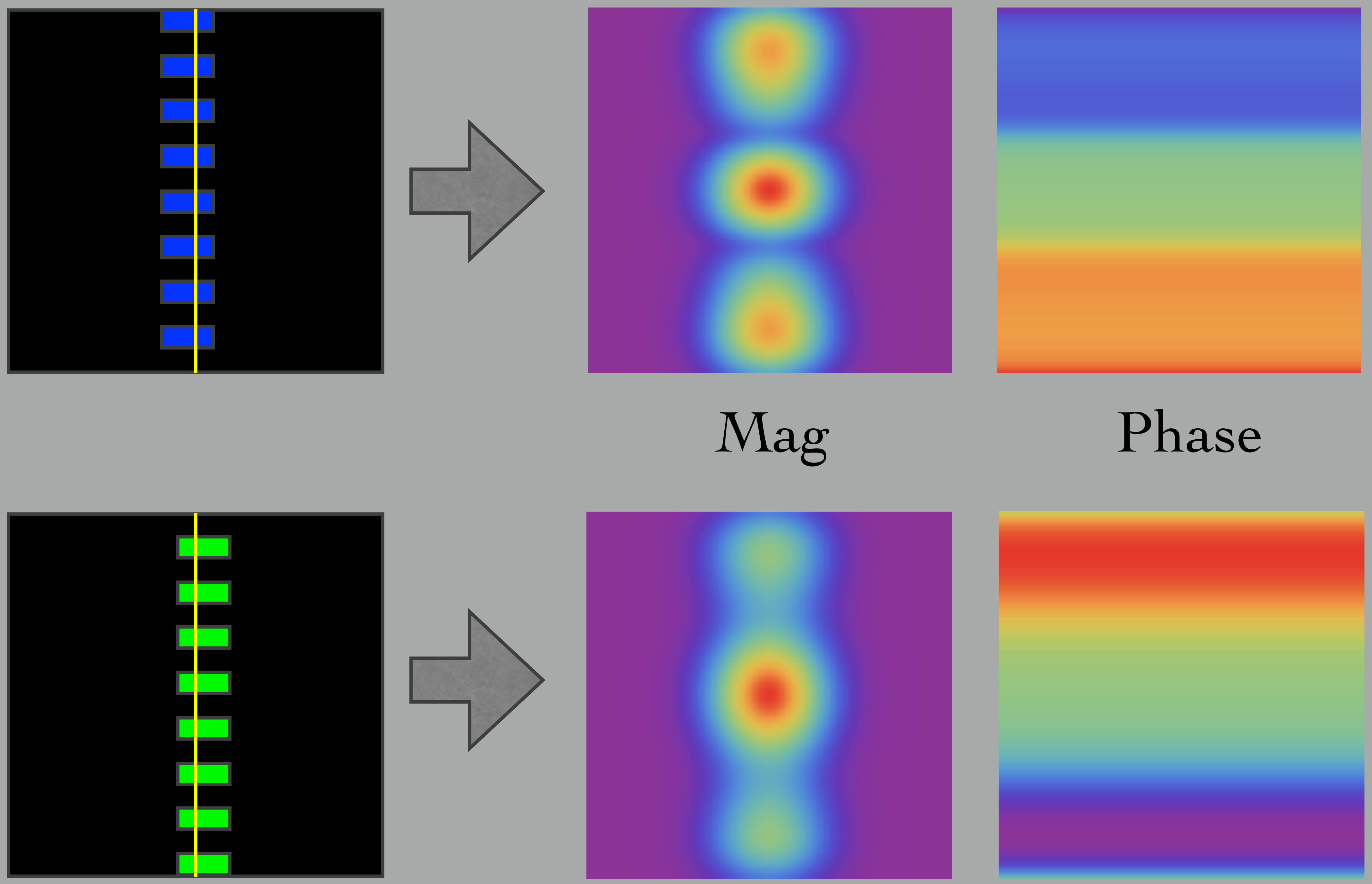
$$FOV_{odd} = \frac{2\pi}{2\Delta k}$$

Nyquist ghosts

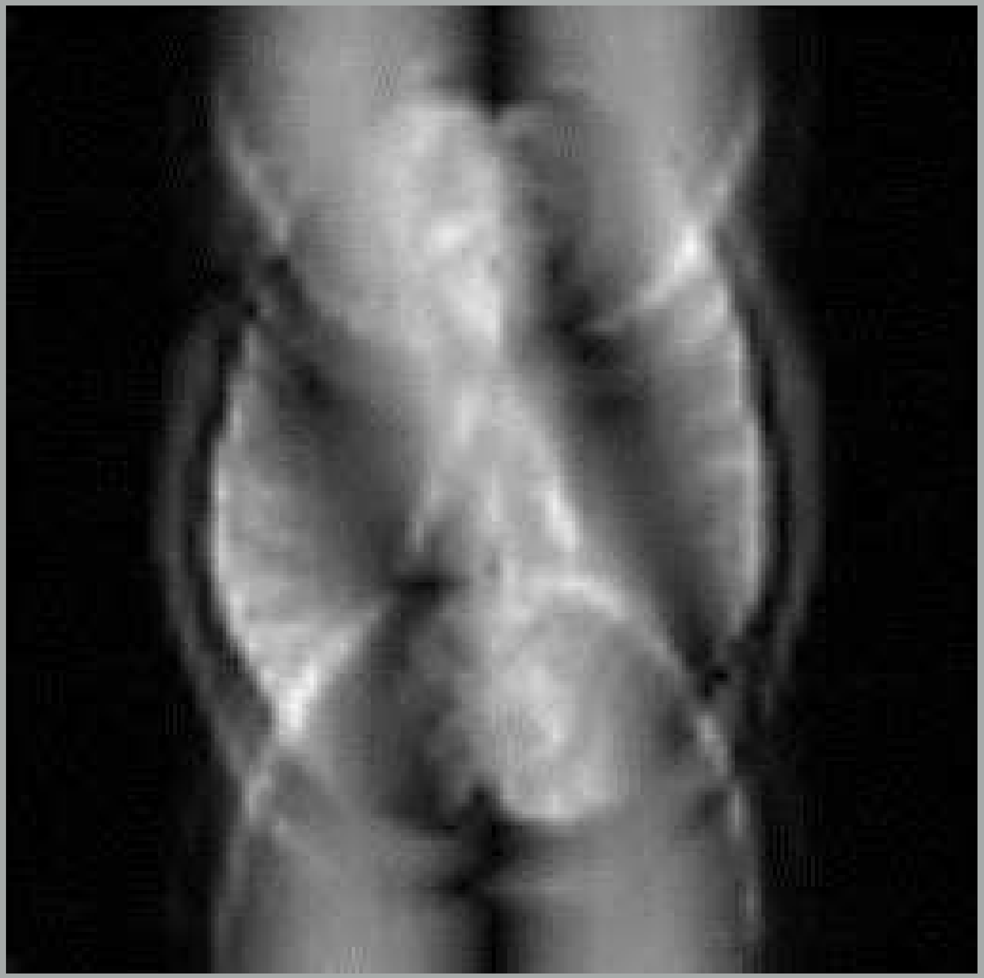




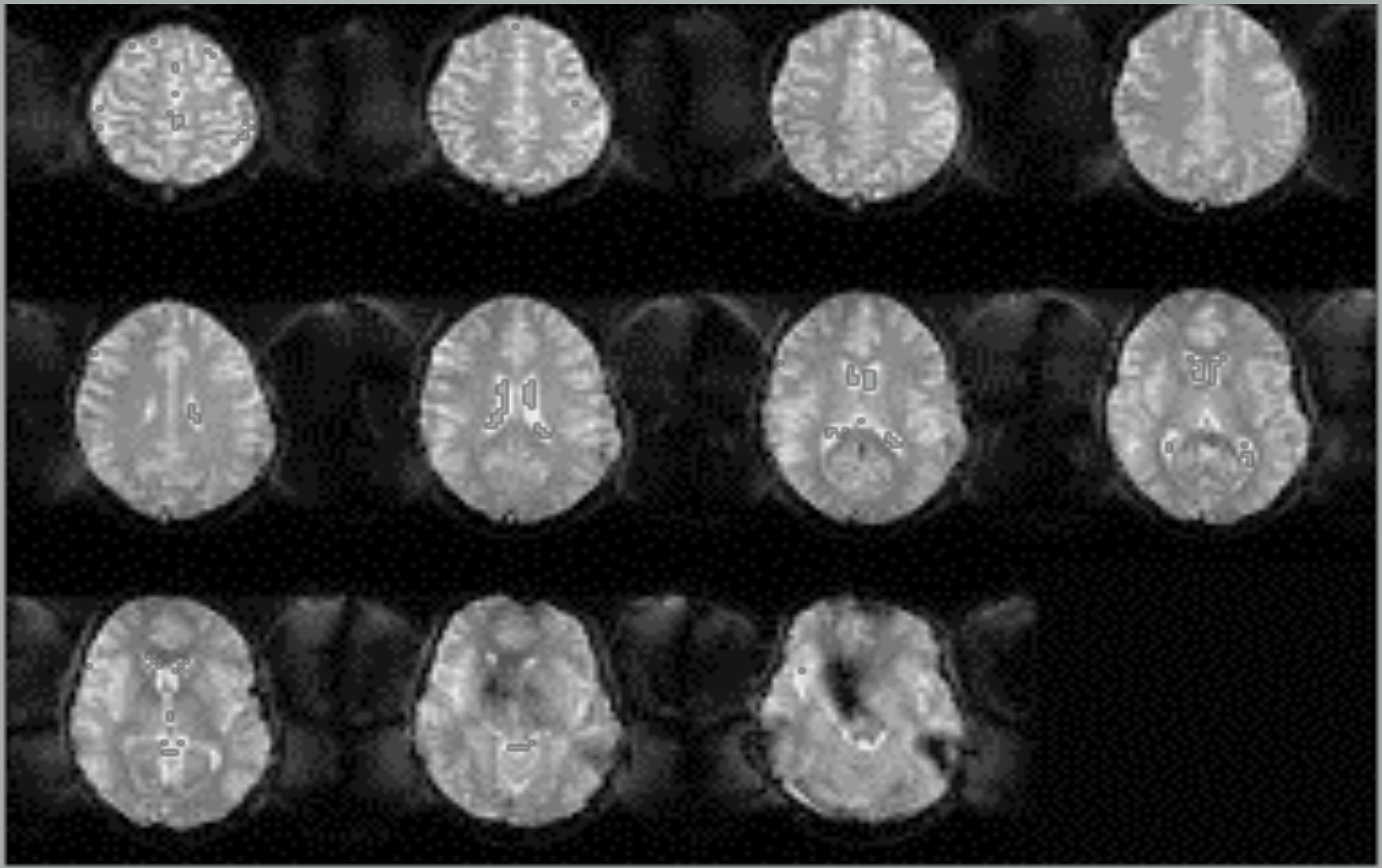
Nyquist ghosts



EPI N/2 ghost



EPI signal dropout



Receiver bandwidth

time between data samples:

$$\Delta t = 8\mu s$$

sampling rate:

$$\frac{1}{\Delta t} = \frac{1}{8\mu s} \approx 128kHz$$

This is the *receiver bandwidth*

If 256 points are collected
total acquisition time is 512 x 8*us* = 4*ms*

Image bandwidth

For a read gradient $G_x = .3G/cm$
creates a modulate across the image of

$$\gamma G_x FOV = 4258Hz/G \times .3G/cm \times 24cm \approx 32kHz$$

This is the *image bandwidth*

Bandwidth-per-pixel

Two spins on opposite sides of the image have precessional rates that differ by $32kHz$

Each of the 256 voxels differ in precessional rate from its neighbor by 32kHz/256 = 125Hz

This is the *bandwidth-per-pixel*

Chemical Shift Artifact

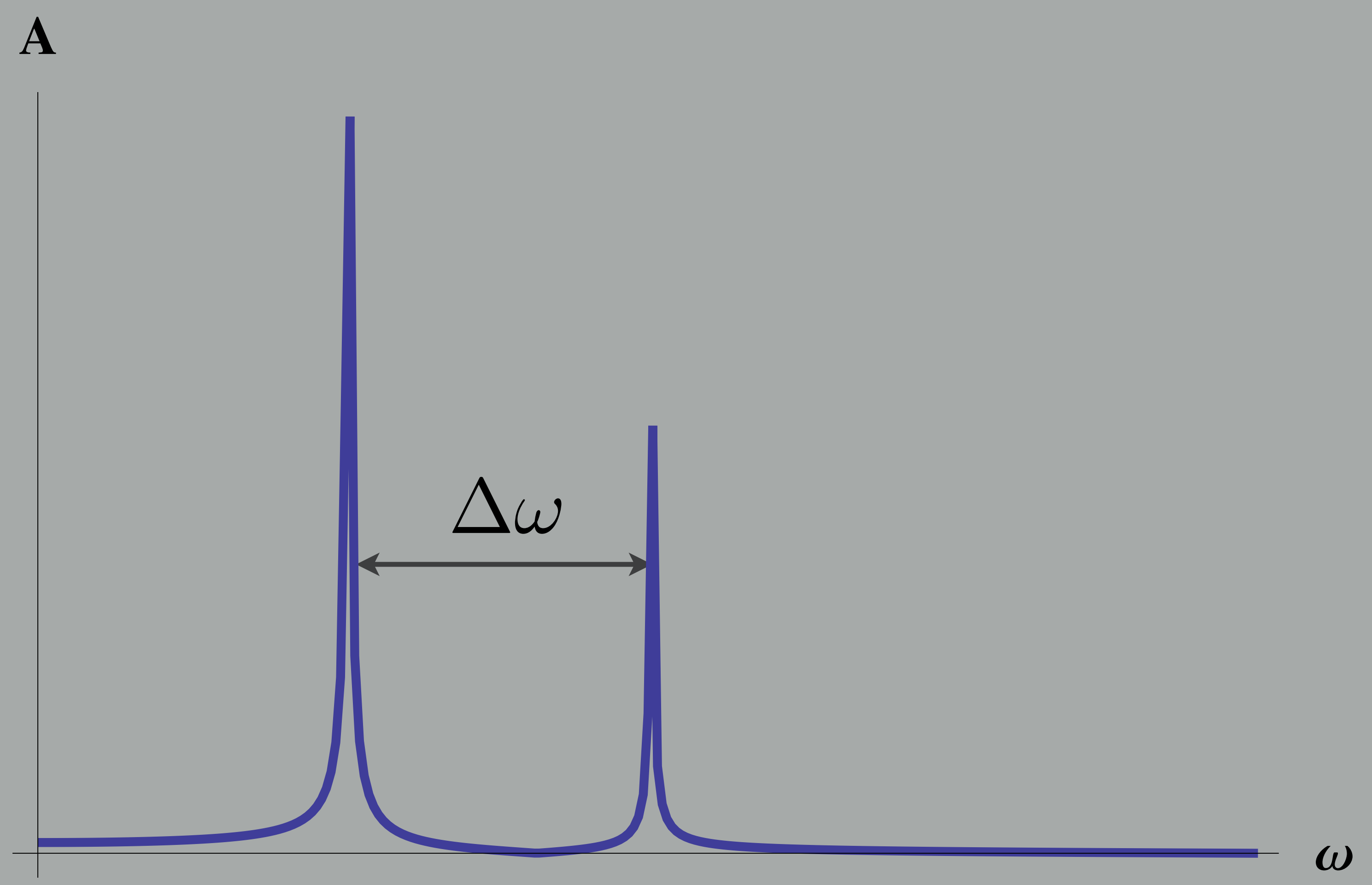
Fat and water have different resonance (Larmor) frequencies by approximately 3.5ppm (parts-per-million)

$$3.5 \times 10^6 \times 42.6MHz/T \approx 150Hz/T$$

So at 3T:

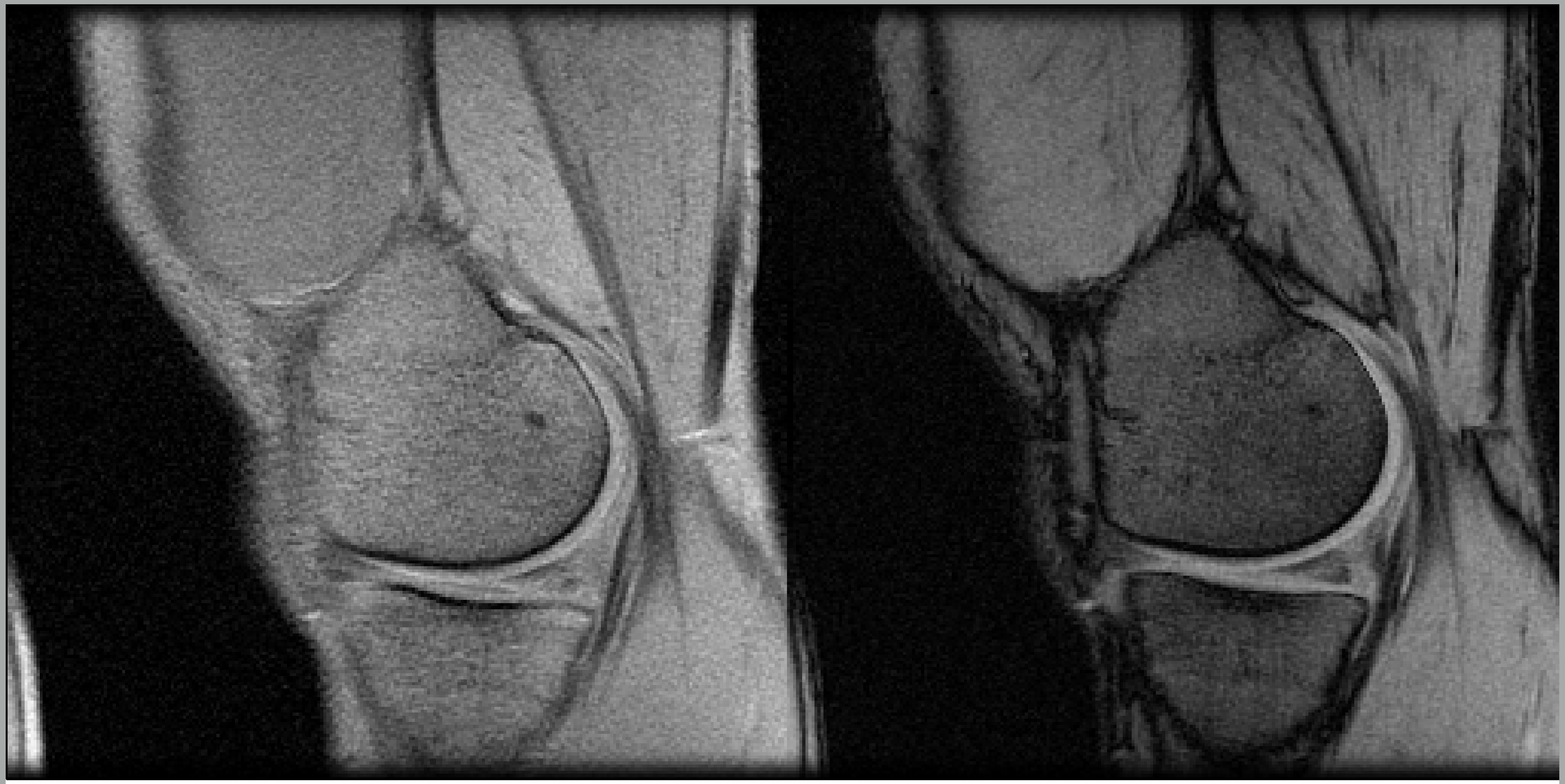
$$3T \times 150Hz/T = 450Hz$$

Chemical Shift Artifact



$$\Delta\omega \approx 440Hz \text{ @ } 3T$$

The uses of chemical shift



In phase (TE = 3.9 ms) Out of phase (TE = 7.0 ms)

Chemical Shift Artifact

Fat is shifted relative to water in the read direction

$$\frac{\text{frequency difference}}{\text{bandwidth-per-pixel}}$$

at 3T for 24cm FOV and 256 pixels:

$$\Delta x = \frac{450 \text{ Hz}}{125 \text{ Hz}} = 3.6 \text{ pixels}$$

Chemical Shift Artifact

Particular bad in EPI which has a very low bandwidth-per-pixel in the phase encoding direction since time between samples is much longer in that direction

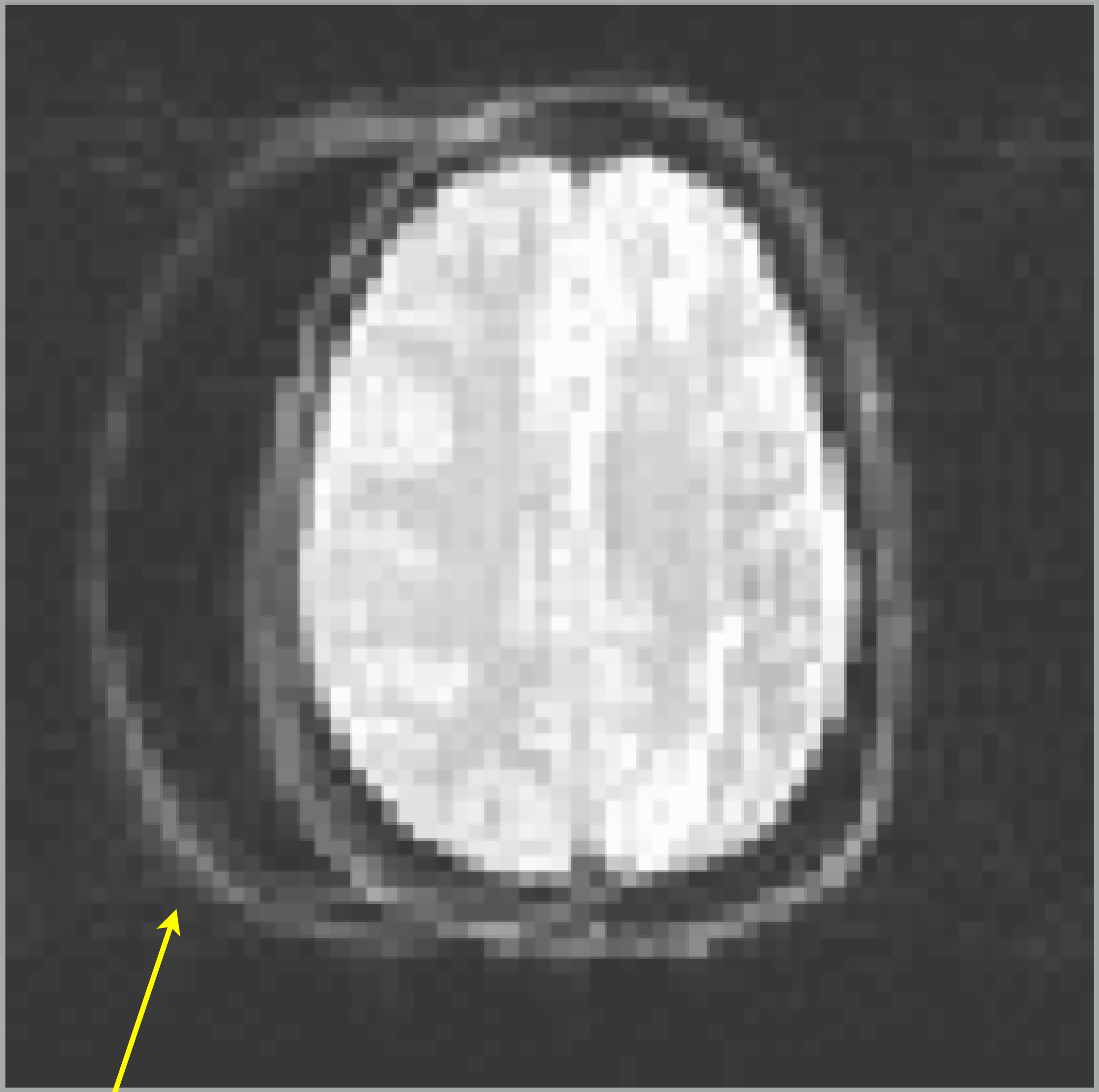
Typically around

15 Hz/pixel

So shift is

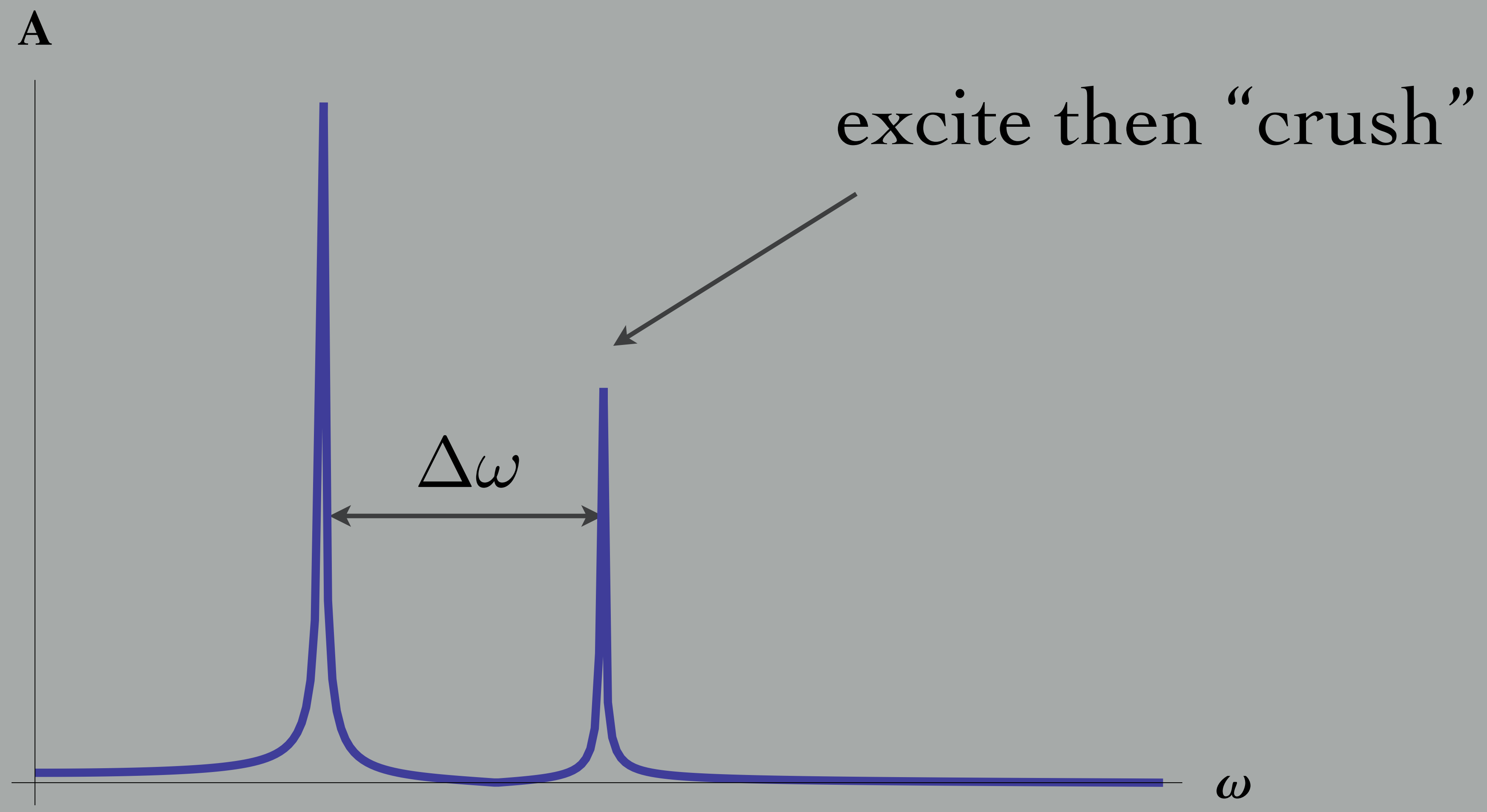
$$450Hz/(15Hz/\text{pixel}) = 30 \text{ pixels}$$

Chemical shift artifacts in EPI



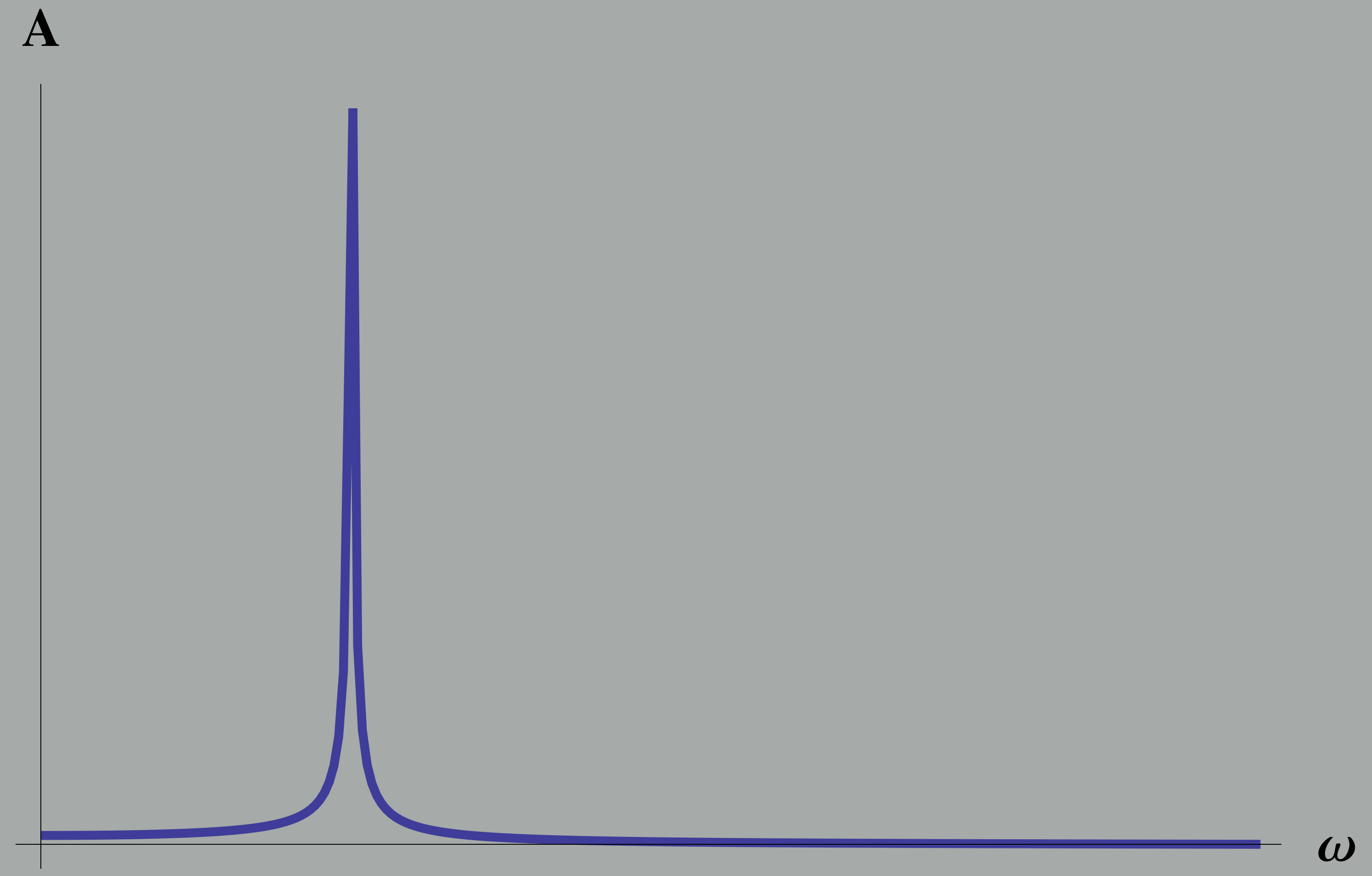
Fat signal

Fat Suppression

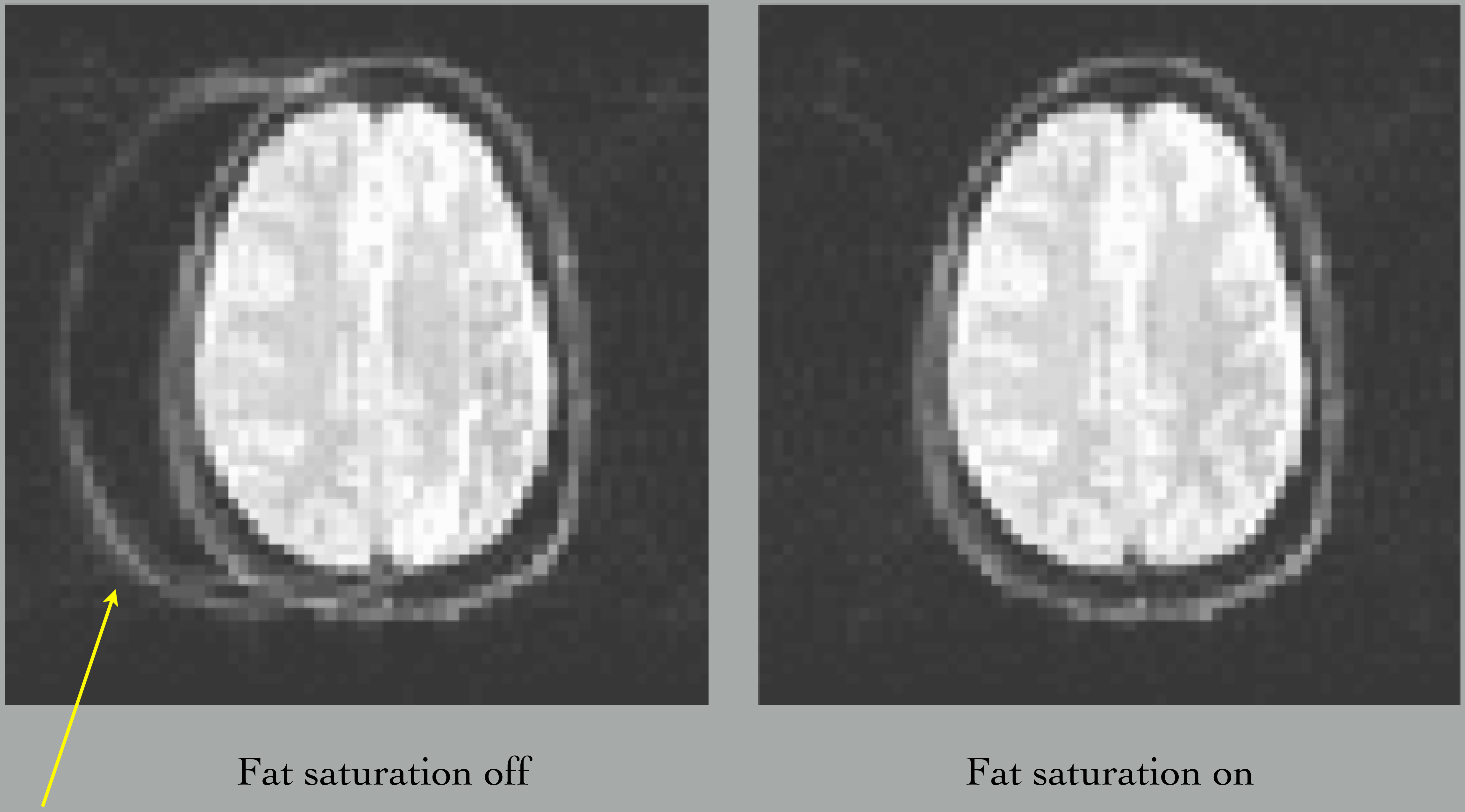


$$\Delta\omega \approx 440Hz \; @ \; 3T$$

Fat Suppression



Chemical shift artifacts

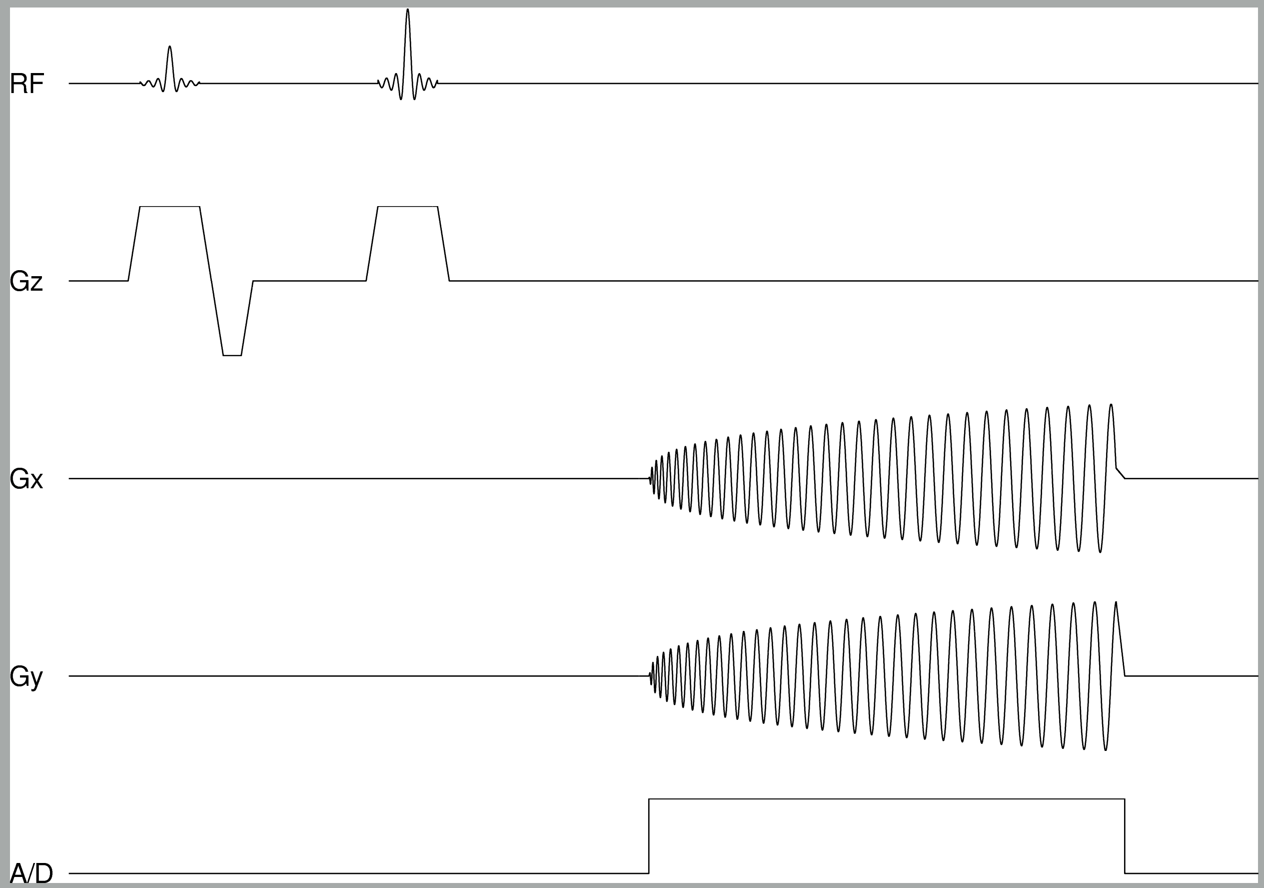


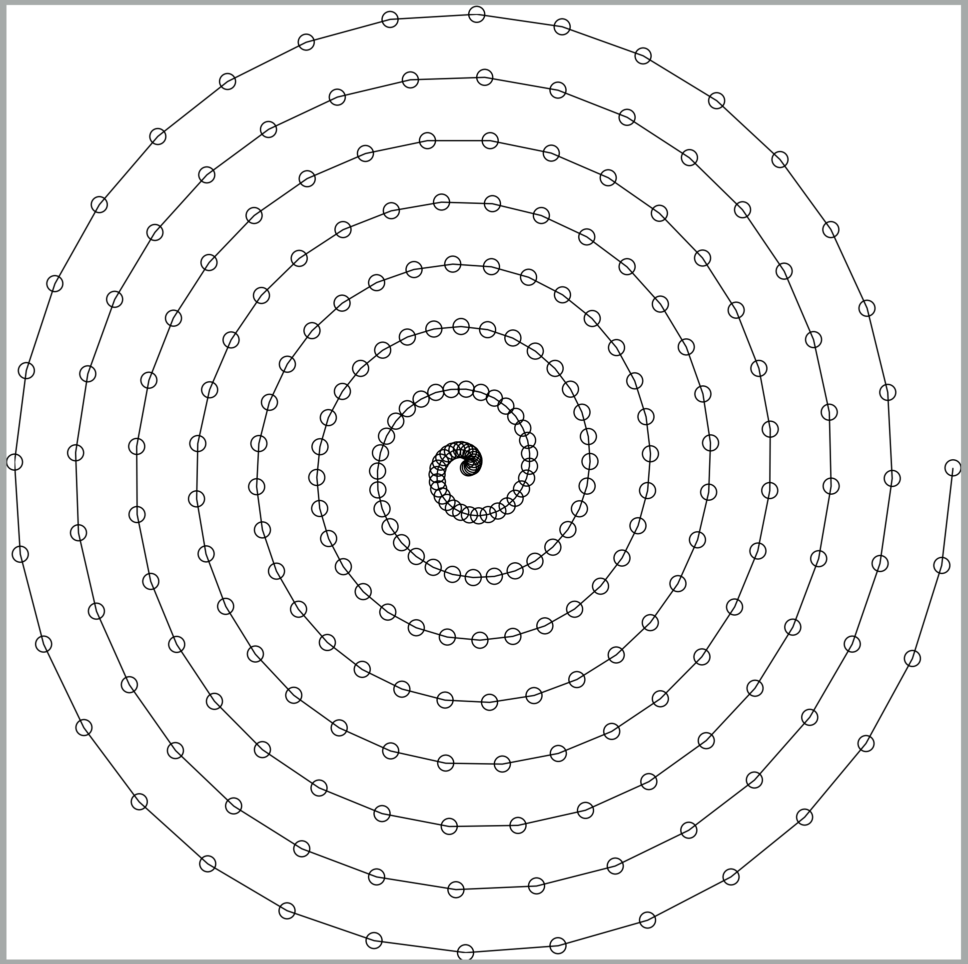
Fat saturation off

Fat saturation on

EPI acquisition

Other trajectories: Spiral Imaging





Spiral Imaging

1. Spiral trajectory less sensitive to motion
2. Artifacts tend to “smear” along trajectory and thus blur rather than alias as in EPI
3. Image reconstruction requires regridding to Cartesian grid for Fast Fourier Transform (FFT)

Spiral Imaging