16 Spatial localization

(**Need to be consistent with units of γG to G**)

16.1 Introduction

In the preceding sections we have seen how the placement of a large number of spins (e.g., a human head) within a large, static magnetic field will result in the generation of a measurable *bulk magnetization* that can be characterized in terms of a simple vector pointing along the direction of the main field. The spins precess about this main field vector with a frequency (the *Larmor frequency*) that is proportional to the local magnetic field via (Eqn 13.10). Application of magnetic field pulses that match the precessional (*resonance*) frequency of the spins allows us to excite the spins, producing signals whose characteristics depend upon the local tissue properties. The process of *magnetic resonance imaging* or *MRI* is to extend this operation to the mapping of the *spatial distribution* of these tissue characteristics.

We also found that by applying another, much smaller, magnetic field along a direction perpendicular to the main field, we could tip the magnetization vector into the plane transverse to the main field, which is called *excitation.* The torque on this magnetization vector by the main field causes it to precess at the frequency associated with the main field, which is the *Larmor frequency.* In order for the tipping procedure to work, then, required that its torque is always perpendicular to the precessing magnetization, and thus this excitation field must also rotate at the Larmor frequency, or *resonance frequency.* Once the magnetization has been tipped into the *transverse plane*, it induced a current in the coils which is the signal that we detect. After the magnetization has been tipped, or excited, and the excitation pulse is turned off, the spins precess in the main field only and the signal decays away, producing the signal called the *free induction decay*, or *FID.* The transverse component decays away by an exponential process characterized by the decay constant T_2, while the longitudinal component grows back according to an exponential recovery curve characterized by the decay constant T_1. In practice, the transverse decay occurs much faster then described by T_2 because of inhomogeneities in the main field that cause spins in different fields to precess at different rates and thus these *isochromats* get out of phase with one another. This process is characterized by the decay constant $T_2* < T_2$. However, the spin dephasing and subsequent fast signal decay can be overcome by the application of a 180° refocussing pulse that causes the different isochromats to come back into phase and produce a signal coherence or *spin echo.*

In short, we have learned how to generate a signal, and refocus static field inhomogeneities at a later time. We have now learned all of the prerequisite knowledge to proceed on how to create an image using nuclear magnetic resonance (NMR), a technique called *magnetic resonance imaging* or *MRI* (**?**, **?**, **?**, **?**). There is really only one new concept that we need to introduce in moving from what we have already learned to creating an image, and that is the effects of a spatially varying magnetic field. More specifically, a magnetic field that varies in a linear fashion across our sample (e.g., the head) and is applied for a specific time interval. This is called a *magnetic field gradient* and the fact that is applied for a time interval (rather than is on continuously) makes it a *pulsed gradient*. The applied spatial variations in the magnetic field are the key to generating magnetic resonance images. Their effects, and how they are combined withe the previously described excitation and detection mechanisms, is the subject of this chapter.

16.2 Magnetic field gradients

Let us return to the NMR experiment and recall that, up to this point, we have considered only the response of spins to a large, static magnetic field B_0. Spins precess according to the Larmor relation (Eqn 13.10) and are excited by the application of an excitation pulse at this resonance frequency. This converts the longitudinal magnetization, which is invisible to the receiver coils, to transverse magnetization, which *is* visible to the receiver coils, and hence produces a signal. Following the application of an excitation pulse, all the excited spins are still precessing with the same precessional frequency $\omega_0 = \gamma B_0$. Now consider what happens if we apply a magnetic field that *varies as a function of position*. Once again we can appeal to the Larmor equation and note that if the magnetic field has a spatial dependence, $B = B(x)$, then the frequency does as well: $\nu(x) = \gamma B(x)$. We will use this fact to create MR images.

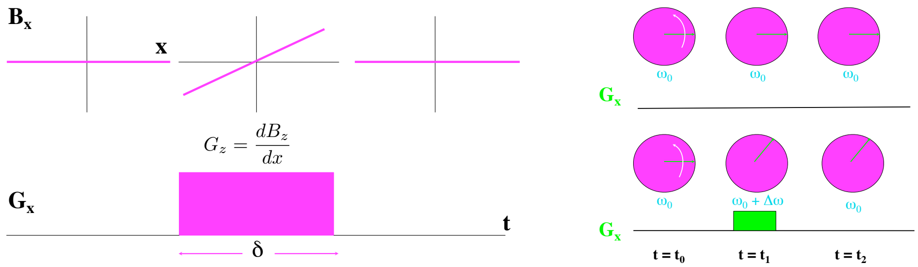
Consider the simplest form of spatial dependence in which the magnetic field varies linearly with spatial coordinate x: $B(x) = Gx$. G is called the *magnetic field gradient* and is in units of magnetic field per distance. Substituting this into Eqn 13.10, the spatial variation in the precessional frequency in the presence of a gradient field is

$$\nu(x) = \gamma G x \tag{16.1}$$

These gradient fields are typically much smaller than the main field, on the order of $2G/cm$. In order to produce gradient fields, special coils, called *gradient coils*, are required. The gradient is applied so that it is symmetric about the center of the magnet, and produces no frequency variations in the center. The frequencies in the presence of a gradient field from a range of positions from $-x_{max}$ to x_{max} are then from $\nu_0 - \gamma G x_{max}$ to $\nu_0 + \gamma G x_{max}$.

Let's begin by considering the specific case of a gradient of constant magnitude that is turned on and off rapidly to create *pulsed gradients*, such as the x-gradient shown in Figure 16.2a Spins at different locations precess at different rates and so become increasingly out of phase with others at different locations, as shown in Figure 16.2.

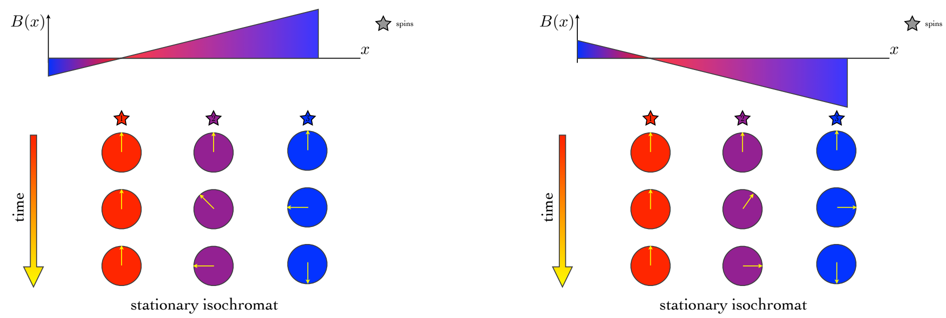
Let's take a closer look at what is really happening when a gradient is applied. In Chapter 4 we introduced the notion of the phase using the analogy of the spinning record (Figure **??**). Let's now revisit that analogy now and see that it is very useful in understanding the effect of magnetic field gradients on spins. Consider Figure 16.3. (Left) Two turntables rotating at the same speed (33 rpm). The two labels "Elvis" and "Jimi" rotate in sync with one another. (Middle) The Jimi turntable is switched to a faster speed (45 rpm) so that the label get ahead of (i.e., out of phase with) the Elvis label. (Right) The Jimi turntable is set back to 33 rpm so that both records rotate at the same speed, but the Jimi record remains at the angle to the Elvis label - the angle or "phase" that it accrued while it was rotating at a different rate. This is shown in Figure 16.3



(a) Gradient pulse. A linearly spatially varying magnetic field. The bottom figure is a simple example of a pulse sequence diagram.

(b) Phase induced by the gradient pulse.

Figure 16.1 The gradient pulse. On the left (a) is shown the magnetic field $B(x)$ as a function of position x (top) for a gradient field $G = \frac{\partial B}{\partial x}$ (bottom) at three time periods. It is off on left. In the middle it is on. And then it is off again on the right. The associated gradient field is shown on the bottom. There is no gradient when the field is off (at left and right) and when it is on, the gradient is a constant magnitude G on for a time δ. On the right (b) is shown the phase at the three time points for a spin that does not experience the gradient (top) and one that does (bottom). The spin in the gradient acquires a phase relative to the spin that is not.

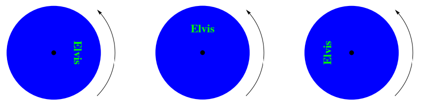


(a) Accrual of spin phase with time in a positive gradient.

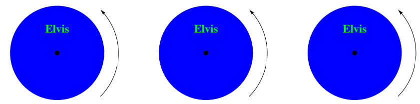
(b) Accrual of spin phase with time in a negative gradient.

Figure 16.2 A positive gradient on the left (a) and a negative gradient on the right (b). Spins at different spatial locations precess at different rates and therefore become increasingly out of phase with others at different locations. The gradients add (if positive) or subtract (if negative) from the much larger field B_o so spins precess faster or slower, respectively, than the Larmor frequency. In the rotating frame, a spin precessing slower appears as if were precessing in the opposite direction (b), but in the lab frame all spins are precessing in the same direction, as they always must

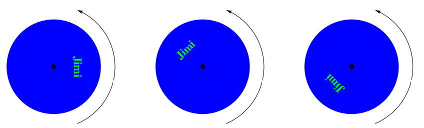
Now let's consider the effect of such a gradient pulse on a spin. The basic relationship of NMR is that the precessional frequency is linearly proportional to the magnetic field: $\omega = \gamma B$. Since the gradient, by definition, applies a magnetic field that varies (linearly) with space, spins at different locations along the direction the gradient is applied are immersed in different magnetic fields, and thus precess at different frequencies: $\omega(x) = \gamma G x$. Consider the simple situation shown



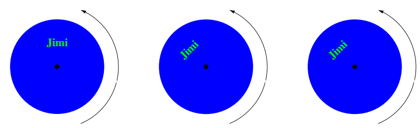
(a) Elvis record at constant $33rpm$ in the lab frame.



(b) Elvis record at constant $33rpm$ in the rotating frame.

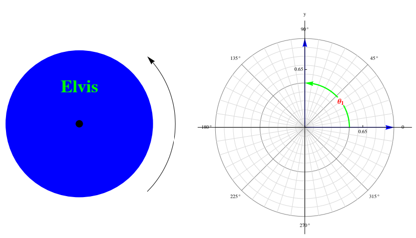


(c) Jimi record at $33rpm$, $78rpm$, $33rpm$ in the lab frame.

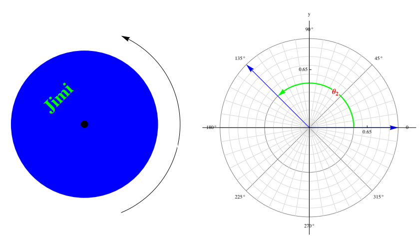


(d) Jimi record at $33rpm$, $78rpm$, $33rpm$ in the rotating frame.

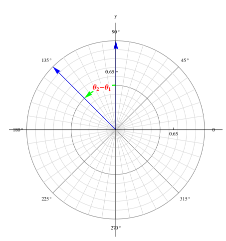
Figure 16.3 Phase shift, phase memory, and the rotating frame. (a) Two record players spinning at the same rate of $33rpm$. (b) The Jimi record is then switched to $78rpm$ and advances relative to the Elvis record. (c) The Jimi record is then switched back to $33rpm$ and now rotates at the same rate as the Elvis record, but is still rotated relative to Elvis. The difference in angle between the two records as a result of the brief period of that rate change of Jimi is the *phase angle* between the two.



(a)



(b)



(c) The difference in angle between the Elvis and Jimi records is the phase difference.

Figure 16.4 The concept of phase - the record player analogy.

in Figure 16.2b where we compare the case of a spin in the absence or presence of a gradient pulse G_x. In the absense of a pulse, the phase (relative to the rotating frame) remains constant. But in the presence of a gradient, a spin in the $+x$ direction feels an increased field, thus an increased frequency of precession. Thus, just as in Figure 16.3, it acquires a phase shift that

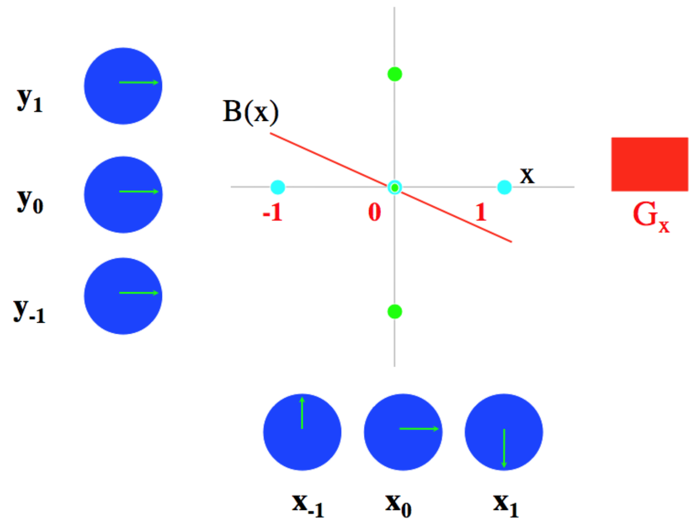


Figure 16.5 The directional dependence of the phase from a gradient pulse. The phase accrual only occurs along the direction of the applied direction, not perpendicular to it.

remains after the pulse has been turned off. It is important to note that this effect occurs only along the direction of the gradient, since that is the only direction along which the magnetic field is changing, as shown in Figure 16.5. This will become important when we talk about diffusion encoding along different "diffusion encoding directions".

The signal that we measure in this experiment, though, is the *sum* of the signal from the three spins. So what do we see during these different time periods? Recall that the signals add like vectors, as shown in Figure 16.6. If we say each spin has a signal of 1 in arbitrary units, we initially have a signal of 3. As the spins become out of phase, the signal decreases. For the example of three spins shown in Figure **??** that are $\phi = \{-180°, 0°, 180°\}$, the signal is zero. (see the right hand column of Figure 16.6). This dephasing is shown in Figure 16.7

Thus, at the completion of the gradient pulse, the spins that were excited by the RF pulse are spread out in frequency because they have felt a spatial variation in fields. The net signal from the transverse magnetization is therefore diminished by the process of phase cancelation. This is just the process of T_2^* relaxation that describes signal loss in the presence of external fields and depicted in Figure **??**. If the gradient area is sufficiently large, there is a complete loss of transverse magnetization. The loss of signal due to spin dephasing in the presence of a gradient is called *gradient spoiling* and is shown in Figure 16.8. This can be made use of in destroying unwanted transverse magnetization (such as might exist after the application of an imperfect inversion pulse). Gradients used for this purpose are called *spoiler*, or *crusher*, gradients.

To summarize:

$$\boxed{\textit{Magnetic field gradients spatially modulate the phase of the magnetization.}}$$

This spatial modulation causes the spin to dephase in a controlled fashion and this will be used to create an image. Of course, this "controlled fashion" is predicated on our instrument being

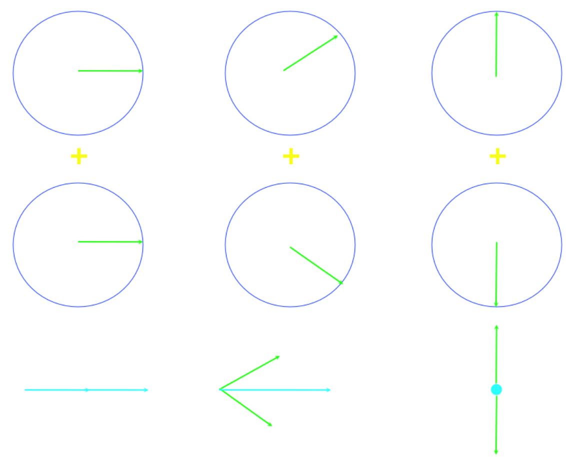


Figure 16.6 Signals from spins add like vectors.

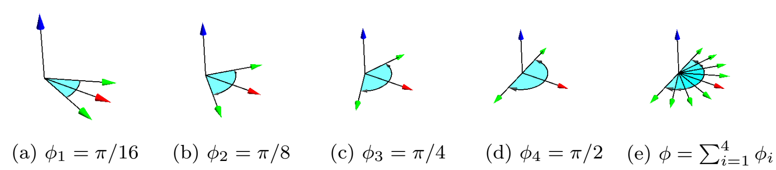


Figure 16.7 Spin dephasing. The four individual isochromats are shown in in (a-d) such as would be found at different spatial location in the presence of a gradient. The signal is the sum overall all spins (i.e., over all spatial locations) and thus can be represented as the combination of isochromats, which is often shown as in (e).

perfect (it's not) and the spins in our subject not moving (they do). If they had then the field they sense during the application of the second gradient depends on where they have moved to, which then requires that we know something about how they have moved. Now we do care about the individual spins, since we need to describe how each spin moves, and so the story become quite a bit more complicated. Enough so to require the writing of this book. One more point that bears repeating, because it will be ubiquitous throughout our discussions.

16.3 An illustrative idealization: Discrete Spins

The spatial variations of the precessional frequency in the sample due to applied magnetic field gradients is of such central importance to MRI that one must have a very strong understanding of its effects, and how it is related to the signal. For this, we shall consider three spins that are at three different spatial locations in the magnet, as shown in Figure 16.9. The collection of all

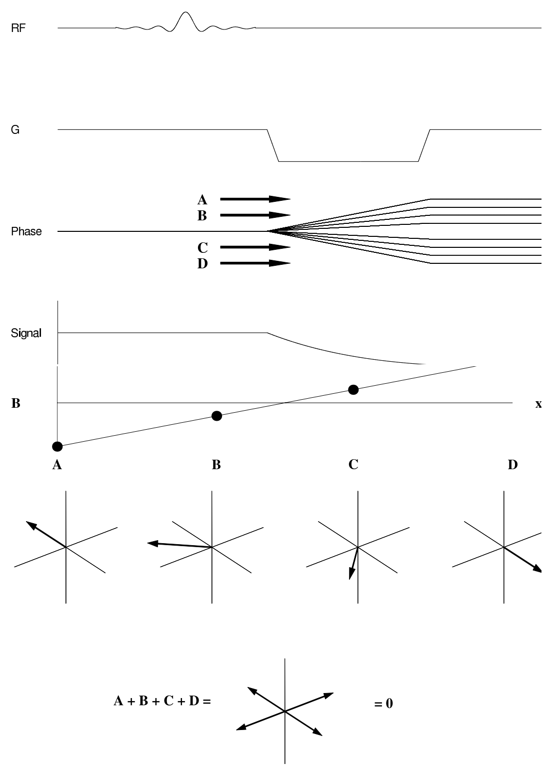


Figure 16.8 The dephasing of spins due to the application of a gradient pulse is called *gradient spoiling* and is useful for destroying unwanted transverse magnetization.

spins precessing at a particular frequency is called an *isochromat*. The additional field at each point due to the gradient is determined by:

$$\Delta B = \frac{dB}{dx}\Delta x = G_x \Delta x \tag{16.2}$$

So the precessional frequency at each point is

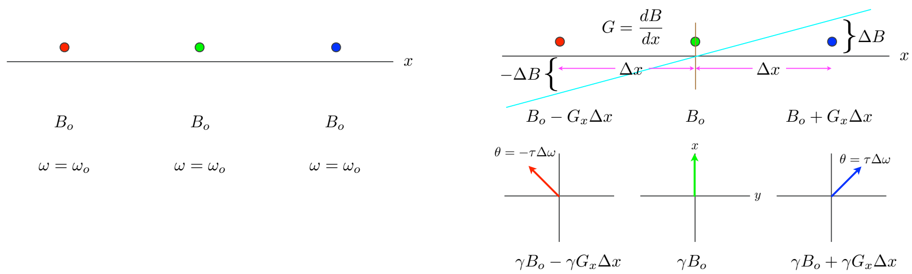
$$\gamma B_o + \gamma \Delta B \tag{16.3}$$

The total signal from the three spins is just the sum of that from each, as shown in Figure 16.10 The signal is simply

$$s(\omega) = \sum_i m_{xy}(x_i) e^{-i\omega_i t} \tag{16.4}$$

where the frequencies are

$$\omega(x,t) = \omega_o + \gamma G_x x \qquad \text{lab frame} \tag{16.5a}$$
$$\omega(x,t) = \gamma G_x x \qquad \text{rotating frame} \tag{16.5b}$$



(a) Three spins aligned along the x-axis in the presence of a static field B_o.

(b) Phase accrual of three spins aligned along the x-axis in the presence of both a static field B_o and a gradient in the x-direction.

Figure 16.9 Three spins distributed along the x-axis a distance Δx apart.

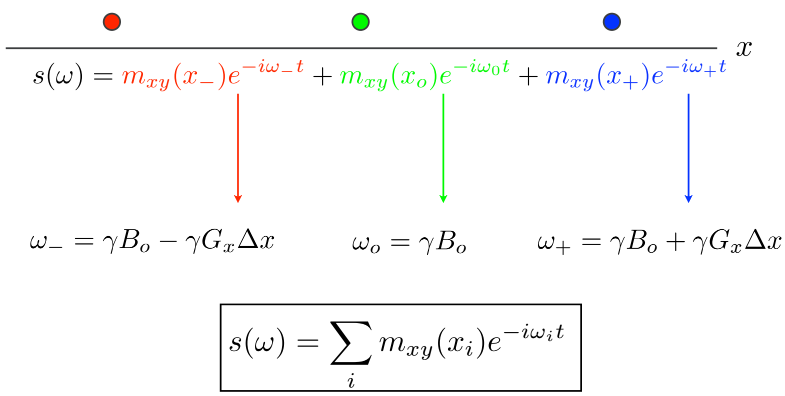


Figure 16.10 The signal $s(\omega)$ from the three spins is the sum of that from each. We use the notation $x_\pm \equiv x_0 \pm \Delta x$. (**Inconsistent notation! Change** m_{xy} **to** m_{perp}).

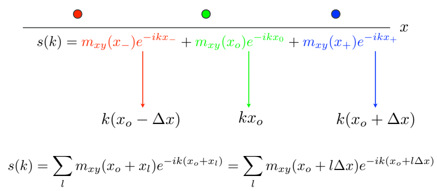
And thus we can rewrite ωt in terms of the *spatial frequency* k as

$$\omega(x,t)t = \underbrace{\gamma G_x t}_{k}\, x = kx \qquad \text{lab frame} \tag{16.6a}$$

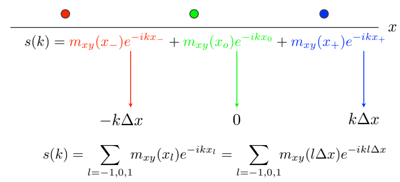
so we can rewrite the signal from the 3 spins Eqn 16.4

$$s(k) = \sum_i m_{xy}(x_i)e^{-ikx_i} \tag{16.7}$$

Not let's redraw the situation depicted in Figure 16.9 in terms of k, as shown in Figure 16.11 So



(a) Lab frame.



(b) Rotating frame.

Figure 16.11 Three spins aligned along the x-axis a distance Δx apart in the presence of both a static field B_o and a gradient in the x-direction. $k = \gamma G_x t$ and $x_\pm = x_o \pm \Delta x$.

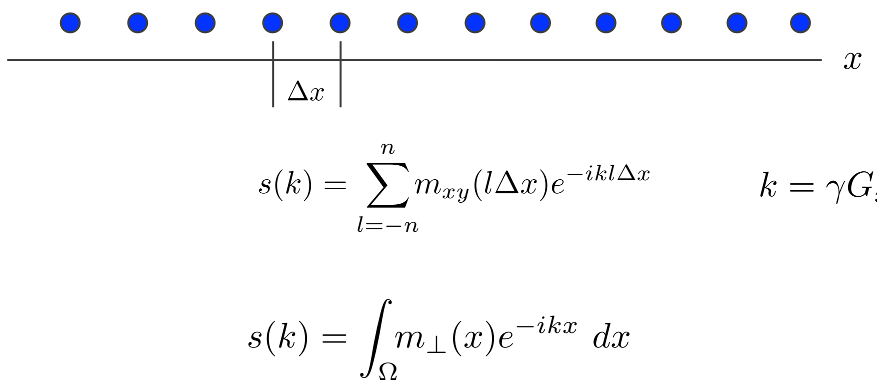


Figure 16.12 n spins aligned along the x-axis in the presence of both a static field B_o and a gradient in the x-direction.

we see the signal can be written for these three spins as

$$s(k) = \sum_{l=-1,0,l} m_{xy}(x_l)e^{-ikx_l} = \sum_{l=-1,0,l} m_{xy}(l\Delta x)e^{-ikl\Delta x} \tag{16.8}$$

from which we see the very important conclusion:

> *The signals from the individual spins add coherently when $k = 2\pi/\Delta x$, i.e., when the k value matches the spatial frequency of the spin distribution.*

Now let's extend this example to n spins aligned along x and equally spaced a distance Δx from one another, as shown in Figure 16.12. This is shown graphically in Figure 16.13

The signal is easily modified from Eqn 16.8 by just extending the range of l:

$$s(k) = \sum_{l=-n}^{n} m_{xy}(l\Delta x)e^{-ikl\Delta x} \tag{16.9}$$

and so again we see that the signal adds coherently when k frequency matches the spatial frequency of the spin distribution.

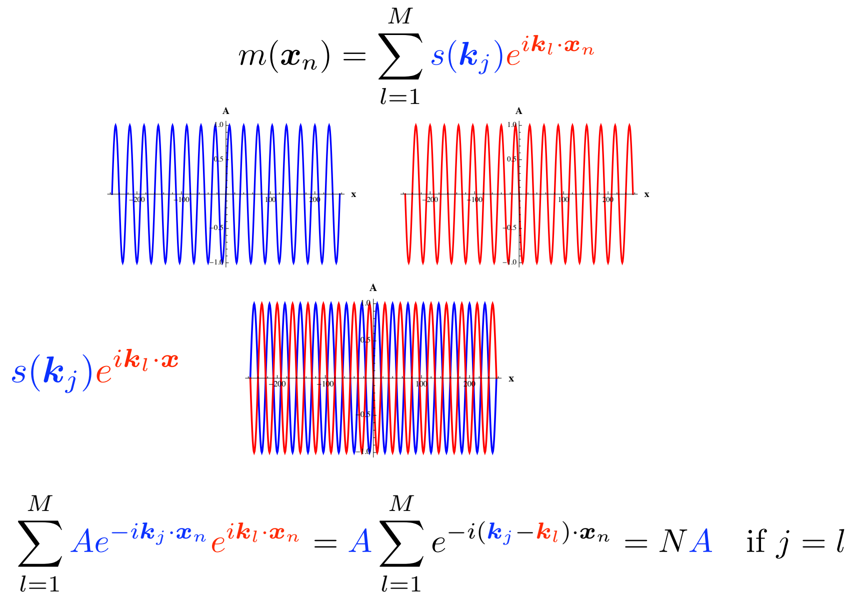


Figure 16.13 The inversion Fourier transform. The signals from the individual spins add coherently when $k = 2\pi/\Delta x$, i.e., when the k value matches the spatial frequency of the spin distribution.

Now, in our usual fashion (Section **??**) we can change from a discrete to a continuous distribution of spins in a spatial region Ω: by making Δx very small and l very large in Eqn 16.9 which results in the continuous form of the signal

$$s(k) = \int_{\Omega} m_{xy}(x)\, e^{-ikx}\, dx \tag{16.10}$$

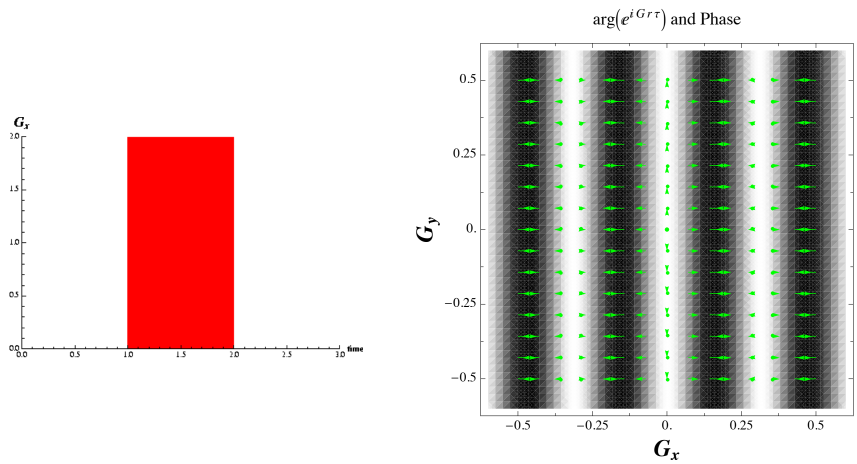
This is still in 1D, since we've only considered a single direction. But it's easy to extend Eqn 16.10 to higher dimensions

$$s(\boldsymbol{k}) = \int_{\Omega} m_{xy}(\boldsymbol{x})\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{x} \tag{16.11}$$

where we've simply replaced the single spatial dimension x with the vector $\boldsymbol{x}$, which represents $\boldsymbol{x}^t = \{x, y\}$ in 2D or $\boldsymbol{x}^t = \{x, y, z\}$ in 3D. The spatial frequency must also be replaced by its equivalent form $\boldsymbol{k}$, which represents $\boldsymbol{k}^t = \{k_x, k_y\}$ in 2D or $\boldsymbol{k}^t = \{k_x, k_y, k_z\}$ in 3D. Note also that now the "multiplication" of $\boldsymbol{k}$ and $\boldsymbol{x}$ takes the form of the dot product (Section 3.10). As we noted before (Section **??**), and is worth emphasizing again,

The signal is the Fourier transform of the transverse magnetization.

Eqn 16.11 is our basic MR signal equation. But now you should be able to look at it and see what it means: we are just adding up the magnetization and its phase at each point and summing them up over all points. And when the k value matches the spatial distribution spins' magnetization, the individual signals add coherently to give a large total signal. Therefore we



(a) An x-gradient pulse.

(b) Spatial modulation of the magnetization phase along x at completion of pulse in (a).

Figure 16.14 Spatial modulation of the magnetization phase produced by a gradient pulse along x generated by an x gradients. The spatial frequency of the phase pattern is $\Delta k_x = \gamma G_x \Delta t$ of the gradient pulse, which is proportional to the area of the gradient pulse.

need to consder this spatial modulation of the phase and how it relates to the imposed gradients and how it interacts with the inherent spatial structure of any object.

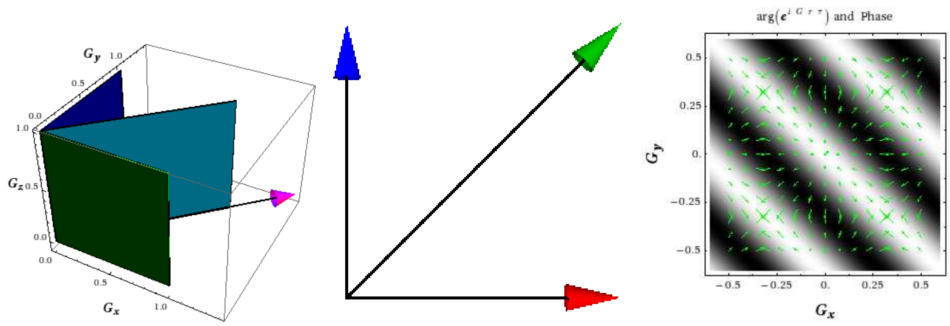
16.4 Spatial modulation of the magnetization phase

Now let's extend our discussion from the 1D problem in the previous section to 2D and consider the effect of applying a magnetic field gradient $\boldsymbol{G}$ along an arbitrary direction $\hat{\boldsymbol{e}}$ in the $x-y$ plane (this can be written $\boldsymbol{G} = G\hat{\boldsymbol{e}}$) for a time Δt. This is called a *pulsed field gradient*. Note that the effects of gradients add like vectors, so that a gradient along any direction in three dimensions can be created by the suitable combination of the three orthogonal (perpendicular) gradients along x, y, and z. A spin at position x will precess through a phase angle $\phi = \omega\Delta t = \gamma G x \Delta t = \Delta k\, x$. The variable $\Delta k = \gamma G \Delta t$ lumps together all the operator controlled variables (G,Δt). It is just the gradient area (its amplitude G times the length of time Δt it is applied), expressed in units of $1/distance$. Therefore at the completion of the gradient pulse, there is a spatial modulation of the phase of the magnetization along the direction $\hat{\boldsymbol{e}}$ of the gradient, with a *spatial frequency* (the distance between the ripplaes) of Δk, as shown in Figure 16.14: Because gradients add like vectors, this phase modulation can be produced in any direction in 3-dimensions. Let's consider how this works in two-dimensions. We construct the $\boldsymbol{k}$ and $\boldsymbol{x}$ vectors

$$\boldsymbol{k} = \begin{pmatrix} k_x \\ k_y \end{pmatrix} = \begin{pmatrix} G_x \\ G_y \end{pmatrix} t \qquad , \qquad \boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{16.12}$$

so that the dot product is

$$\boldsymbol{k} \cdot \boldsymbol{x} = k_x x + k_y y = G_x x t + G_y y t \tag{16.13}$$



(a) Gradients add like vectors. (b) The vector representation of the gradient in (a). (c) The spatial modulation of the magnetization.

Figure 16.15 Gradients add like vectors. A two dimensional example. (Here we use the same amplitude and extend the time for display purposes. It is the gradient area that matters.) Spatial modulation of the magnetization phase produced by a gradient pulse along an arbitrary direction in the $x - y$ plane generated by a combination of orthogonal (x and y) gradients. The spatial frequency Δk of the phase pattern is area $\Delta k = \gamma G \Delta t$ of the gradient pulse.

so we see that the effect of the separate gradients is that their effects add like vectors in the phase $\boldsymbol{k} \cdot \boldsymbol{x}$. An example is shown in Figure 16.15(a). The effect of the gradient direction on the spatial modulation of the magnetization is shown in Figure 16.15(c).

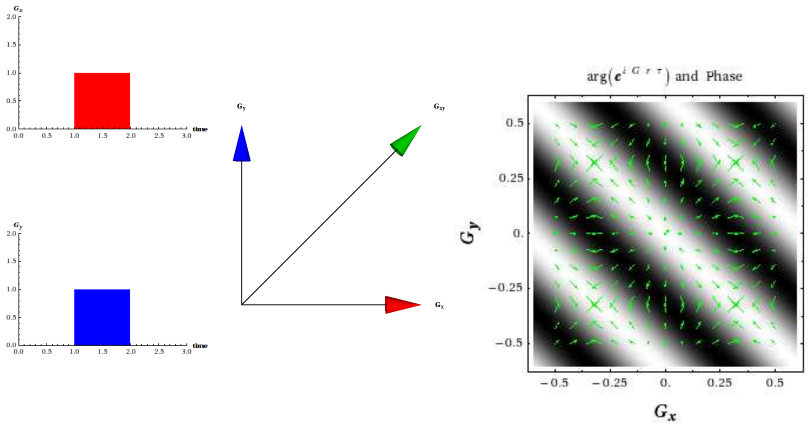
The effect of the gradient area on the spatial modulation of the magnetization is shown in Figure 16.16.

The effect of the gradient direction on the spatial modulation of the magnetization is shown in Figure 16.17.

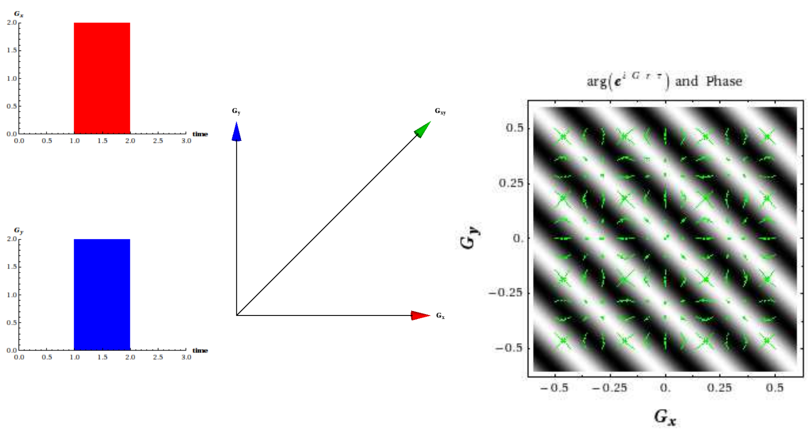
16.5 The Gradient Echo

When a gradient field is applied, spins in different locations precess at different rates. The total signal from all of these spins decays quickly because the spins in a volume become out of phase. Consider what happens, however, if a gradient is applied for a time Δt_1 but is followed by a gradient of opposite sign applied for a time Δt_2. Spins at position x that precess through an angle $\Delta\phi = \gamma G x \Delta t_1$ from the first gradient, will precess through an angle $\Delta\phi = -\gamma G x \Delta t_2$ from the second gradient. Therefore when $\Delta t_1 = \Delta t_2$, i.e., the gradients are reversed but on for the same duration, the total phase is zero. Note that this is independent of the location. Therefore, at the end of the second gradient all the spins come back into phase and the signal is regained. This process is shown in Figure 16.18. Note that it is really the product $G\Delta t$ that matters. This is the area under the gradient. Therefore the same effect can be achieved if, for example, the second reversed gradient is half as strong as the first but is twice as long in duration. The retrieval of this coherence by the application of a gradient of equal and opposite area is called *gradient refocusing*. If this refocusing is done during the period of data acquisition, the generated signal from this coherence is called a *gradient echo*.

To review, the three effects that result from the application of a pulse field gradient, 1) spatially modulated magnetization, 2) gradient refocusing, and 3) gradient spoiling, are integral compo-



(a) The phase for $\{G_x, G_y\} = \{1, 1\}$.



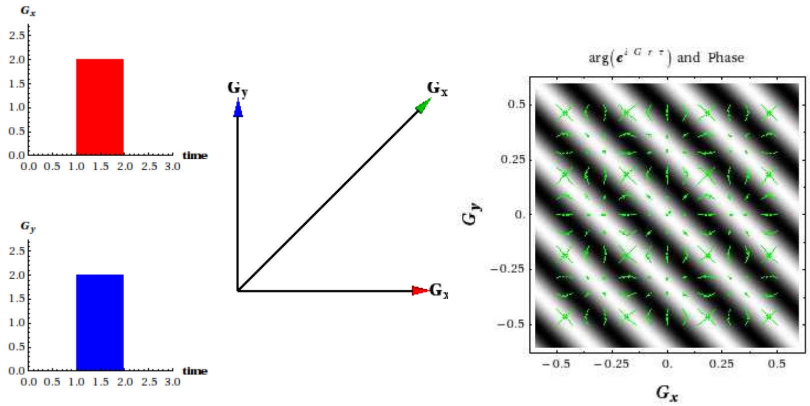
(b) The phase for $\{G_x, G_y\} = \{2, 2\}$.

Figure 16.16 Increasing the gradient areas (either by increasing their relative amplitudes by the same amount,as is done here, or by increasing their duration by the same amount)

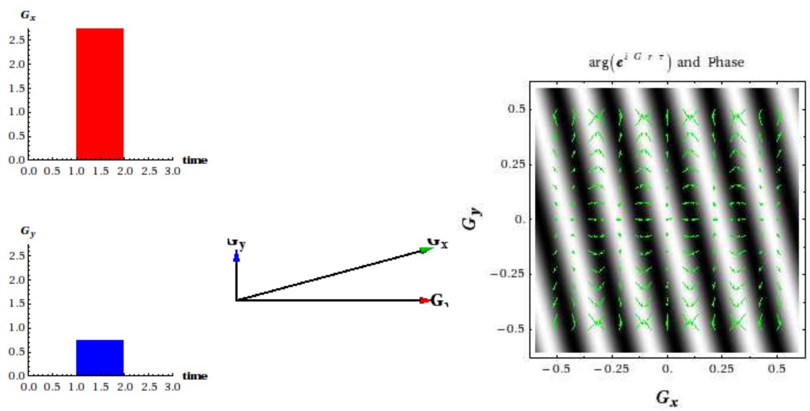
nents of the basic MR imaging pulse sequence. The first encodes spatial information, the second allows high signal during data acquisition, and the third helps limit unwanted signal components.

The effect of two consecutive gradient pulses of equal and opposite sign on three spins at three different spatial locations along the direction $\hat{\boldsymbol{e}}$ of the gradient: $x = \{x_o - dx, x_o, x_o + dx\}$ where x_o is the spatial location that the additional field due to the gradient is 0. Therefore spins at $x_o - dx$ precess slightly slower than those at x_o (and $x_o + dx$), and so their phase lags behind that of the spin at x_o while those at $x_o + dx$ precess slightly faster and so their phase gets ahead of that of the spin at x_o (and $x_o - dx$). This is shown in Figure 16.19.

So we see that the application of a gradient of the same but opposite area (why area?) causes the spins to come back into phase with one another, because the phase increase (decrease) caused



(a) The phase for $\boldsymbol{G}_{xy} = \{1, 1\}$.



(b) The phase for $\boldsymbol{G}_{xy} = \Re_{\theta}(-30°)\boldsymbol{G}_{xy}$.

Figure 16.17 Changing the gradient direction changes the direction of the phase modulation across the object.

by the additional field at location x is exactly matched by the decrease (increase) by the same gradient of opposite sign. This signal regaining by the unphasing and rephasing by equal and opposite gradients is called a *gradient echo*. Of course, in an actual voxel, there are many, many spins. But that doesn't matter - the effects shown in Figure 16.19 occurs for each individual spin. So we can plot the signal from a voxel and see that the same signal decrease then increase occurs there as well, as shown Figure 16.20.

Thus, if an equal but opposite gradient is applied, we get a gradient echo which means that the spins that decayed due to T_2^* from the application of the first lobe of the gradient all come back into phase and we get back a signal. Of course, the relaxation by both T_1 and T_2 occurs in the time between these lobes. The gradient echo is of fundamental importance to MRI for the following reason:

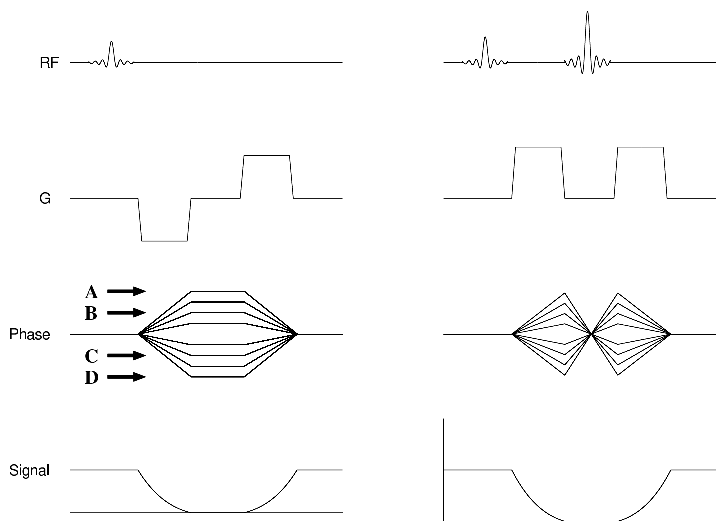
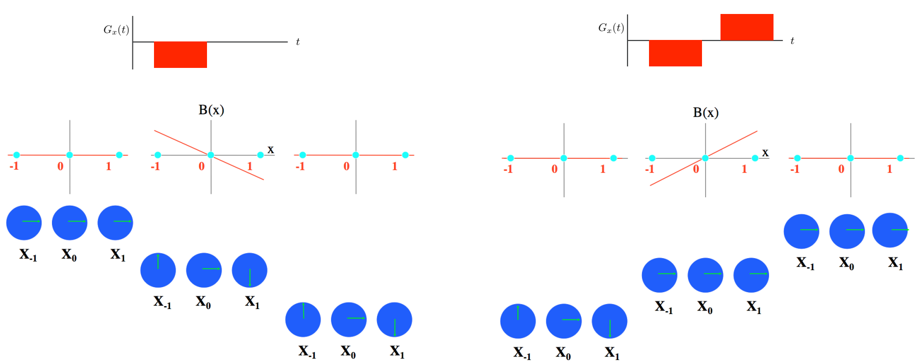


Figure 16.18 The dephasing caused by the application of a gradient pulse, as in Figure 16.8, can be undone and the transverse coherence regenerated by the application of a gradient of equal but opposite sign. This is called *gradient refocusing* and produces a *gradient echo*

.



(a) The spatial variation of phase in a gradient ....

(b) ... followed by the reversed gradient.

Figure 16.19 The bipolar gradient pulse.

> *While we can collect the signal from the FID, delaying its collection by the use of the gradient echo enhances the dependencies of the signal on the physical influences of the system, such as relaxation and diffusion, which, in their dependencies on the timings of the pulse sequence, can then be altered so as to create contrast between tissues for which these physical parameters differ.*

In a full pulse sequence, it is desireable to collect data from as much of the gradient echo as

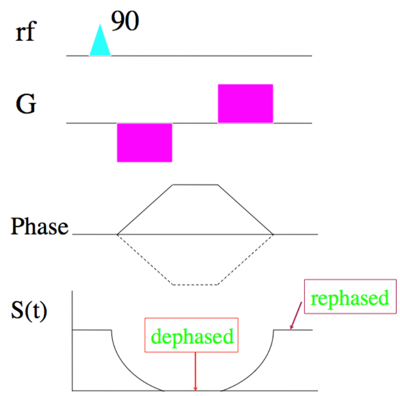


Figure 16.20 The bipolar pulse and the gradient echo.

is "visible" - that is, while it is forming and while it decays away. Below we will see how spatial encoding is performed, but for now it is enough to notice that if we made the second lobe of the bipolar gradient twice as long as the first as shown in Figure 16.21, the echo would occur precisely in the *middle* of the second lobe, since that is when the positive gradient area equals the negative gradient area, just our condition for the gradient echo.

16.6 Combining the gradient and spin echos: The Effective Gradient

Please recall that in a spin echo experiment the two lobes of the bipolar pulse have the same sign because the 180° pulse flips the spin (see Section **??**) so our present discussion on the gradient echo holds true, but you should get used to seeing a gradient configuration shown in Figure 16.22

and still recognize it as producing a gradient echo. Spin echo experiments also have gradient echos in them, if that's what you're wondering.

16.7 Image Formation

We have already seen in Section 14.14 what happens when there are variations in the z component of the field: the magnetization vector precesses at an angular frequency corresponding to the field it is in, and such variations produce a shift in the frequency. In the context of variations of the main field in Section 14.14, these produced frequencies that were always precessing at a different rate, because we were assuming the static field inhomogeneities were indigenous to the system. As such, these field variations change the local Larmor frequency and so we speak of these inhomogeneities as producing a range of *resonance* frequencies, because it is during excitation (which needs be at the Larmor frequency) that this comes into play. But more generally we can consider what happens if we have, in addition to the main field $B_o\hat{z}$ (assumed now to be perfectly homogeneous), a general magnetic field in the z-direction that is a function of position r in three-dimensional space $B_z(r)$. The local frequency will then be a function of position, from

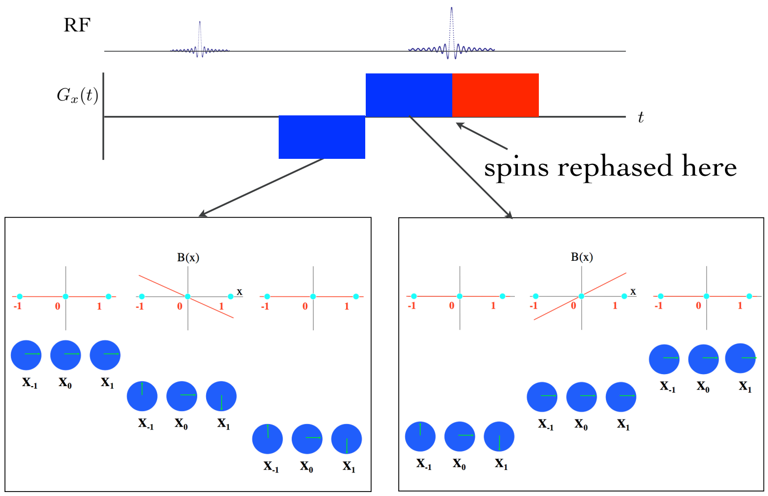


Figure 16.21 The gradient echo in a pulse sequence.

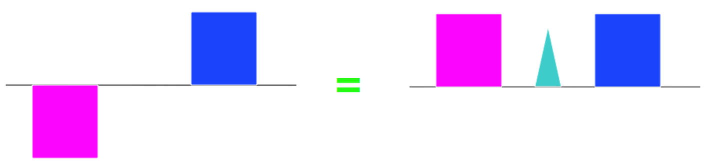


Figure 16.22 The gradient echo in a spin echo experiment: the effective gradient (**need to label pulse with a** 180°)

Larmor's theorem:

$$\omega(r) = \gamma B_o + \gamma B_z(r) = \omega_o + \omega(r) \tag{16.14}$$

The simplest form of variation is that the field varies linearly with position:

$$\boldsymbol{B}_z(r) = \boldsymbol{G}_r(t) \cdot \boldsymbol{r} \tag{16.15}$$

where

$$\boldsymbol{G}_r = \frac{\partial \boldsymbol{B}_z}{\partial r} \tag{16.16}$$

is called the *gradient* of the magnetic field. Note that it is the spatial variation of the z-component of the field. Note also that the variations of the field involve the dot product (Section 3.10) of the gradient with the vector $\boldsymbol{r}$. This is important - it means that the field varies *along*, i.e., parallel to, the gradient. This field variation then causes a spatial variation in the frequency, from Eqn 14.59,

$$\omega(\boldsymbol{r}, t) = \omega_o + \gamma \int_0^t \boldsymbol{G}_r(\tau) \cdot \boldsymbol{r}(\tau)\, d\tau \tag{16.17}$$

The temporal variation of spatial parameter $\boldsymbol{r}(t)$ shows that, as expected, the signal equations depend on the motion of the spins, such as flow or the subject of this book, diffusion. For now, we will only consider stationary spins so that $\boldsymbol{r}$ is *not* a function of time. In this case Eqn 16.17 becomes

$$\omega(\boldsymbol{r}, t) = \omega_o + \boldsymbol{k} \cdot \boldsymbol{r} \tag{16.18}$$

where we define the new parameter

$$\boldsymbol{k} \equiv \gamma \int_0^t \boldsymbol{G}_r(\tau)\, d\tau \tag{16.19}$$

In the rotating frame of our signal detection, the signal Eqn 14.58 then takes the form

$$s(t) = \int_\Omega \boldsymbol{m}_\perp(\boldsymbol{r}, t) e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\, d\boldsymbol{r} \tag{16.20}$$

The transverse magnetization is proportional to the spin density $\rho(\boldsymbol{r}, t)$ and so the signal equation can be written

$$s(t) = \int_\Omega \rho(\boldsymbol{r}, t) e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\, d\boldsymbol{r} \tag{16.21}$$

This is the most standard form that of the signal equation. Note that Eqn 16.16 is a vector quantity and as we have done many times now before, we can put this into an orthogonal coordinate system of our choosing (Chapter 2), and create an arbitrary vector $\boldsymbol{r}$ from its components. In our standard Cartesian system, this just means that $\boldsymbol{G}_r$ can be created from the three components along the three axes $\{x, y, z\}$:

$$G_x = \frac{\partial \boldsymbol{B}_z}{\partial x} \quad , \quad G_y = \frac{\partial \boldsymbol{B}_z}{\partial y} \quad , \quad G_z = \frac{\partial \boldsymbol{B}_z}{\partial z} \tag{16.22}$$

This type of gradient field is what a typical scanner produced using a special sets of coils designed to make fields that are very linear (an engineering problem of no small magnitude!). Since for a gradient that is constant (i.e., not a function of time) within the time interval $[0, t]$

$$\boldsymbol{k}(t) = \gamma \boldsymbol{G}_r \int_0^t d\tau = \gamma \boldsymbol{G}_r t \tag{16.23}$$

in which case the signal Eqn 16.20 takes the specific form

$$s(t) = \int_\Omega \boldsymbol{m}_\perp(\boldsymbol{r}, t) e^{-i\gamma \boldsymbol{G}_r\cdot\boldsymbol{r}t}\, d\boldsymbol{r} \tag{16.24}$$

As we noted before, the signal is the Fourier transform (Section **??**) of the transverse magnetization. And now this should be clear in comparison with Eqn **??**.

But the signal is just the data we collect. What we want to determine is the spatial map of the transverse magnetization $\boldsymbol{m}_\perp(\boldsymbol{r}, t)$. This is our *image*. But from our discussion in Section **??**

we know that if $s(t)$ is the Fourier Transform of $\boldsymbol{m}_\perp(\boldsymbol{r},t)$, then $\boldsymbol{m}_\perp(\boldsymbol{r},t)$ can be recovered, or *reconstructed*, by application of the Inverse Fourier Transform:

$$\boldsymbol{m}_\perp(\boldsymbol{r}) = \int s(t)e^{i\gamma \boldsymbol{G}_r \cdot \boldsymbol{r}t}\, d\boldsymbol{t} \tag{16.25}$$

The signal observed just at the end of the pulse is the sum of the signals from all spins that were excited. These spins now have a spatially modulated transverse magnetization phase. This brings us to the central concept of MR imaging. Recall the two main facts regarding Fourier analysis (which have nothing to do with MRI): 1) Any pattern can be described by the sum of sinusoids, 2) Correlating the pattern with test sinusoids tells us the contribution of the different spatial frequencies to that pattern. Thus we arrive at a central concept in MRI:

The image is the Inverse Fourier Transform of the data.

Notice the very important fact that both the data and its Fourier Transform are *complex* (Chapter 4). One of the fascinating aspects of MRI is that the data is complex, and that the phase has meaning. The magnetization is a real quantity and so its Fourier Transform should also be real[1]. But there are a variety of influences that can make this not the case, such as field imperfections, bulk motion, flow, etc. For this reason, the image is usually created by taking the magnitude of the magnetization $|\boldsymbol{m}_\perp(\boldsymbol{r})|$. Later we will find that these imperfections can create distortions of the image $\boldsymbol{m}_\perp(\boldsymbol{r})$ but can also be measured to correct the distortions. There are *many* practicalities and subtleties to the so-called *Fourier relationship* between the data and the image, but the basic idea is that you collect the data, take its Fourier Transform, take the magnitude of that, and there's your image.

We have just shown that a gradient pulse produces a spatially modulated wave of a single spatial frequency $\boldsymbol{k}$ and that the MR signal is the *sum* of the product of this modulation and the spin density. But this is precisely the operation of the Fourier Transform! Therefore, the data for a given gradient pulse, that is, a given wave of spatial frequency k, are simply *the contribution of that spatial frequency to the object*. We can therefore map out the contribution of many spatial frequencies by applying many gradient pulses of different k, that is, of different areas. When this process is finished, we have as our *data* the amplitudes of the contributions of the different k values. For this reason, the spatial frequency space is called *k-space* and the data in MR imaging are measurements of *k-space*. The *image* is then the object created from the many contributions of these waves of different spatial frequency. This is just the Fourier Transform of the data. Thus we can summarize MR imaging as follows: The gradient pulses map out the Fourier Transform of the object, and we therefore recover the object by *inverse* Fourier transformation the data. The brightness of a pixel is determined from the *magnitude* of the Fourier transformed data. But the image also contains a *phase* which represents the angle through which spins have precessed by the time of the echo. The phase contains important information because it is affected by a number of physical parameters, including field variations and flow.

Thus the image and the data form a *Fourier transform pair* that we saw in Section **??**. This is illustrated graphically in Figure 16.23. How the spatial frequency components contribute to the image can be easily visualized by exaggerating a single k-space point in the data, as shown in Figure 16.24.

A single point in *k-space* clearly represent a single spatial frequency, as seen in the spatial

[1] This is in Section **??**, which is not done yet!

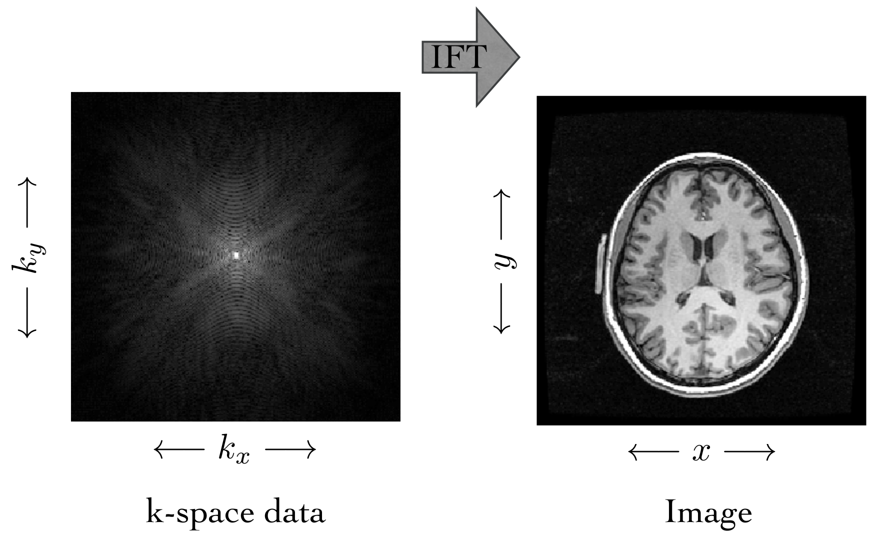


Figure 16.23 The data and the image form a Fourier transform pair.

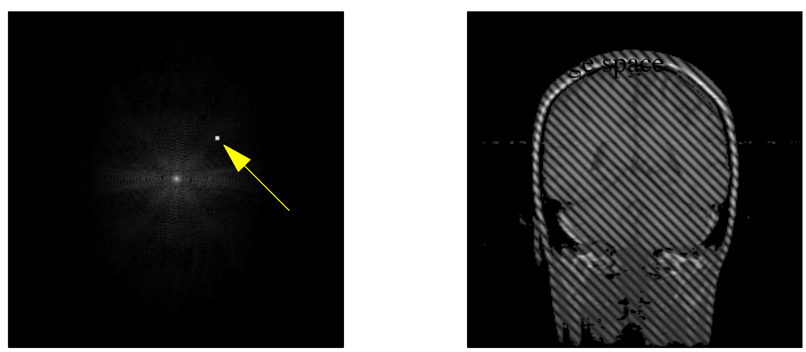
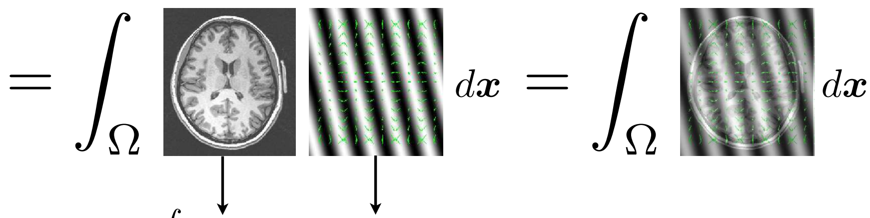


Figure 16.24 The perpendicularity (orthogonality) of the sinusoidal waves that compose a shape result in the fact that a spatially modulated wave is produced by a single point in the data from k-space (left) of an image. This is shown by artificially amplifying the amplitude of a single data point, the effect of which is to produce a wave of a single spatial frequency in the image (right). The k-space data on the left and the image data on the right are related by Fourier transformation (FT).**The yellow arrow needs to be from $k = 0$ to the point, and then explain that the angle is the same angle as the direction of the waves.**

modulation of the image intensity. This is just the modulation created by the gradient pulse in Figure 16.14. It is worth noting that the angle between the center of *k-space* and the amplified point is the same as the angle of the wave in the *image*. This is a manifestation of the *Fourier Projection Theorem* (**?**).

So we can understand the data Eqn 16.11 in the following graphical form: the magnetization (the image) is multiplied by a phase generated by the gradients and summed over all space, as



$$s(\varphi) = \int_\Omega m_\perp(\boldsymbol{x}, t) e^{-i\varphi(\boldsymbol{x},t)}\, d\boldsymbol{x}$$

Figure 16.25 Graphical depiction of the MR signal.

$$m_\perp(\boldsymbol{x}) = \int s(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{k}$$

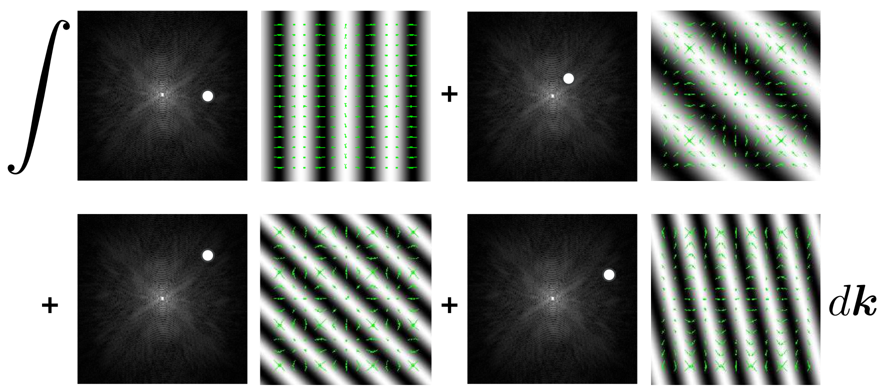


Figure 16.26 Graphical depiction of the MR image formation.

shown graphically in Figure 16.25 Conversely, we can understand image formation (reconstruction) as the inverse process, as depicted in graphically in Figure 16.26

16.8 Slice selective spin echo imaging

One might gather from the previous discussion that the way MR images are acquired is simply to apply many combinations of bipolar pulses of the three orthogonal gradients $\{G_x, G_y, G_z\}$, thus creating a net gradient in many directions, thereby gathering up enough samples in k-space to reconstruct the image. In fact, one could do this, but it is not the most efficient way to collect data. The technique that is considered in some ways to be the most "basic" sequence is, in fact, an excellent example of the versatility of the methods of MRI, because, although the result is a volume of data where the contrast between tissues has been manipulated, the data in all three directions $\{x, y, z\}$ are all collected in a different manner. Two of the directions (defined to be

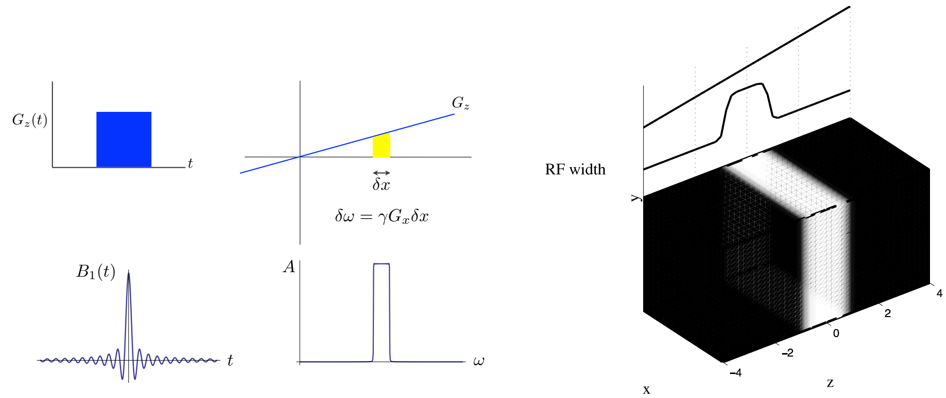


Figure 16.27 Slice selection. Application of an rf pulse with a small bandwidth of frequencies in the presence of a gradient field excites spin within that range of frequencies. Since this range of frequencies in the tissue is caused by a gradient pulse, it is equivalent to a spatial width of excited spins and, hence, a slice is excited.

"in-plane" or $x - y$) are spatially encoded using the gradient induced spatial phase variations we just covered. The third, or "slice", dimension is defined by a new concept we introduce here, which is *slice selection*. In these so-called *slice selective* acquisitions, the excited region of a subject is made to be a plane (e.g., a slice) so that that is all the spins that are actually visible to the coil. The spatial encoding with the gradients thus only affects these visible spins, and so the experiment is essentially 2-dimensional in nature. These are combined with the spin echo sequence of RF pulses in order to refocus T_2^* effects, and the result is the *slice selective spin echo* acquisition. Of course, the gradient echos are always present so don't be fooled by the terminology. Spin echo images really mean spin *and* gradient echo images. Later we will discuss experiments in which the entire subject (or "volume") is excited and the gradient induced phase in all 3-dimensions is used. These are called *volume* acquisitions. It turns out that it is usually more efficient not to use a spin echo for these acquisitions. This leaves on the gradient echos, and so these will be called *gradient echo* acquisitions. More about those later in Chapter 18.

16.8.1 Slice selection

We saw that it was possible to excite spins precessing at the Larmor frequency by the application of an RF pulse at the Larmor frequency. Consider now what happens if an RF pulse is applied while a gradient field is also being applied. Let us assume that the RF pulse, rather than being at a single frequency ν_0, has a small frequency range or *bandwidth* $\Delta\nu$ about ν_0. Applying the RF pulse then excites all spins that are within this frequency range. As the frequency range corresponds to a range of positions (because of the gradient field), the result is that a width, or *slice*, of spins is excited. This process is called *slice selection*. After slice selection, only the spins in the slice are visible to the receiver coil. This is shown in Figure 16.27.

Note that after slice selection has taken place, there is a spatial modulation of the magnetization across the slice due to the slice selection gradient. This will result in a signal loss within the excited slice due to dephasing. This situation can be ameliorated by the application of a refocusing gradient immediately following the slice selection.

(**Put slice selective equation here!**)

16.8.2 Phase encoding

The basic concept behind phase encoding was introduced previously when the signal immediately following a gradient pulse was examined. It was shown that a gradient of area $\Delta k = G\Delta t$ produces a spatial modulation of the transverse magnetization and that the net signal is then sensitive to only that spatial frequency component in that object. In other words, the phase distribution *encodes* the spatial frequency. Now recall that real objects (e.g. knees) contain many spatial frequencies. To form an image of an object is to measure the range of spatial frequencies in the object. This is accomplished simply by applying many gradient pulses of different areas, and therefore different k values. This is the process of *phase encoding*. The most straightforward way to apply this scheme is to repeat the pulse sequence many times, each time changing only the area of the phase encoding gradient. One standard method of achieving this is to vary the gradient area by fixing the gradient width and to increment the gradient amplitude. This is the *spin warp* scheme used in the standard implementation of many pulse sequences, for example, the spin echo pulse sequence shown in Figure 16.30. The time interval that the sequence is repeated is just the repetition time, T_r. Thus the total scan time in this method is just the number of phase encoding steps times the repetition time.

(**Put phase encoding equation here!**)

16.8.3 Frequency encoding

Consider now what we would see if, instead of applying successive phase encoding gradients of increasing area $k = \gamma G n\Delta t$, $(n = 0, 1, .., n_{max})$, we apply a gradient of maximum area $\Delta k = \gamma G n_{max}\Delta t$ and look at the signal when the gradient area is at increments of $k = \gamma G\Delta t$. What we would see at the successive steps $(n = 0, 1, .., n_{max})$ would be precisely the same thing that we observe after successive phase encoding steps, since we are just observing the influence of increasing gradient areas of the same amount applied to the spins. This process of *frequency encoding* is, in fact, exactly the same process as phase encoding, as shown in Figure 16.28. Both measure the total signal after the application of a gradient, thereby measuring the k component defined by the gradient area and hence that spatial frequency component of the object.

In practice, the frequency encoding gradient of area $k_{max} = \gamma G n_{max}\Delta t$ is preceded by a negative “precompensation” gradient of half this area. The precompensation pulse produces a total area at the beginning of the frequency encoding gradient of $(-n/2)\Delta k$. Therefore, at the center of the read gradient, when its area equals that of the precompensation pulse, a gradient echo occurs, since all spins come into phase. This is the point $k = 0$. The phases around this point are therefore symmetric, running from $(-n/2)\Delta k$ at the beginning (as a result of the precompensation pulse), to $(n/2)\Delta k$ at the end, since an equal area of gradient has been applied after $k = 0$. The results would therefore appear to be a redundant acquisition of the spatial frequencies from $-k_{max}/2$ to $k_{max}/2$. As we will see later, this is not necessarily the case.

Another way to look at frequency encoding is as follows: When the read (x) gradient is turned on during data collection, spins precess with a frequency that is proportional to their position x. The detected signal is the sum of signals from the spins along the x direction. The amplitude

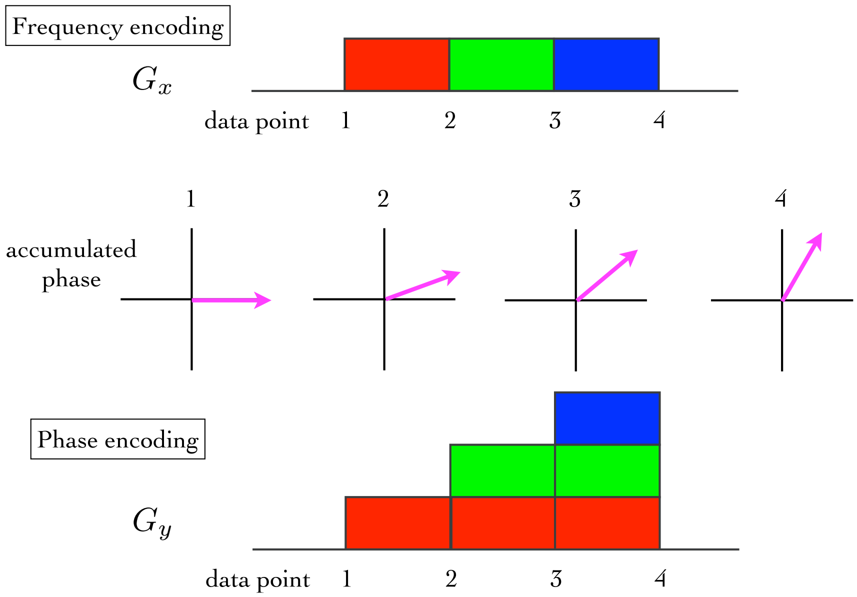


Figure 16.28 The equivalence of frequency and phase encoding. The spatial frequency encoding is proportional to the gradient area. This may be accomplished either with brief pulses of varying area (phase encoding, bottom) or by applying a constant gradient but sampling the data at intervals of equal gradient area (frequency encoding, top).

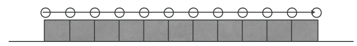
of the signal from the different locations is the amplitude of the different frequencies, which is found by Fourier transformation of the signal. The amplitude of the signal at each frequency is the total amplitude of all spins at the corresponding x position, and thus is the sum over every y position for a given x position. This is called the *projection* along y.

(**Put frequency encoding equation here!**)

16.9 The basic pulse sequence components

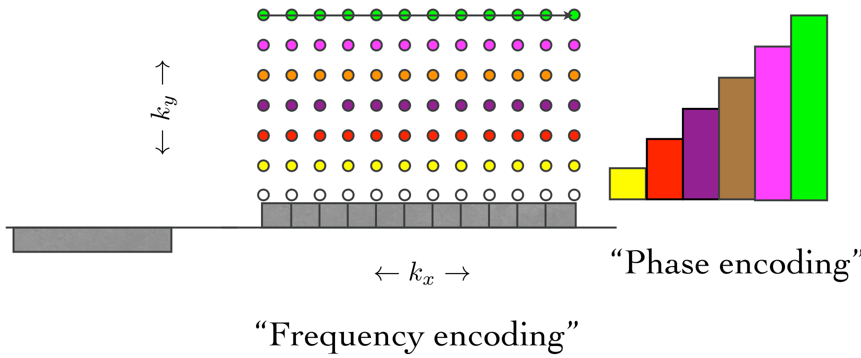
We now have all the basic components to construct an imaging pulse sequence. In Figure 16.30 is shown the standard pulse sequence diagram for the basic spin echo pulse sequence. The horizontal axis is time, and the vertical axis is amplitude. Each of the vertical lines represents the separate component of the scanner over which the operator has control: the gradients on each physical axis (x, y, z), the RF pulse, and the data acquisition (denoted A/D or *analog-to-digital* conversion since the received analog signal is converted to digital, or numerical, form for storage on the system computer).

Reading from left to right, in increasing time, the three operations of slice selection, phase encoding, and frequency encoding occur in sequence. These gradients are applied along perpendicular directions that are traditionally labeled z, y, and x, respectively. The slice select block consists of an RF pulse in the presence of a z gradient which is immediately followed by a com-



$$k_x = \gamma G_x t = \gamma G_x n \Delta t$$

(a) Frequency encoding.



(b) Frequency and phase encoding.

Figure 16.29 The gradient pre and post.

pensating (refocusing) z gradient (half the area, opposite sign) that excites a slice of spins. One step in the phase encoding (y) gradient is then applied, followed by a compensation pulse of area $-(n/2)\Delta k$ that occurs *before* the read gradient (hence, *pre-compensation* pulse) followed by a frequency encoding gradient twice the area ($n\,\Delta k$) of the pre-compensation pulse. During the frequency encoding gradient, the RF receiver is turned on and data are acquired at n points. For this reason, the frequency encoding gradient is often called the *read* gradient This particular pattern of gradients in which the phase encoding gradient is incremented for each repetition is called the *spin warp* technique In this method, one spatial frequency component in the y-direction is collected for *every* one of the k components collected in the x direction by the process of frequency encoding.

(**Put the full equation for phase and frequency encoding here!**)

The gradient echo is designed to be at the center of the data acquisition to increase the signal intensity by generating a large coherence of the spins. It is important to realize that this phenomenon is occurring even in a spin echo experiment - the spin and gradient echoes are usually (though not necessarily) both set to occur at the center of the data acquisition. Thus, although the terminology of *spin echo* and *gradient echo* pulse sequences suggests their mutual exclusivity, in fact the spin echo sequence always includes a gradient echo.

16.10 The k-space trajectory

A very important concept in MRI is the location of the sequential samples in k-space. This path in k-space, or *k-space trajectory*, traced out by the imaging scheme is really the defining characteristic of a pulse sequence, and what MR physicists spend a lot of time designing. The

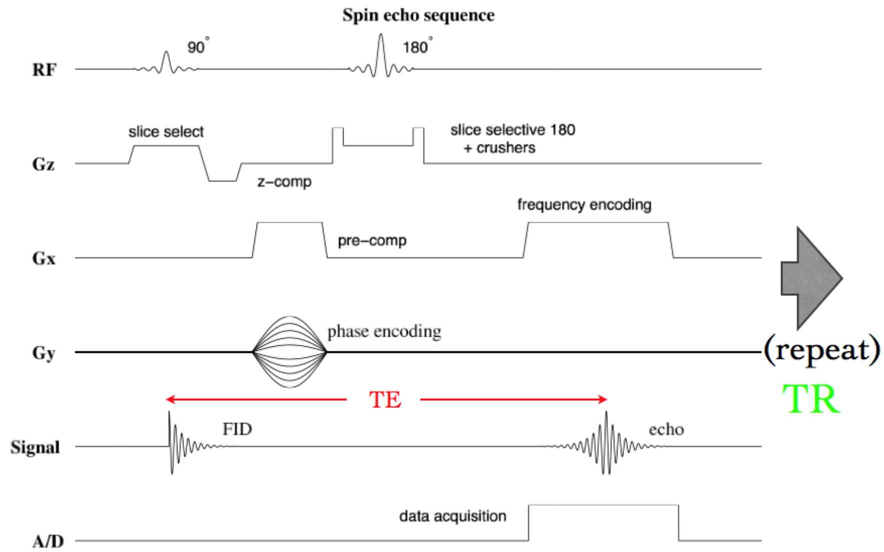


Figure 16.30 Basic spin echo pulse sequence. The radio frequency (rf) pulses include the 90° slice selective excitation and the 180° slice selective spin echo. The gradient G_z includes the slice selection pulse and the accompanying refocusing or compensation pulse for the 90° pulse, and the slice selection pulse, bracketed by crusher gradients, for the 180° pulse. The gradient G_y performs the phase encoding. The signal consists of the FID following the excitation and the echo (consisting of both a spin and a gradient echo). Data are collected during the frequency encoding gradient so only the echo signal is detected.

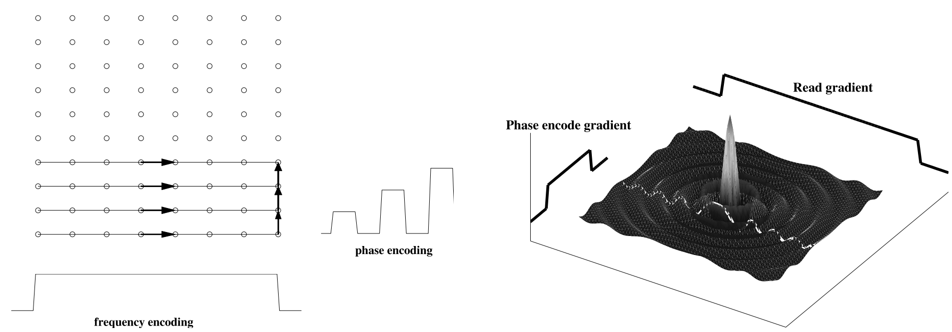
pulse sequence details, such as those shown in Figure 16.30, are just that - details. Given the software and hardware characteristics of a particular scanner, the pulse sequence components are really just composed of basic building blocks combined to give what one is really after - a particular k-space trajectory.

For example, in the conventional spin echo pulse sequence using the spin-warp scheme (Figure 16.30), the k-space trajectory is a simple raster pattern, as shown in Figure 16.31.

This k-space trajectory is advantageous because it gives samples that are on a simple Cartesian grid with equally spaced points. Since the processing to reconstruct the image is a Fourier Transform, images can be very rapidly reconstructed by acquiring samples that are powers of 2 in both x and y dimensions (recall the discussion from Section **??**) which then gives a simple Cartesian grid with equally spaced points whose length is a power of 2 in each dimension. The image can then be reconstructed by simply taking the 2D FFT of the data.

For the Cartesian trajectory, the trajectory depends upon the maximum value of k sampled, denoted k_{max}, and the separation Δk of the sample points in k-space. These two choices have a very important affect on the final image. The highest spatial frequency sampled, k_{max}, determines the high spatial frequency of the object that can be deteced, and thus is related to the image *resolution* δx:

$$\delta x = \frac{\pi}{k_{max}} \tag{16.26}$$



(a) All of the k_x components are collected during the application of the read gradient following the application of a single phase encoding pulse that encodes a single k_y component. Each phase encoding pulse moves the trajectory to a new k_y line.

(b) The path of the trajectory in k-space is along the k_x direction for a fixed value of k_y.

Figure 16.31 K-space trajectory for the basic spin-warp imaging technique.

The smallest spatial frequency determines the extent of the image, that is, the *field-of-view* or *FOV*

$$FOV = \frac{2\pi}{\Delta k} \tag{16.27}$$

But the image characteristics are also determined by the actual sequence of collection of the k-space points, the so-called *k-space trajectory*. For example, the phase encoding ordering can have a profound influence on the image contrast. For example, sampling from the center of k_y outward ($k_y = 0, \Delta k_y, -\Delta k_y, 2\Delta k_y, -2\Delta k_y, ..., k_{y,max}, -k_{y,max}$), called *centric* phase encoding, effectively shortens the time between the excitation and echo center and thus provides T_1 weighting. Sampling from the highest to lowest k_y components ($k_{y,max}, ..., -k_{y,max}$), called *linear* phase encoding, increases the time between the excitation and the echo center, thus producing T_2 weighting.

The interaction between physical parameters and the final image is to a large degree dependent upon the k-space trajectory, which can thus be tailored to manipulate these effects. This will play a critical role, for example, in our estimation of motion that degrades multi-shot diffusion imaging methods.

16.11 Resolution and Field-of-View

Recall that the center of k-space occurs at the center of data acquisition during the application of the frequency encoding gradient, since that is where the sum of the total gradient area from the first half of the read period and the precompensation gradient is to zero. That is, the frequency encoding collects k-space components from $-k_{x,max}$ to $k_{x,max}$. Similarly, the phase encoding steps through a range from $-k_{y,max}$ to $k_{y,max}$. The center of k-space contains the low spatial frequencies and the spatial frequencies become progressively higher as in a *radial* direction outward from the center of k-space.

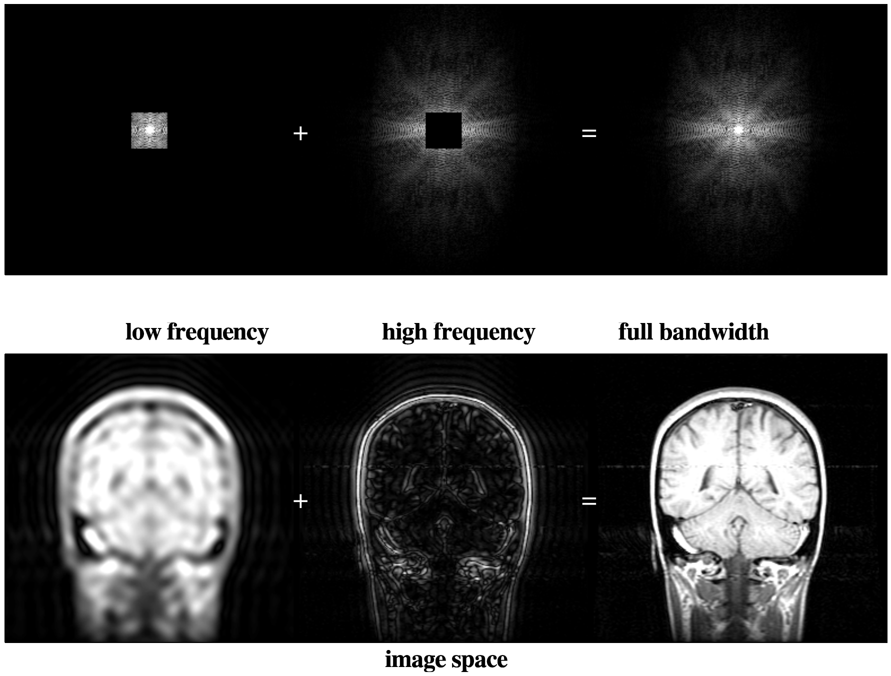


Figure 16.32 Building up the MR image. The low spatial frequencies are at the center of the k-space data (top row, left) and increase radially outward, with the high spatial frequencies residing near the outer portion (top row, center). The image is the sum of the low and high frequency components (right). An important property of the Fourier transform is that the Fourier transform of the sum of the k-space data is identical to the sum of the Fourier transforms of the separate k-space components. That is, $FT[k_{low}] + FT[k_{high}] = FT[k_{all}] = I_{all}$ is the same as $I_{low} + I_{high} = I_{all}$.

This can be illustrated by using the aforementioned property of Fourier transforms (Chapter **??**): The Fourier transform of the sum of the k-space data is identical to the sum of the Fourier transform of the data. We can thus decompose the data into the low and high spatial frequency components by taking the central and outer portions of the data, respectively, and looking at the Fourier transforms (i.e., the images) of these components. The sum of these *images* is identical to the actual image. This is shown in Figure 16.32. Notice that the farther out from the center of k-space the data are collected, the sharper the image. The higher the spatial frequencies that are collected, the better edges are resolved, so the better neighboring structures can be resolved.

The *resolution* Δx of the image, defined as the smallest structure that can be resolved, is therefore related to the inverse of the highest spatial frequency collected:

$$\Delta x = \frac{\pi}{k_{max}} \tag{16.28}$$

Notice also that the low resolution image on the left of Figure 16.32 has strong banding patterns that appear emanate from the bright edges. This is the Gibbs ringing (Figure 11.5) produced by the high contrast thin structures of cartilage. Because only the low spatial frequency data are used to form this image, structures with sharp edges are not well represented, and the result is a strong intensity ripple at these edges. The highest spatial frequency determines the smallest observable spatial variation, i.e., the resolution. What about the lowest spatial frequency? The lowest spatial frequency in the data represents the *largest* observable spatial variation. This represents the entire visible region, also called the *Field of View*, or *FOV*.

$$FOV = \frac{2\pi}{\Delta k} \tag{16.29}$$

16.12 Bandwidth

The relationship between space and temporal frequency means that the image dimensions can also be described in terms of frequency. This is a useful description when we discuss off-resonance effects. In Equation 16.29, the minimum spatial frequency Δk, and hence *FOV*, is determined by the gradient strength and the time Δt between the collection of consecutive data points: $\Delta k = \gamma G_x \Delta t$. Δt is called the *sampling interval* and its inverse, $1/\Delta t$, is called the *sampling rate*. Typically, the *FOV* is adjusted by changing the gradient strength since the sampling rate is fixed by the receiver data acquisition hardware. A typical sampling interval is one data point every 4 μs, that is, a sampling rate of $1/(4\ \mu\text{s}) = 256kHz$. This sampling rate is also equal to the range of frequencies to which acquisition is sensitive, or the *bandwidth*. If 256 data points are collected, the total *acquisition time* is $256 \times 4\ \mu\text{s} = 1$ ms. The *receiver bandwidth* is the extent of frequencies to which the receiver is sensitive and this must be greater than the image bandwidth. A read gradient of $G_x = .3G/cm$ creates a modulation across a $FOV = 24$ cm of $\gamma G_x FOV = 4258Hz/G \times .3G/cm \times 24cm \approx 32kHz$. This is the *image bandwidth* which means that two spins on opposite sides of the image have precessional rates that differ by $32kHz$. Each of the 256 voxels differ in precessional rate from its neighbors by the *bandwidth per pixel* of $32kHz/256 = 125Hz$.

The description of the image in terms of bandwidth and bandwidth per pixel is quite useful in the understanding of certain types of artifacts, such as chemical shift and susceptibility variations, in which frequency variations interfere with the spatial mapping procedure to create image distortions, as discussed subsequently. This description in terms of frequencies will prove particularly handy when discussing the effects of frequency variations in the field or due to the presence of difference chemical species with different resonance frequencies, as we shall later see in our discussion of echo-planar imaging (Section **??**) and its associated artifacts (Chapter 19).

Suggested reading