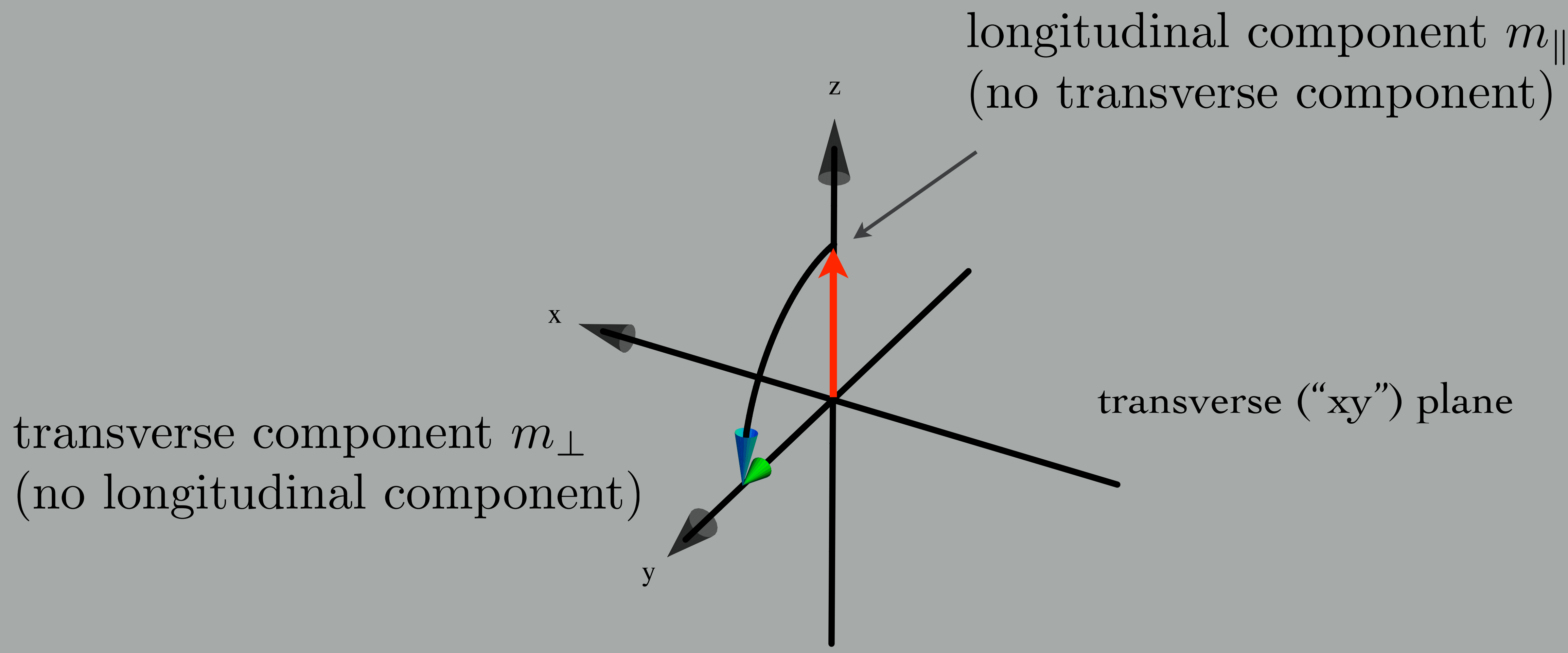
Lecture 5
Magnetic Resonance Imaging: Image Formation

Lecture Summary

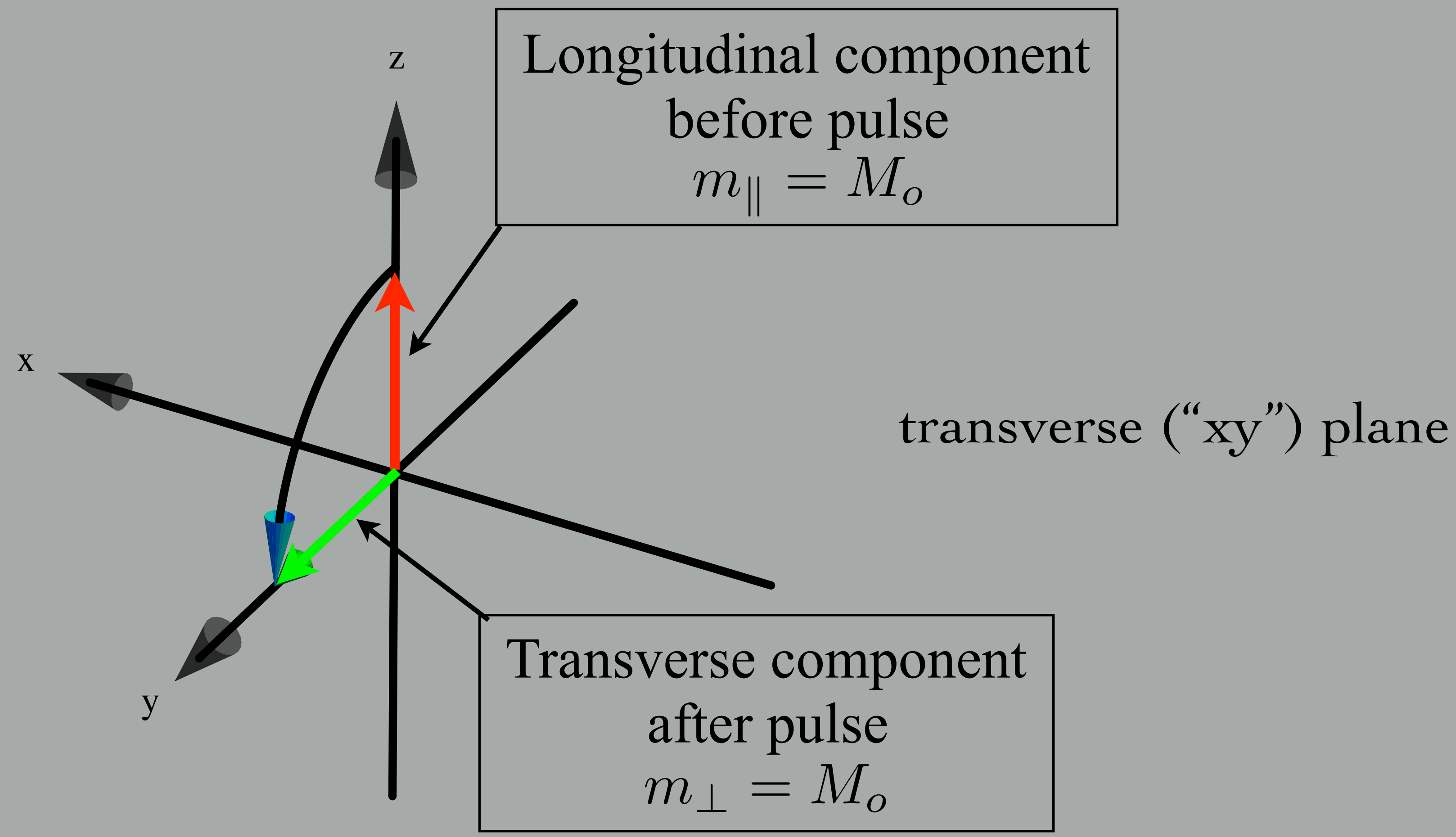
1. The Spin Echo
2. Magnetic field gradients
3. Spatial modulation of phases
4. The Gradient Echo
5. The NMR signal
6. The NMR image

Excitation



90° RF pulse

Excitation



90° RF pulse

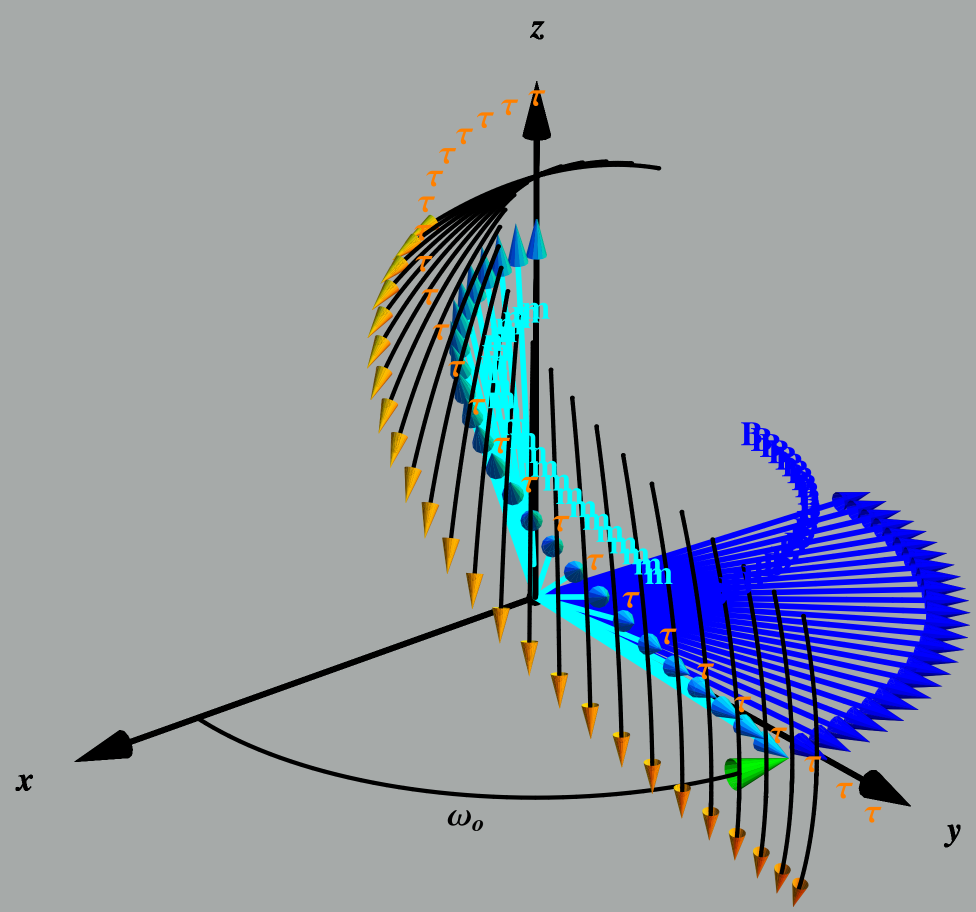
Rotating Frame

lab frame

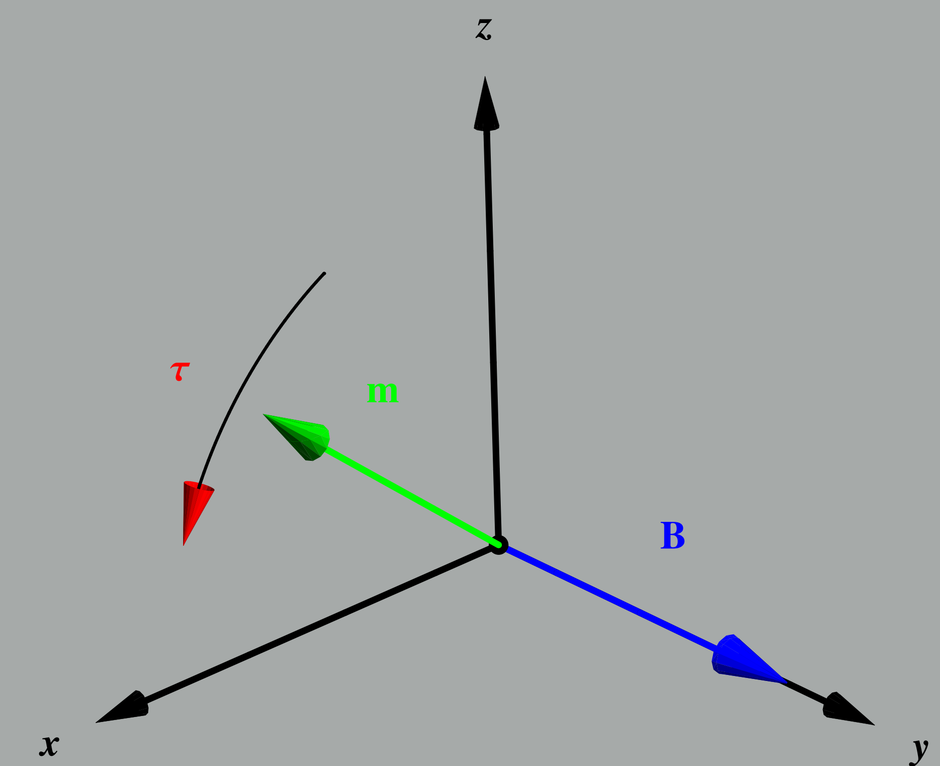
rotating frame

$$\omega_o = \gamma B_o$$

Excitation



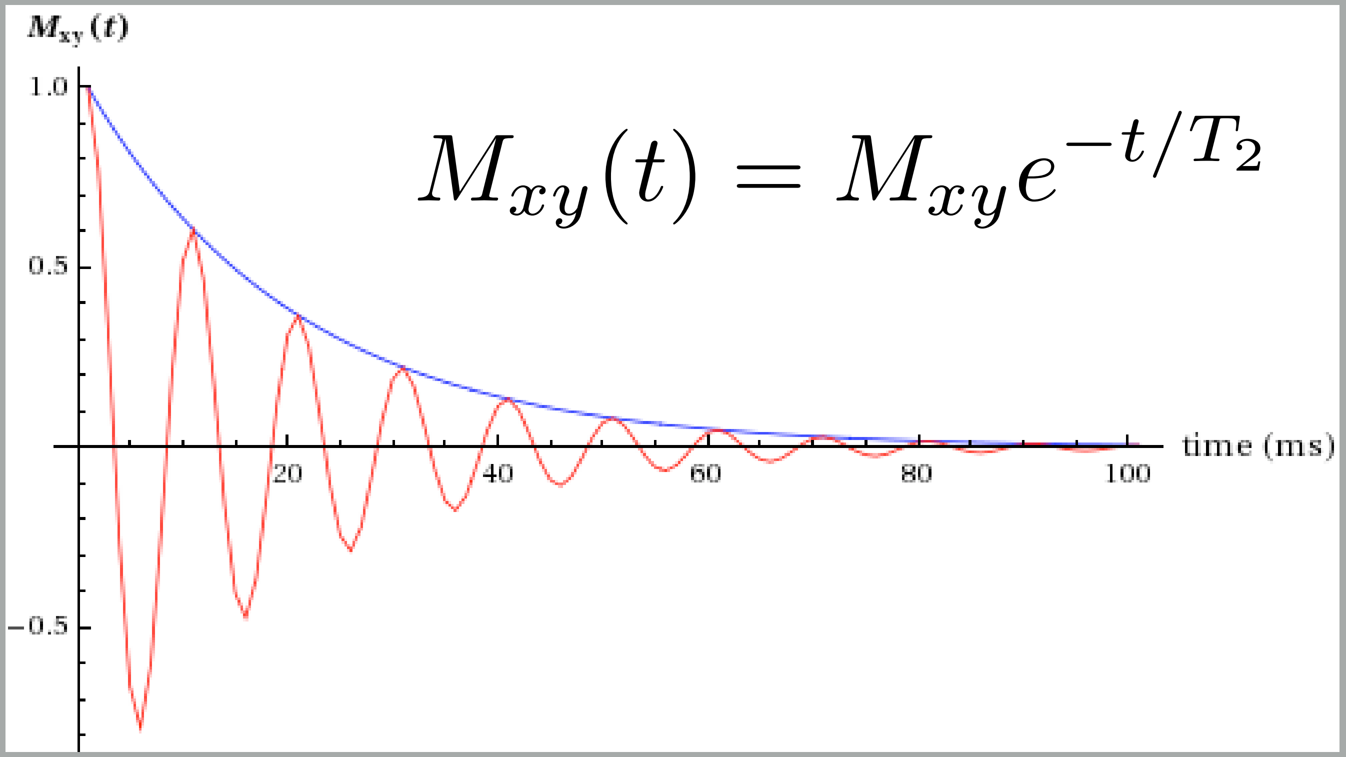
lab frame



rotating frame

A magnetic field B_1 applied along the y' axis that rotates at frequency $\omega_o = \gamma B_o$ relative to the lab frame is called the *rf excitation pulse*, since ω_o is in the radio-frequency (MHz) range.

Free Induction Decay



Free induction decay for a single isochromat
blue=on resonance
red=off resonance

T_2 is irreversible intrinsic transverse decay

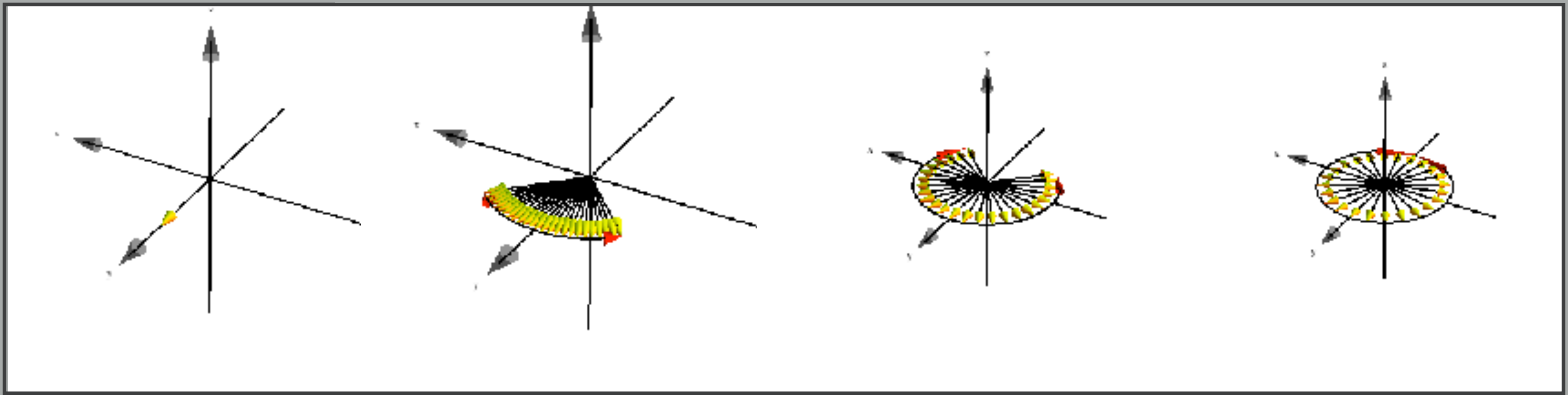
Free Induction Decay

Immediately following termination of
excitation pulse:

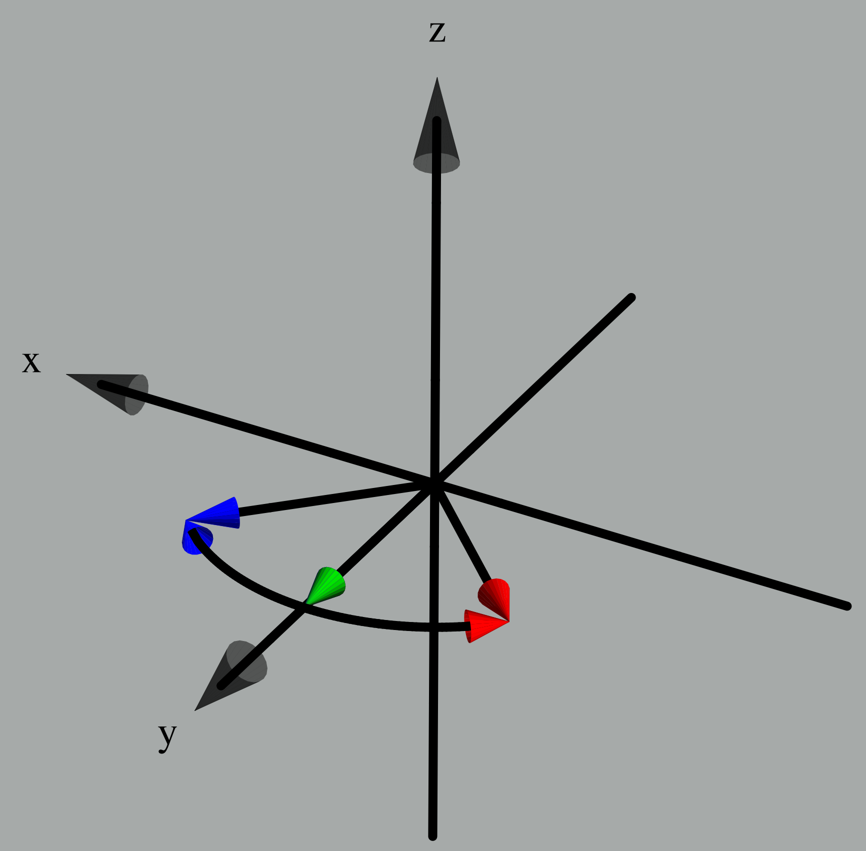
1. Spins precess in main field ***Free*** for excitation pulses
2. There precession generates current in RF coils by the Faraday's Law of ***Induction***
3. This signal diminishes exponentially due to ***Decay*** of the transverse component

This is called
Free Induction Decay

Free Induction Decay

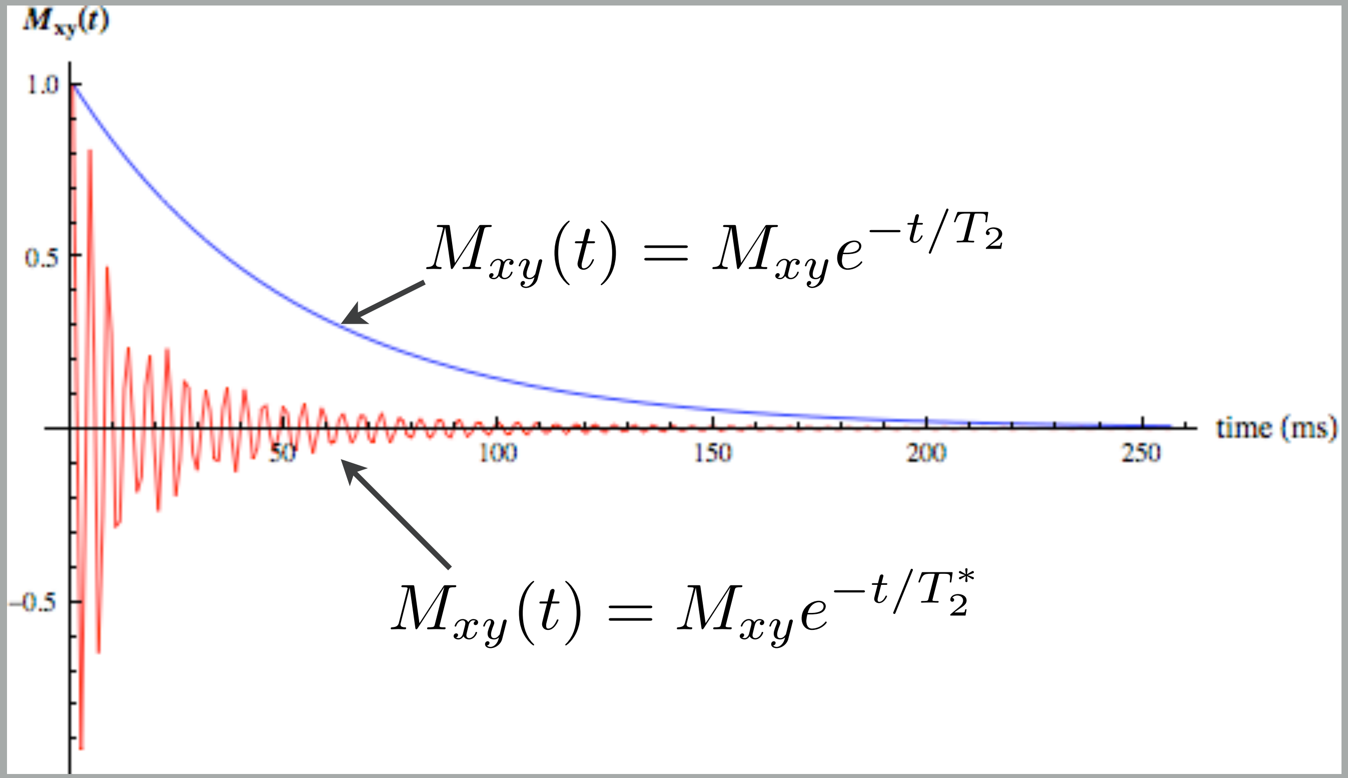


T_2^* dephasing



$$t = \tau$$

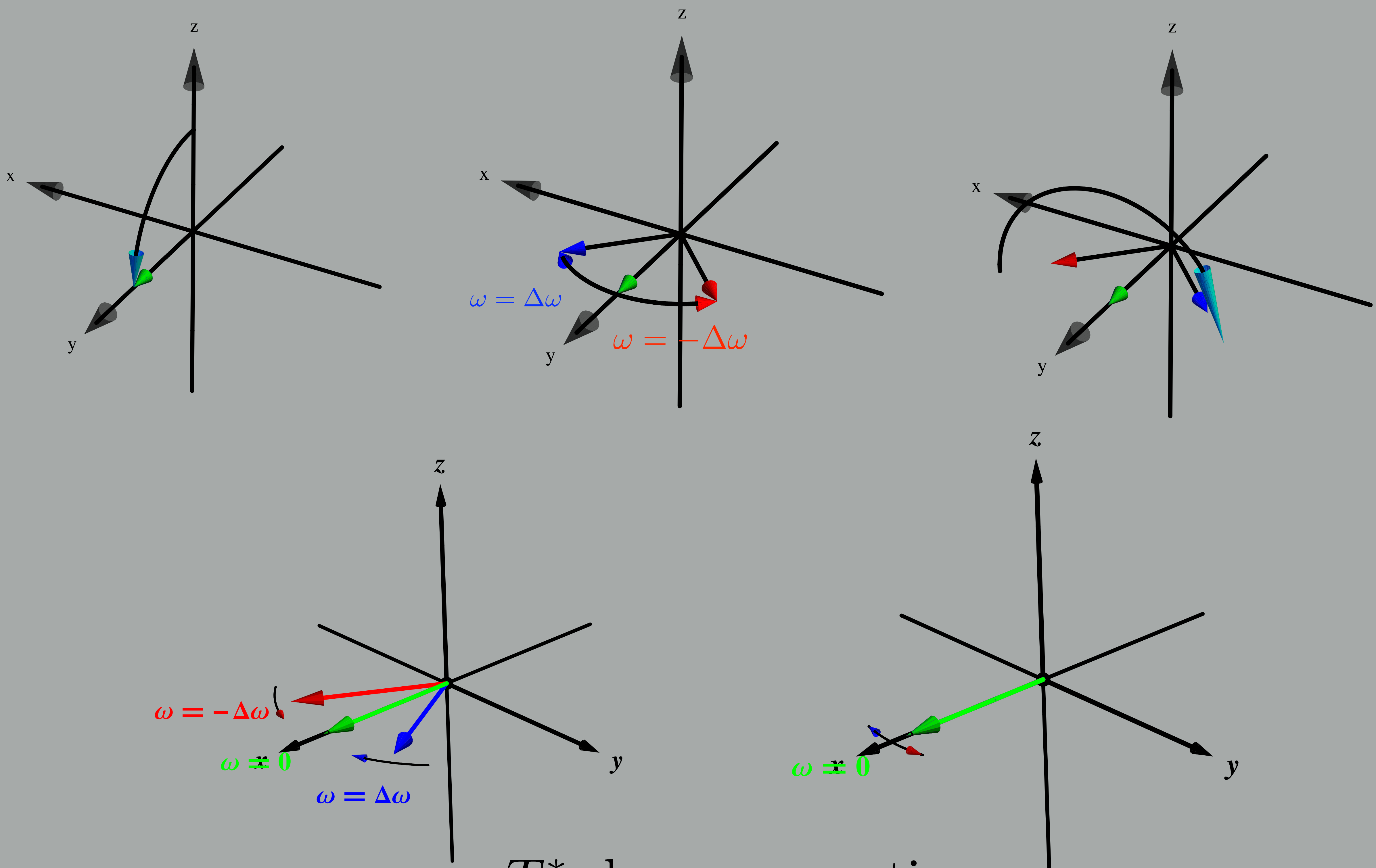
Free Induction Decay



T_2 is irreversible intrinsic transverse decay

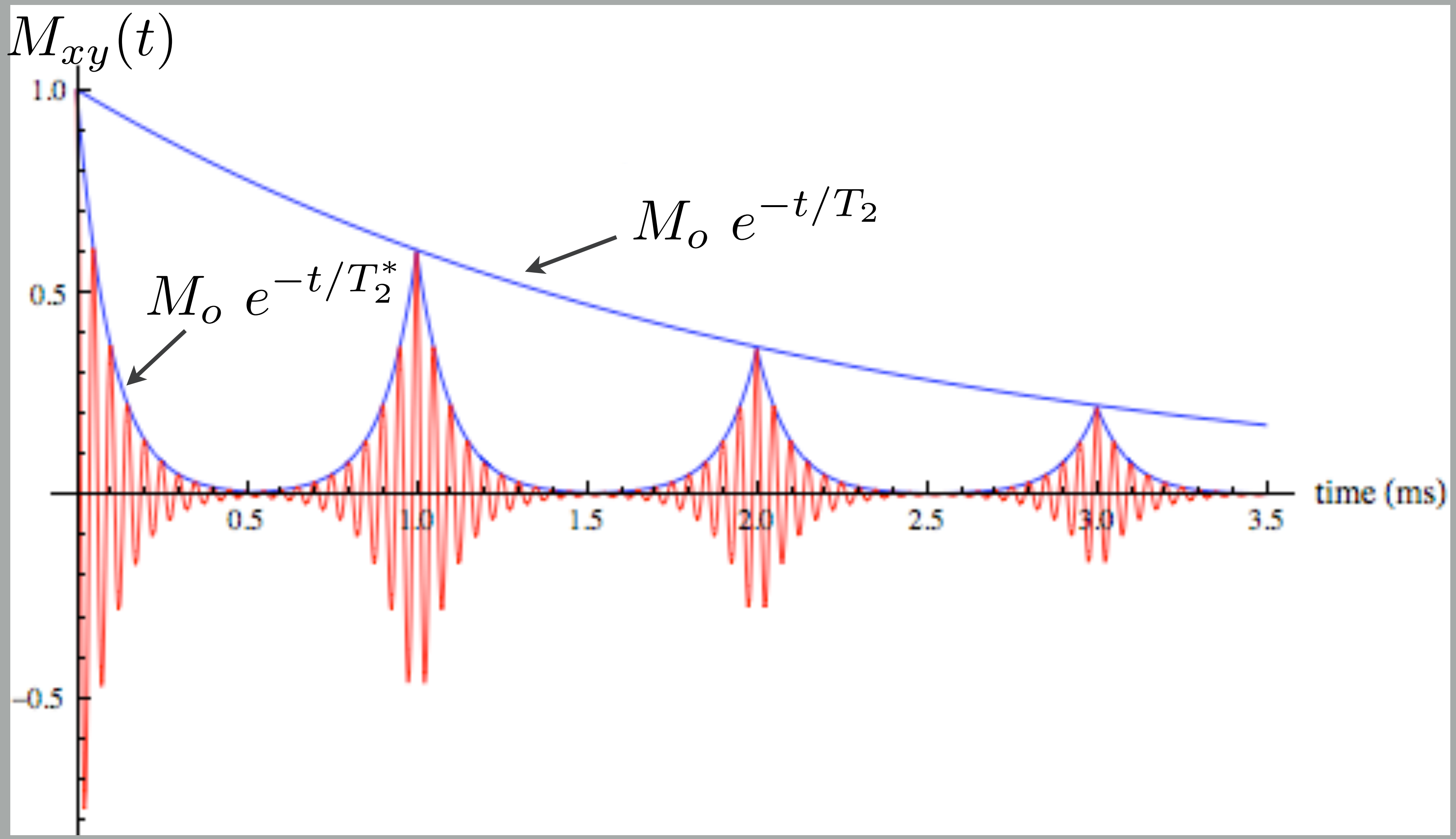
T_2^* includes reversible decay due to field inhomogeneities

Spin Echo



T_2^* decay correction

Multiple spin echos



The NMR signal

$$s(\omega) = \int_{\Omega} m_{\perp}(\boldsymbol{r}, t) e^{-i\varphi(x,\tau)} \, d\boldsymbol{r}$$

where $\quad \varphi(x, \tau) = \gamma \int_0^{\tau} B(x, t) \, dt$

The signal is the *Fourier Transform* of the transverse magnetization

Traditional units

$$\boxed{\omega = \gamma B}$$

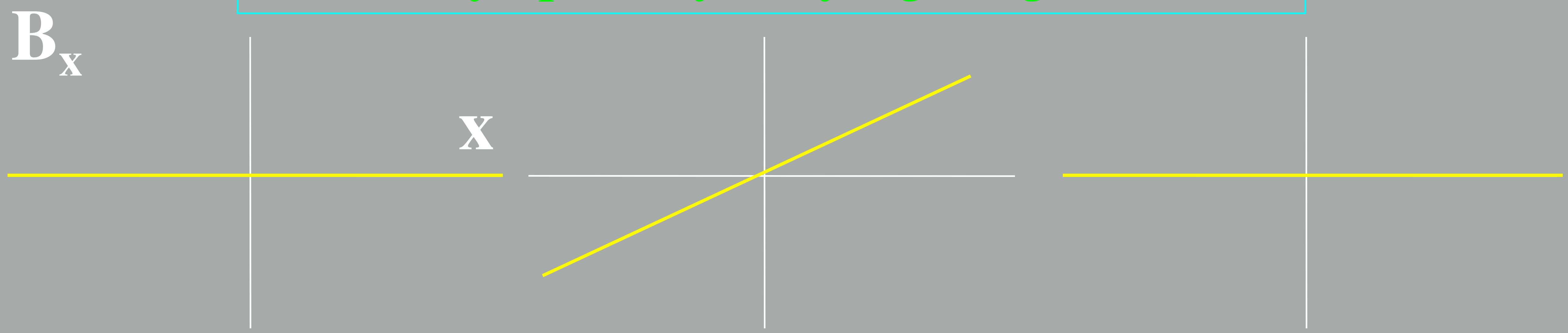
$$[\mathrm{Hz}] = \left[\frac{\mathrm{Hz}}{\mathrm{Tesla}}\right] [\mathrm{Tesla}]$$

$$\boxed{\omega = \gamma G x}$$

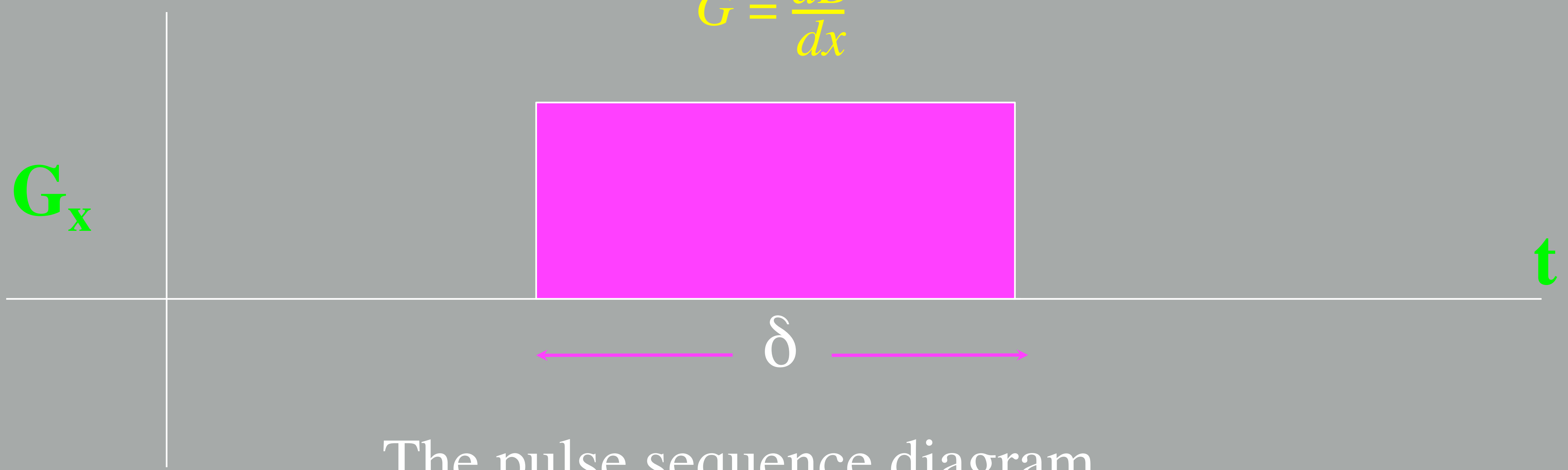
$$[\mathrm{Hz}] = \left[\frac{\mathrm{Hz}}{\mathrm{Gauss}}\right]\left[\frac{\mathrm{Gauss}}{\mathrm{cm}}\right] [\mathrm{cm}]$$

Magnetic Field Gradient

A linearly spatially varying magnetic field

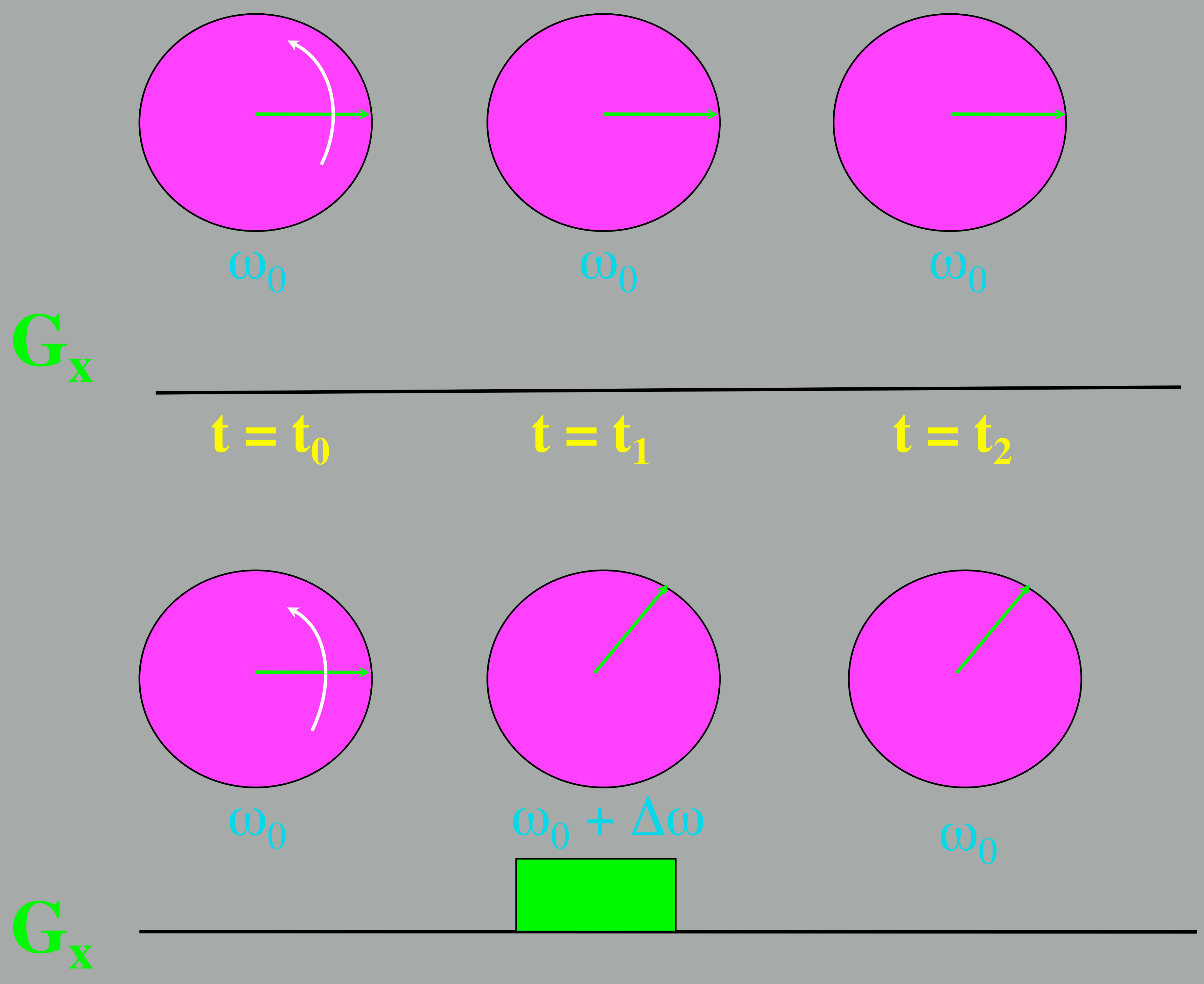


$$G = \frac{dB}{dx}$$

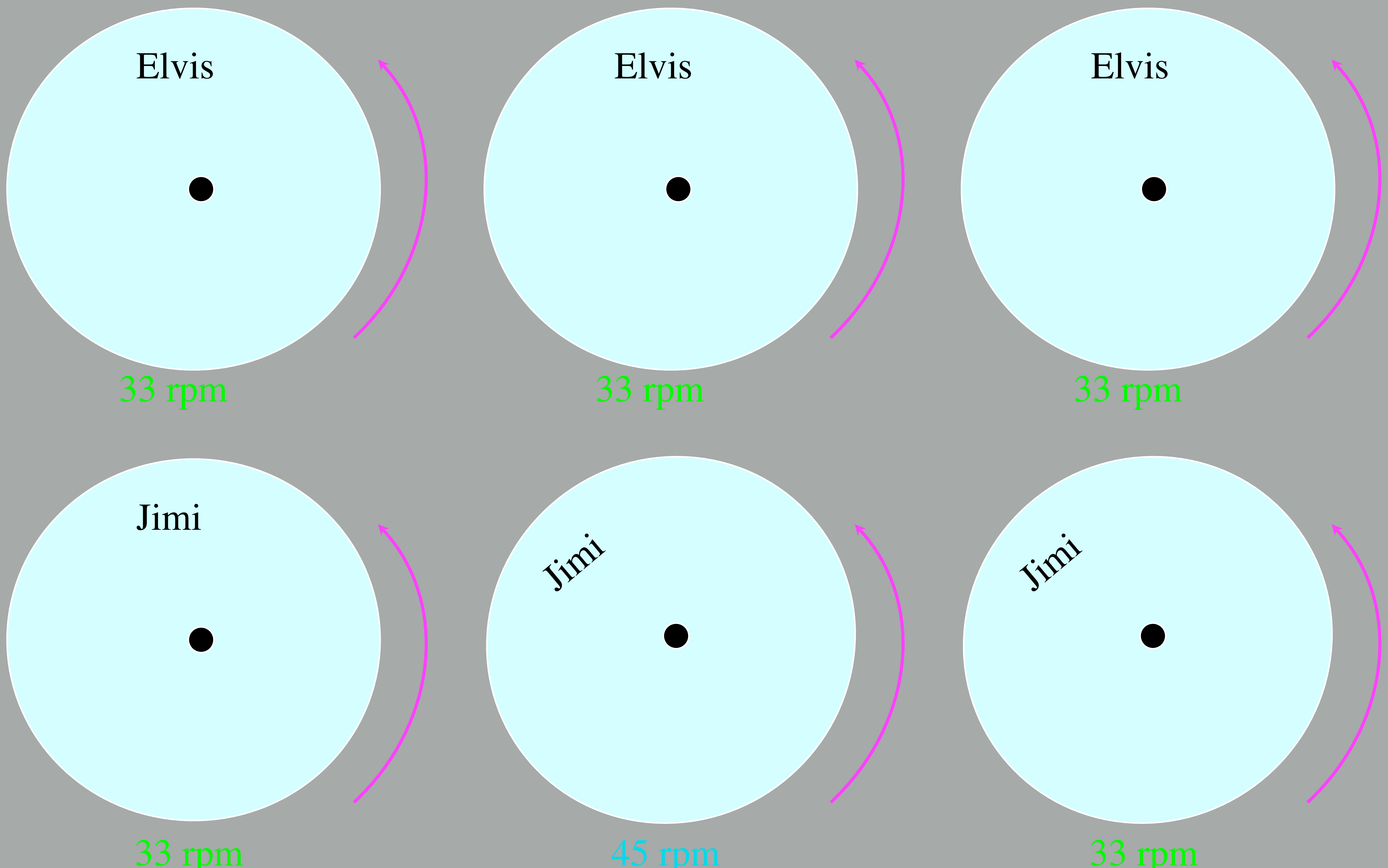


The pulse sequence diagram

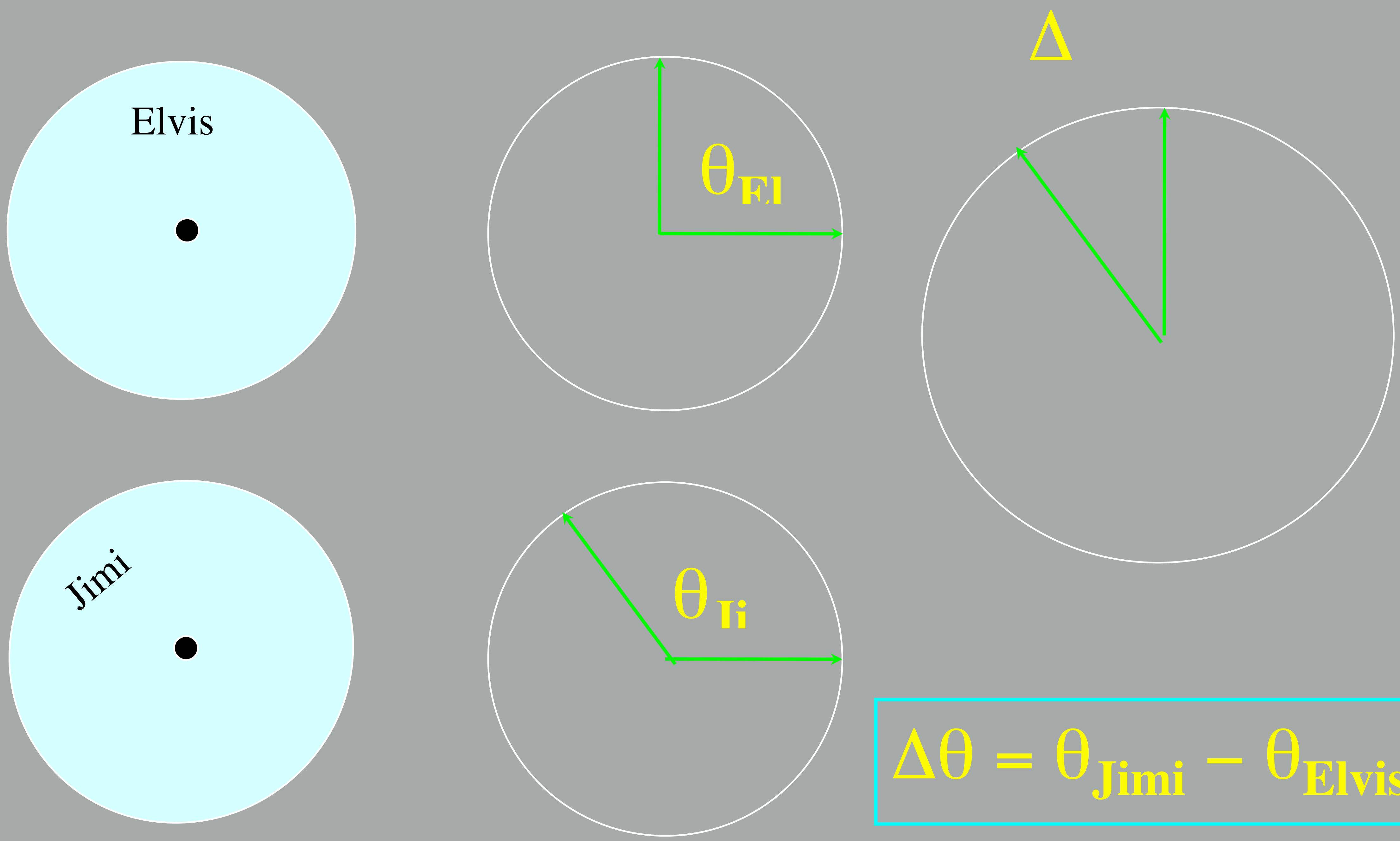
Gradient and phase



Phase Memory

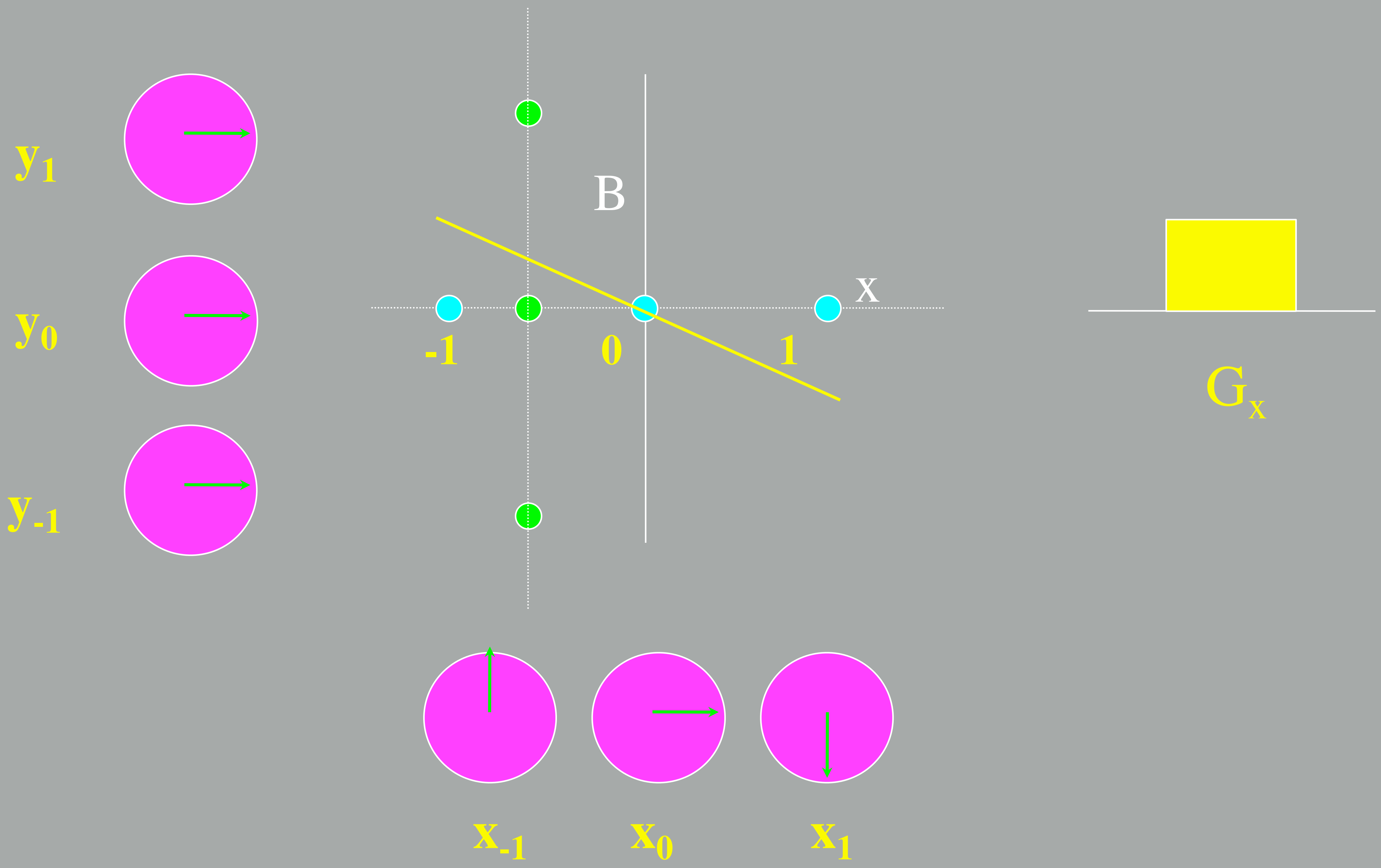


Spin Phases

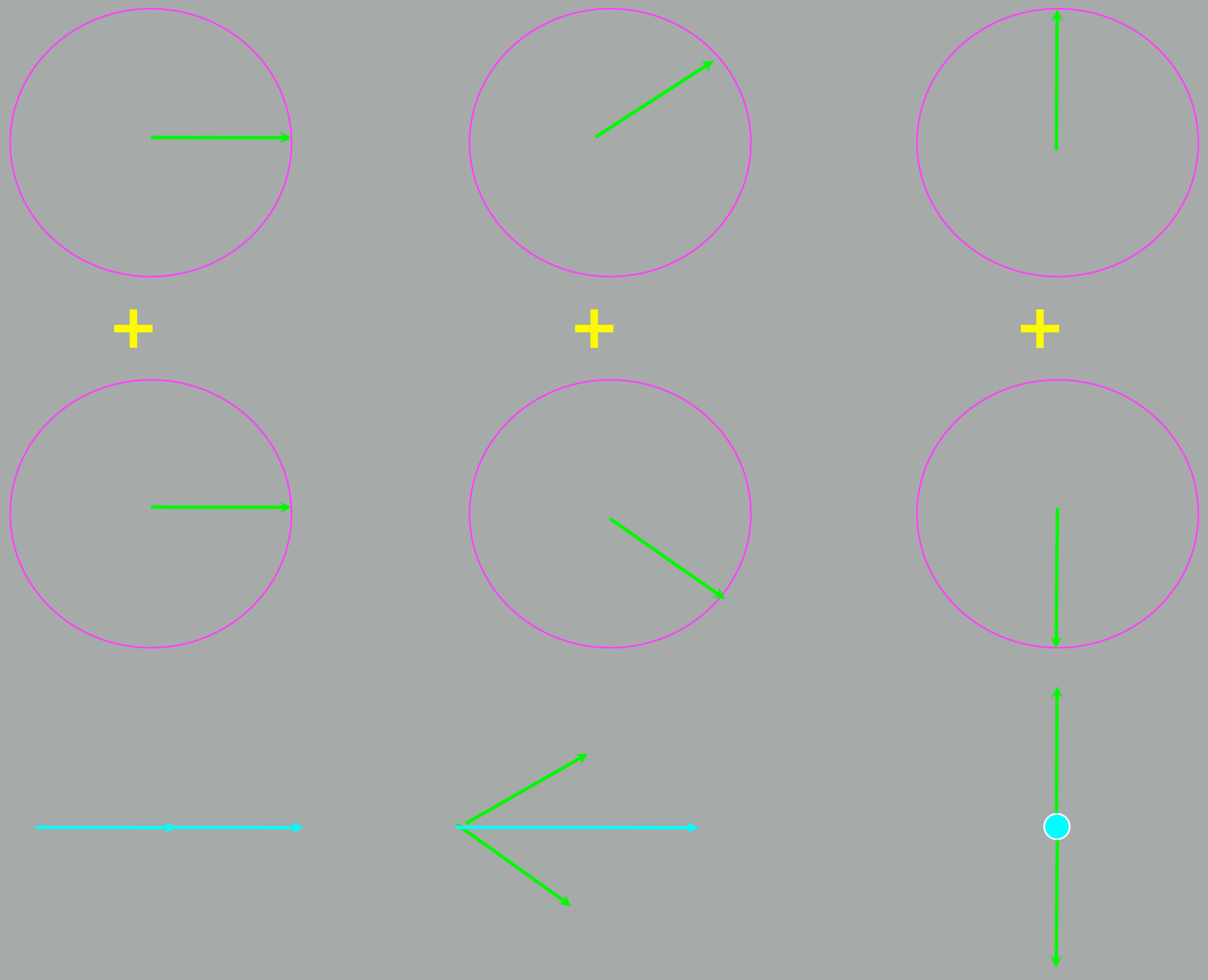


$$\Delta\theta = \theta_{\mathrm{Jimi}} - \theta_{\mathrm{Elvis}}$$

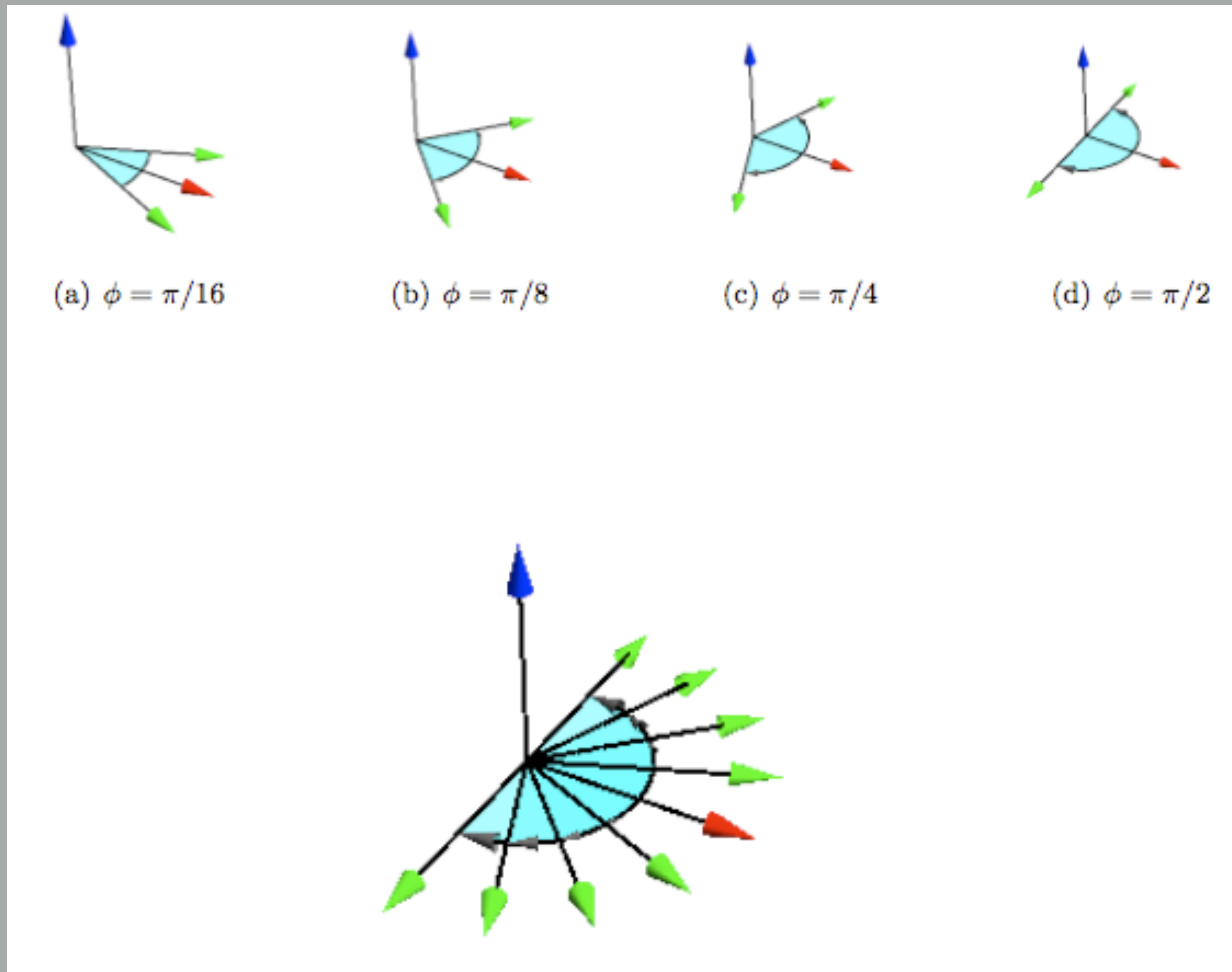
Spatial variations of phase along gradient



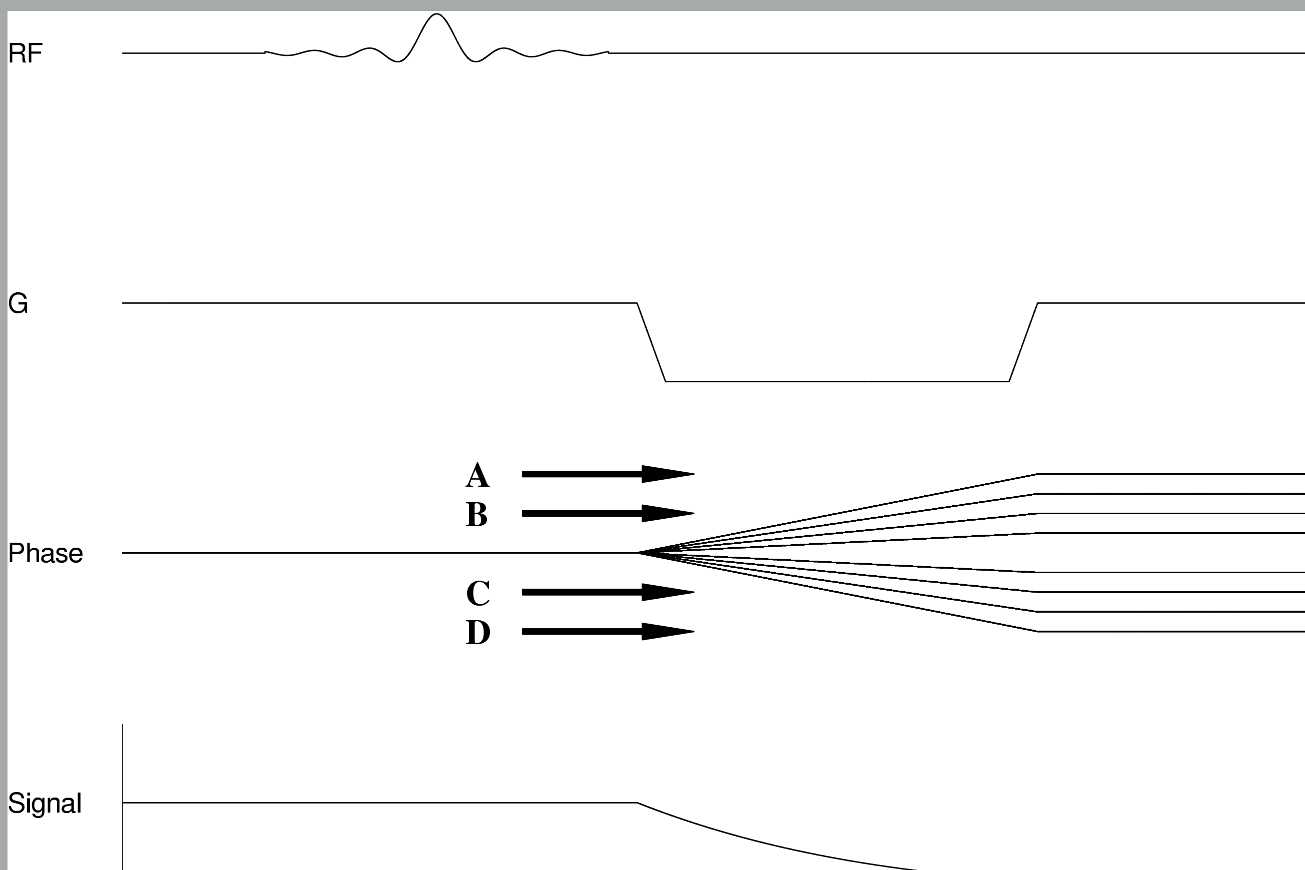
Signals add like vectors

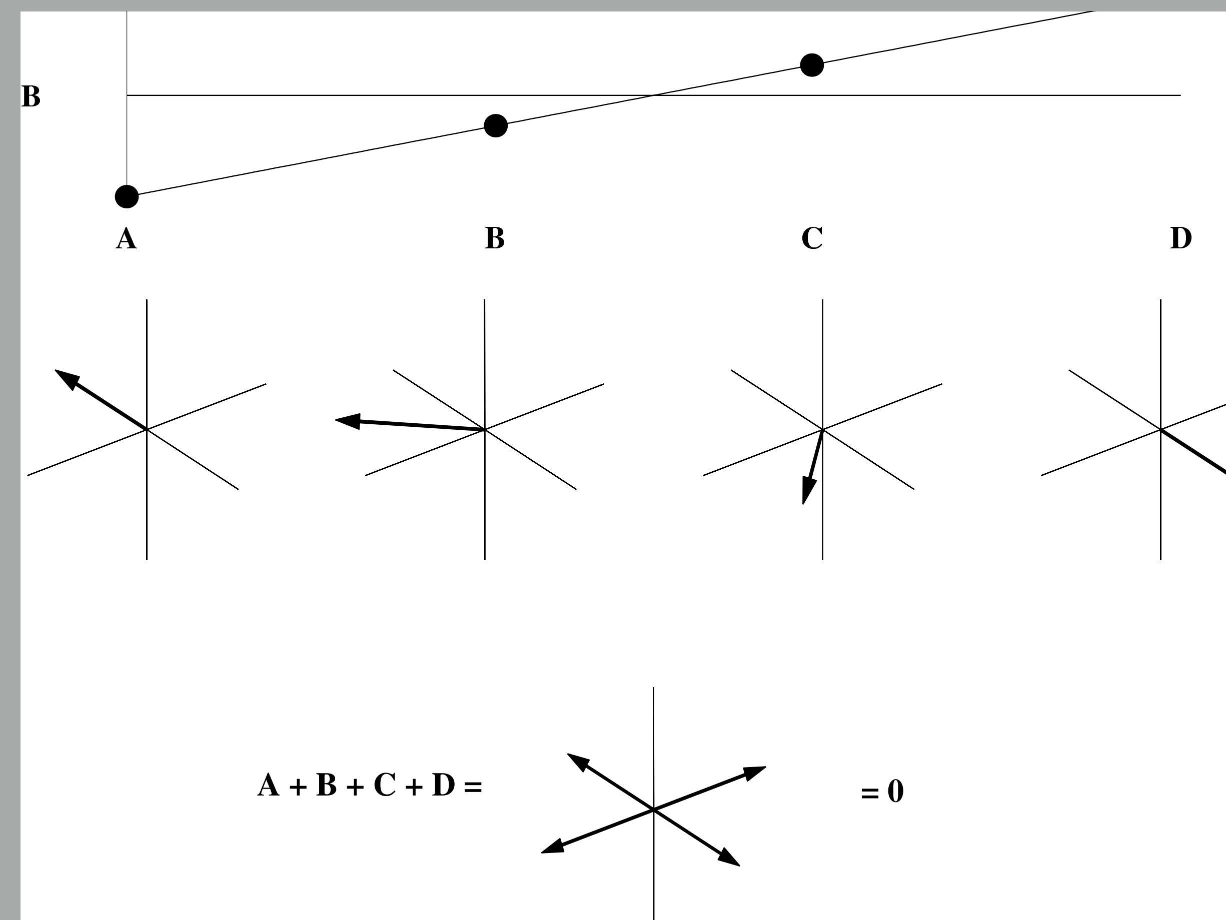


Spin dephasing

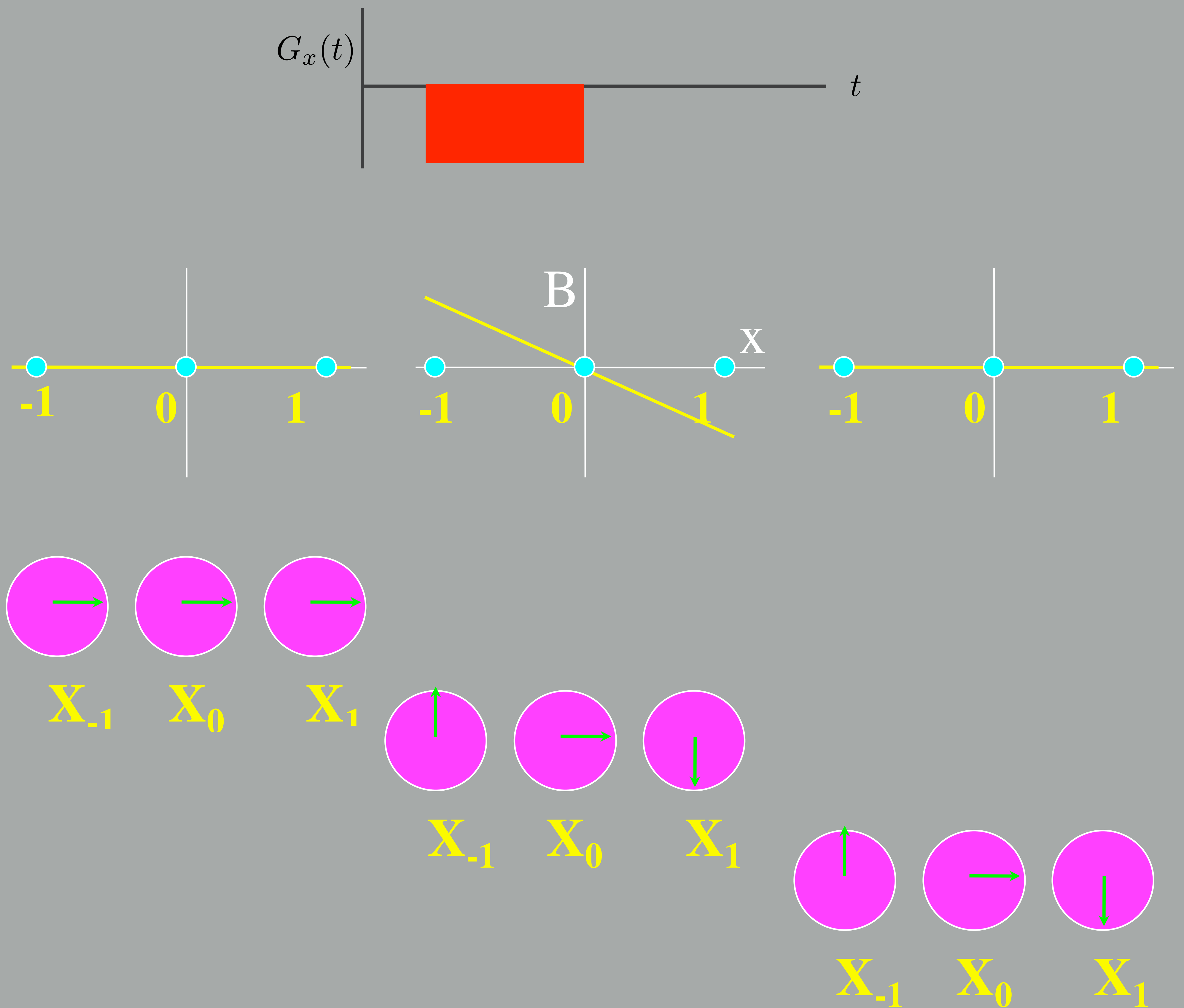


Spin dephasing

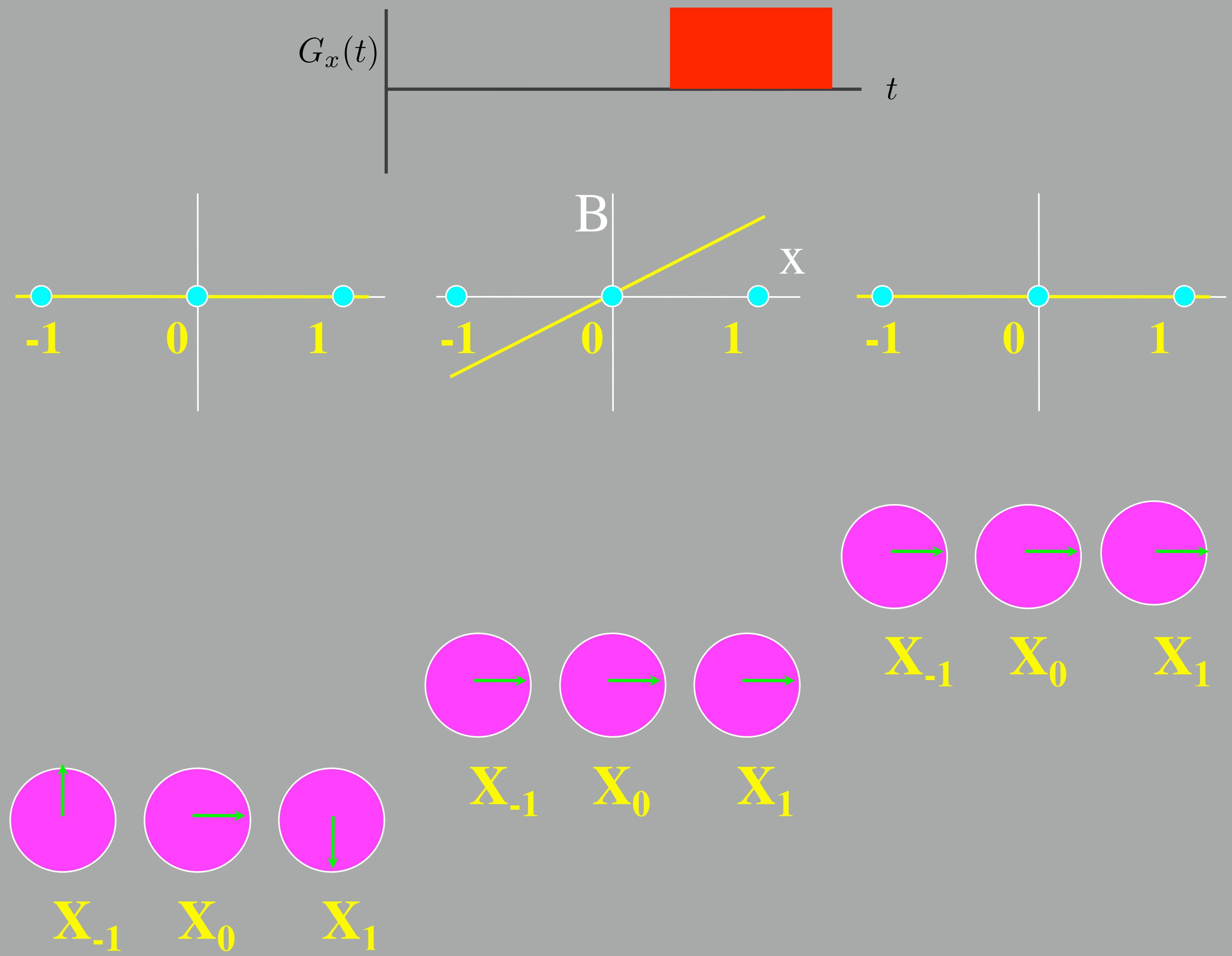




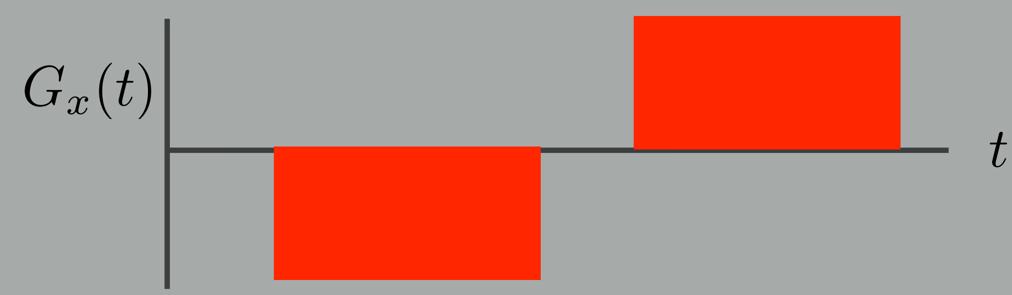
A negative gradient ...

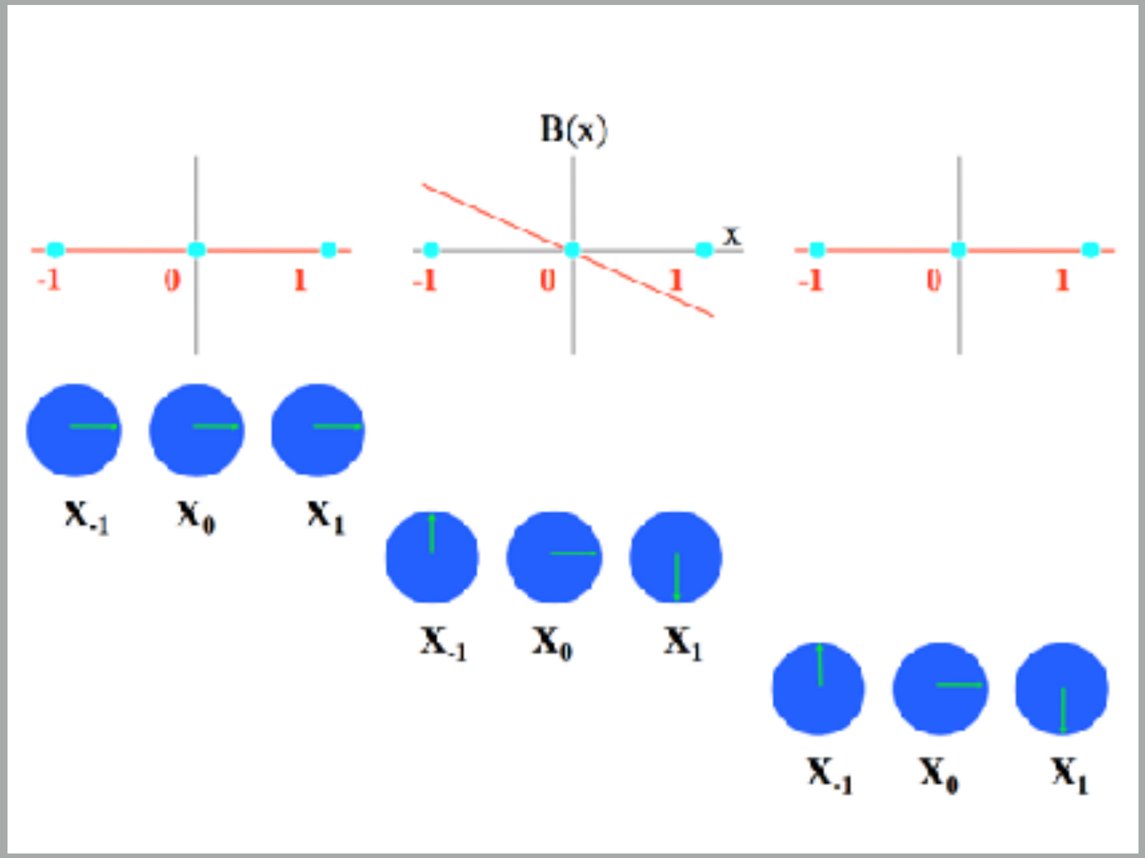


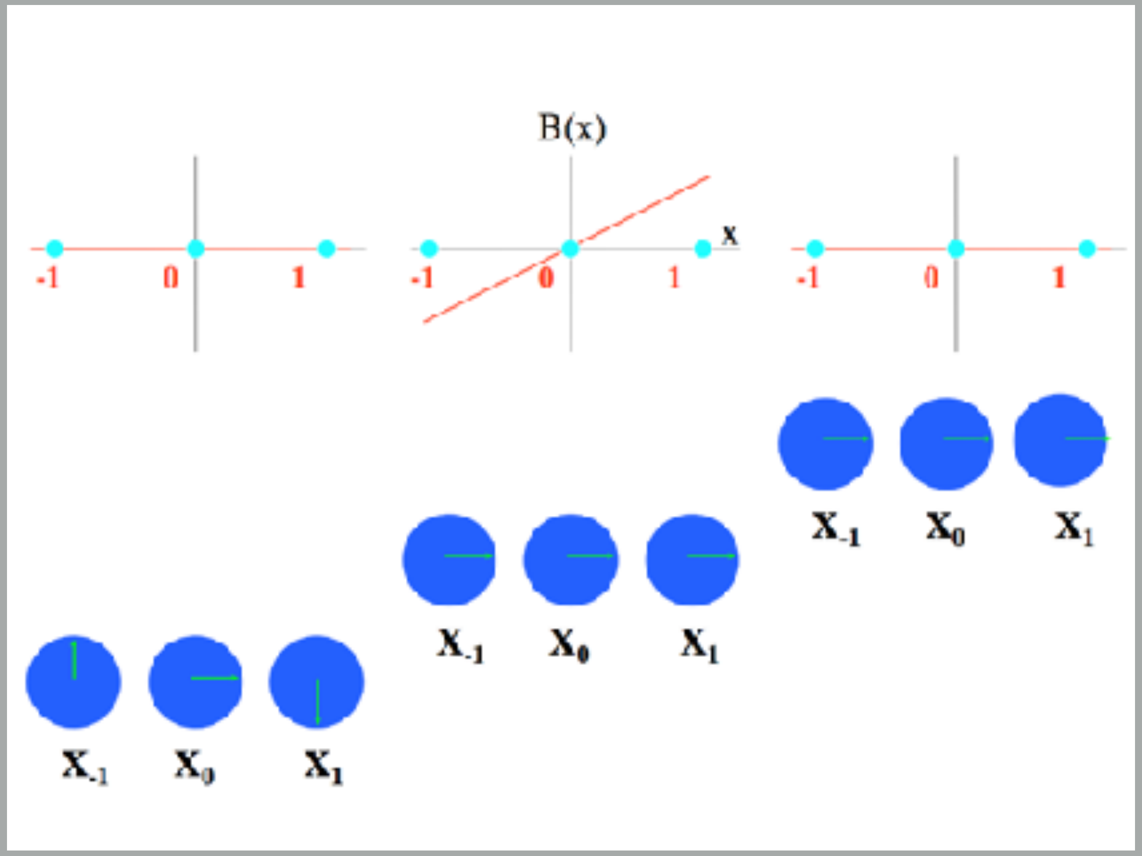
...followed by the reversed gradient



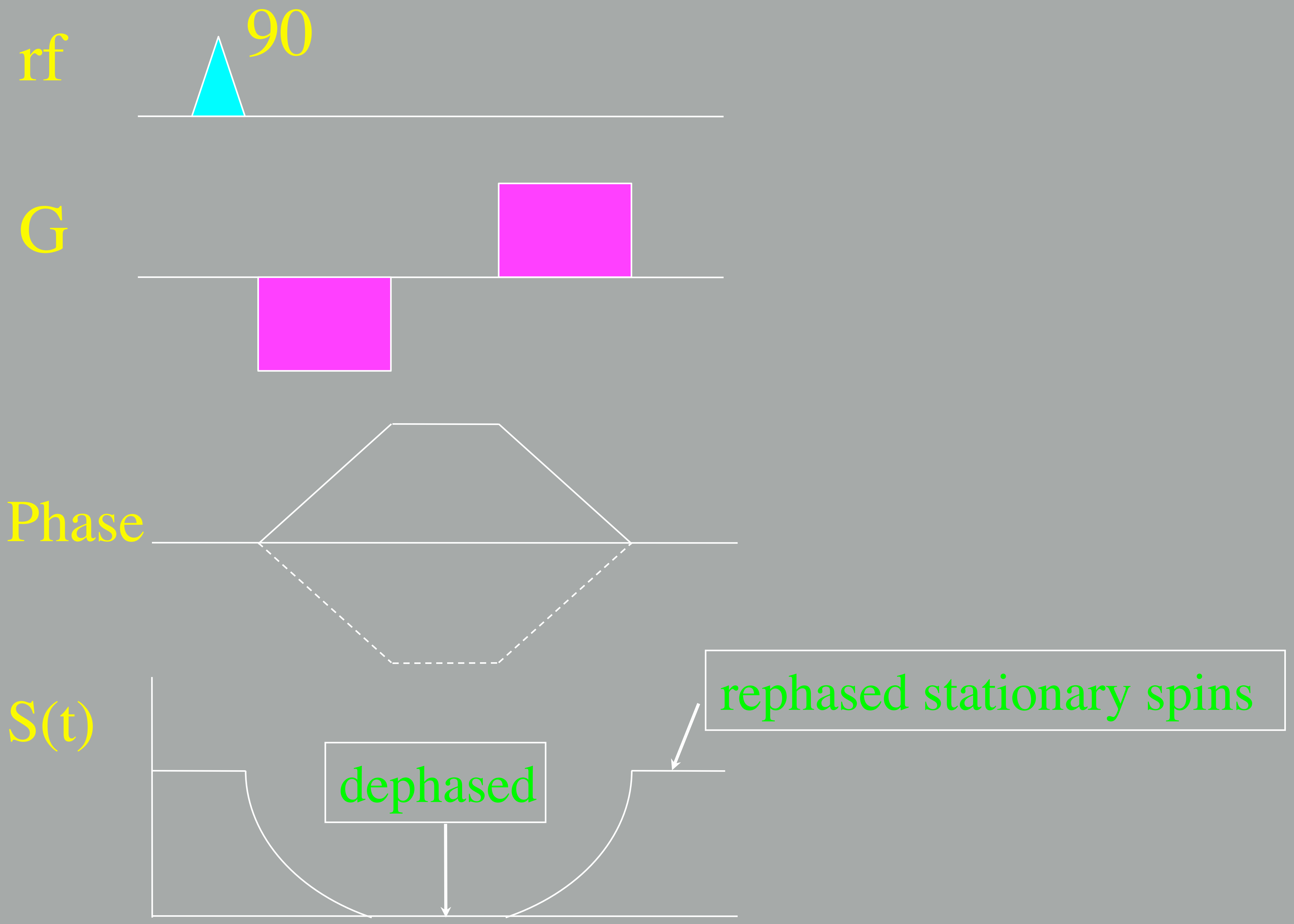
The Bipolar Gradient



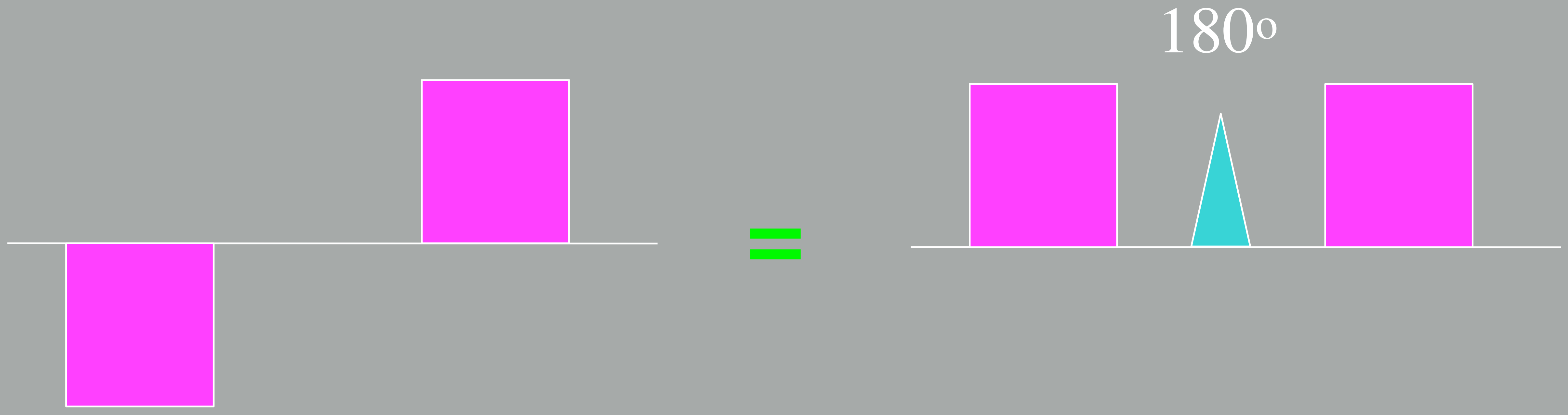




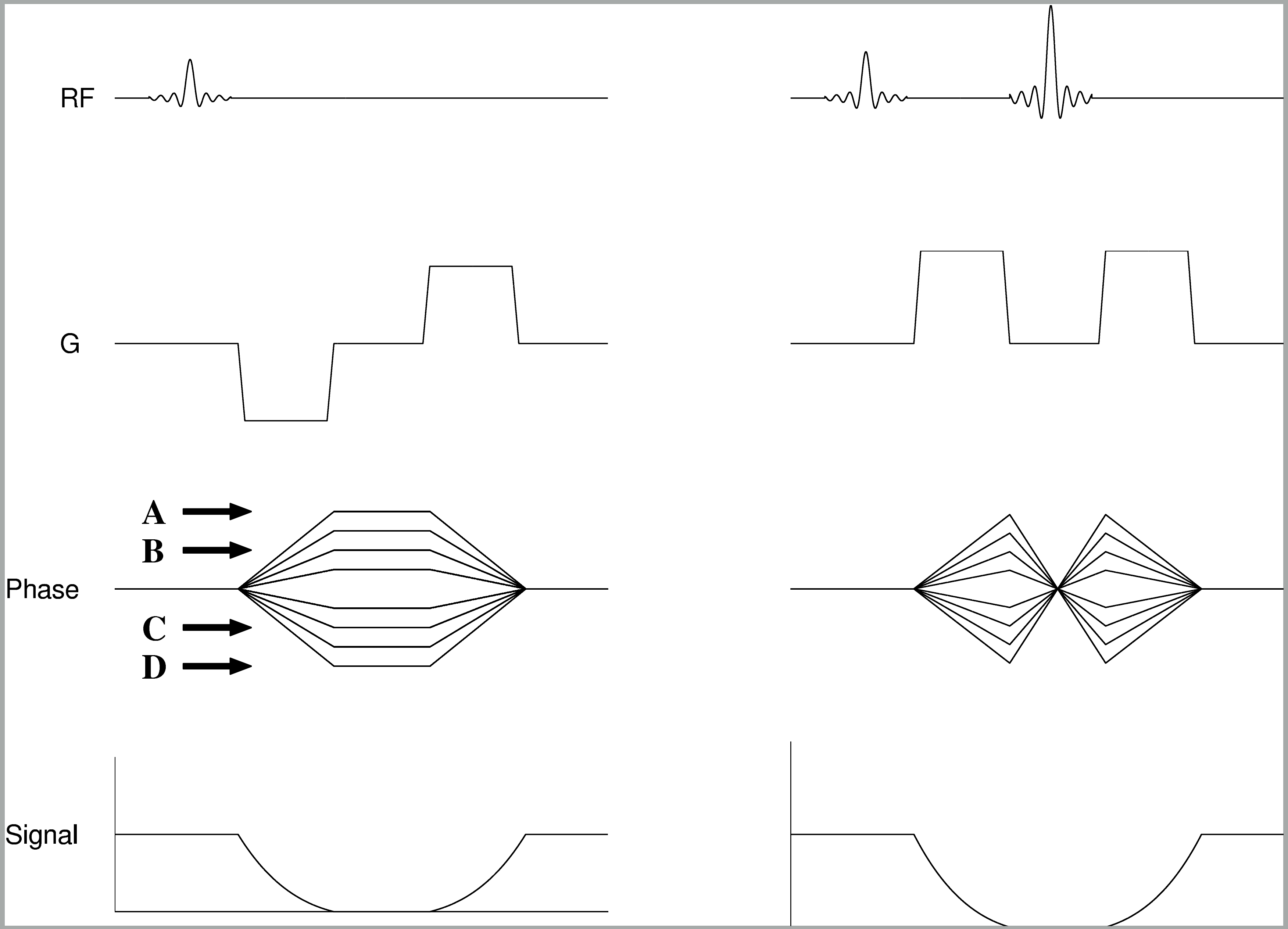
Gradient echo



Effective gradient



The effective gradient



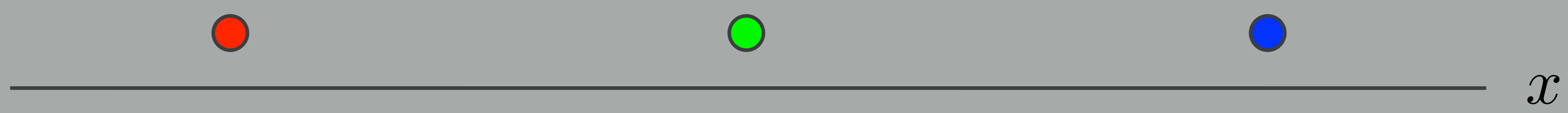
Isochromats

$$\boxed{\omega = \gamma B}$$

All spins precessing at a particular frequency are called an *isochromat*

(same frequency = same "color")

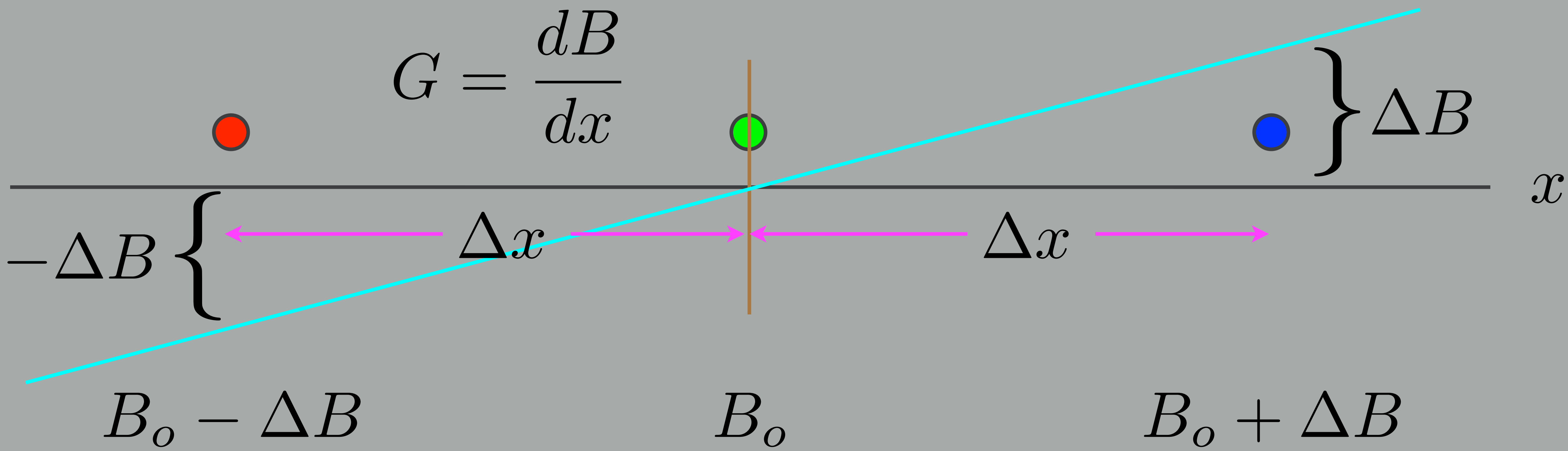
A tale of 3 isochromats



B_o B_o B_o

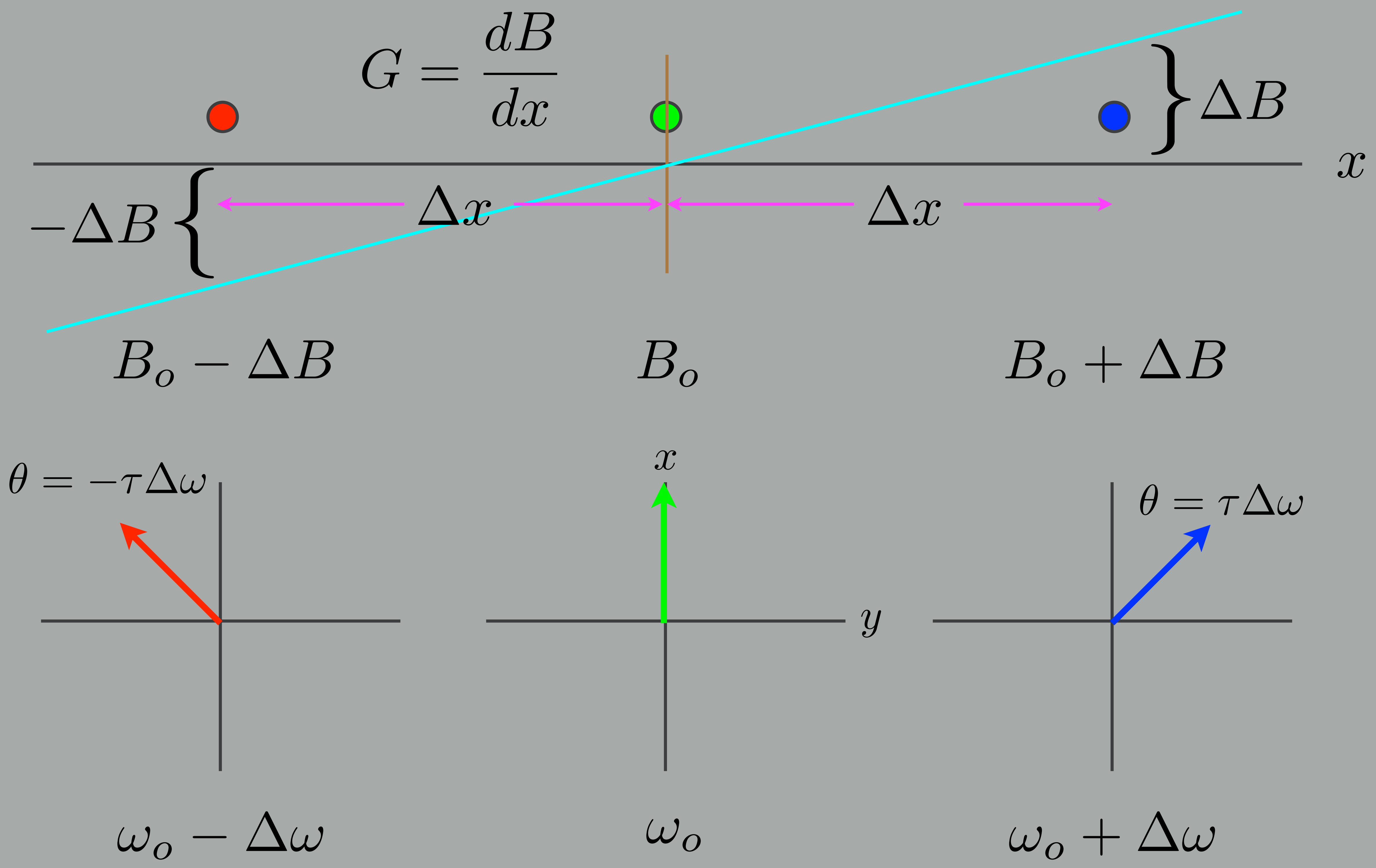
$\omega = \omega_o$ $\omega = \omega_o$ $\omega = \omega_o$

A tale of 3 isochromats

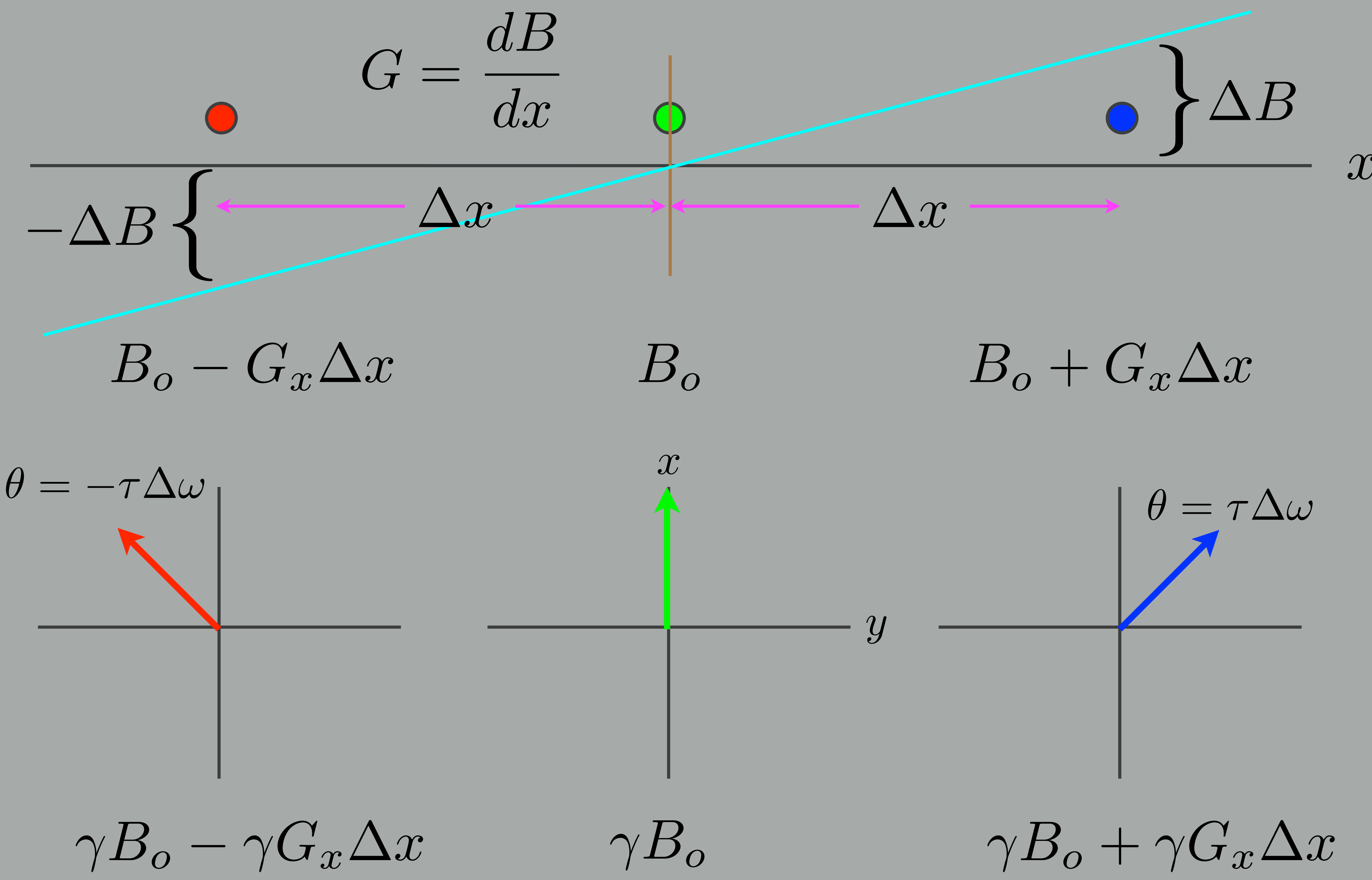


$$\Delta B = \frac{dB}{dx}\Delta x = G_x \Delta x$$

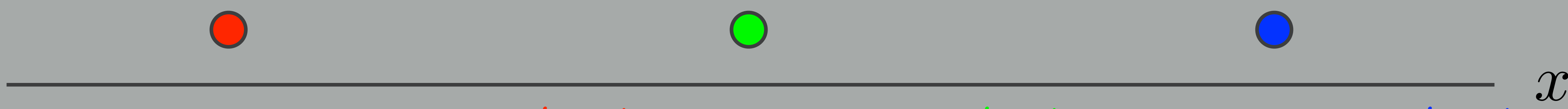
A tale of 3 isochromats



A tale of 3 isochromats



The NMR signal



$$s(\omega) = m_{xy}(x_-)e^{-i\omega_- t} + m_{xy}(x_o)e^{-i\omega_0 t} + m_{xy}(x_+)e^{-i\omega_+ t}$$

$$\omega_- = \gamma B_o - \gamma G_x \Delta x \qquad \omega_o = \gamma B_o \qquad \omega_+ = \gamma B_o + \gamma G_x \Delta x$$

$$s(\omega) = \sum_i m_{xy}(x_i)e^{-i\omega_i t}$$

$$x_\pm \equiv x_o \pm \Delta x$$

The NMR signal (lab frame)

$$s(\omega) = \int_\Omega m_{xy}(x) e^{-i\omega(x,t)t} dx$$

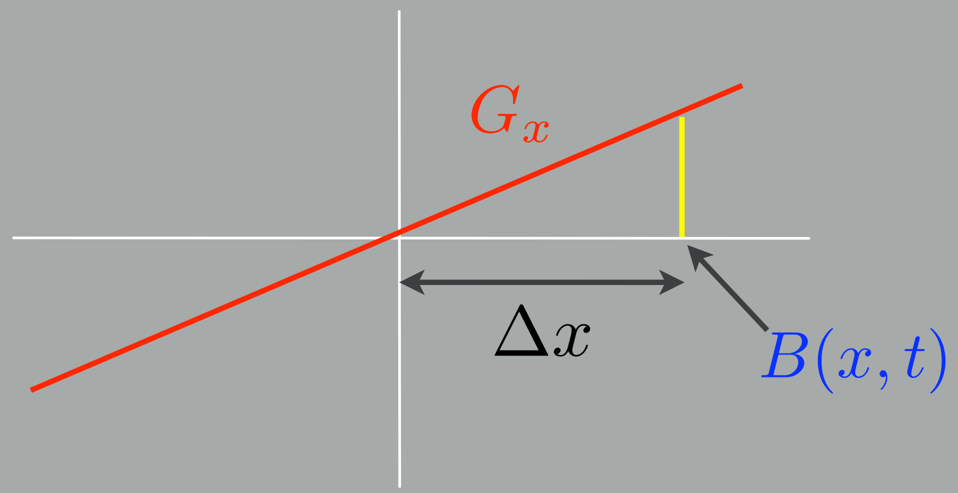
where

$$\omega(x,t) = \omega_o + \gamma G_x x$$

The NMR signal (rotating frame)

$$\varphi(x,\tau) = \gamma \int_0^{\tau} B(x,t)\, dt$$

$$B(x,t) = B_o + \frac{\partial B}{\partial x} dx = B_o + G_x \Delta x$$



the magnetic field *gradient*

The phase

$$\varphi(x,\tau) = \gamma \int_0^\tau B(x,t)\,dt = \underbrace{\gamma B_o \tau}_{\varphi o} + \gamma \int_0^\tau G(t) x(t)\,dt$$

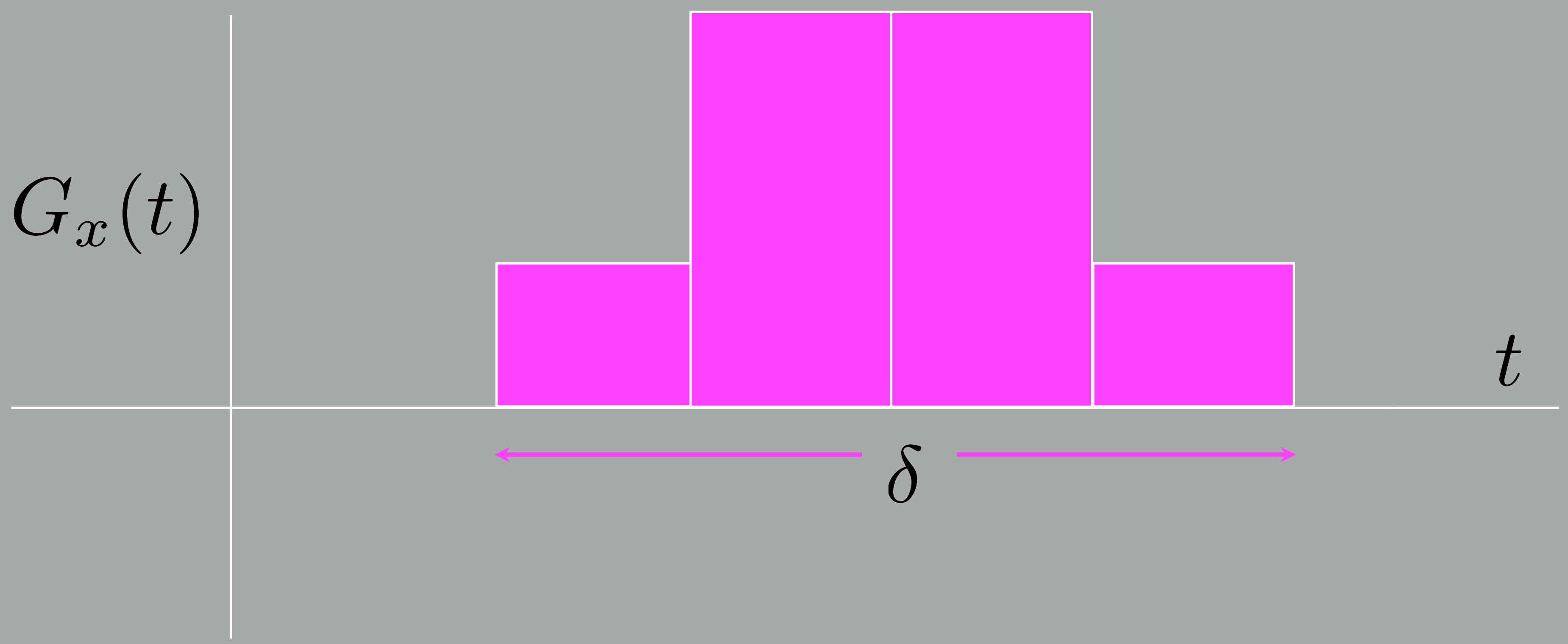
laboratory frame

$$\varphi(x,\tau) - \varphi_o = \gamma \int_0^\tau G(t) x(t)\,dt$$

rotating frame

The phase in the rotating frame

$$\varphi(x,\tau) = \gamma \int_0^\tau G(t)x(t)\,dt$$

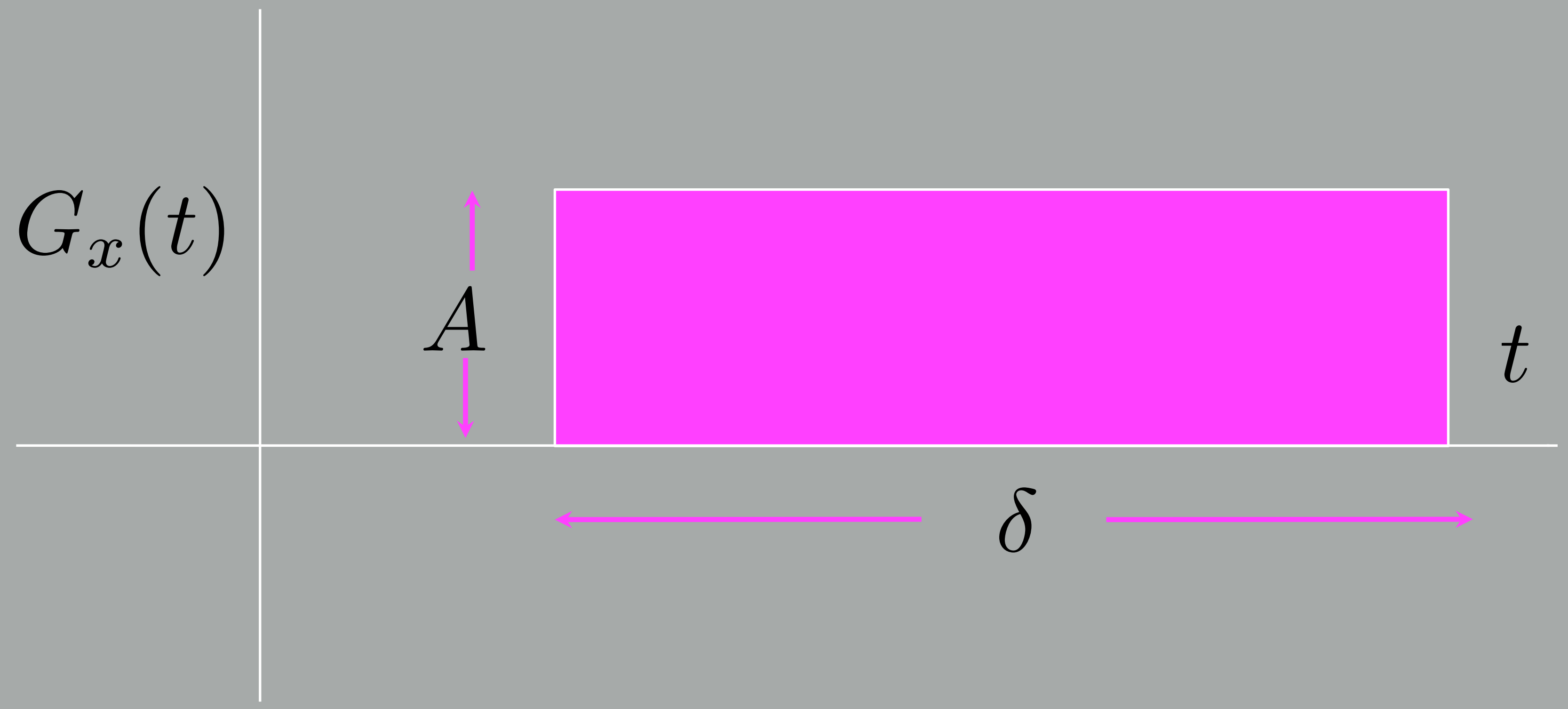


Simplification #1: fixed location, known gradient time

$$\varphi(x,\delta) = \gamma x \underbrace{\int_0^\delta G_x(t)\,dt}_{k}$$

The phase in the rotating frame

$$\varphi(x,\tau) = \gamma \int_0^{\tau} G(t)x(t)\,dt$$



Simplification #2:
constant gradient amplitude

$$\varphi(x,\delta) = \gamma x A \int_0^{\delta} dt = \gamma x \underbrace{A\delta}_{k}$$

The NMR signal (rotating frame)

$$s(\omega) = \int_{\Omega} m_{xy}(x) e^{-i\omega(x,t)t} dx$$

where

$$\omega(x,t) = \gamma G_x x$$

The NMR signal

$$s(\omega) = \int_{\Omega} m_{xy}(x) e^{-i\omega(x,t)t} dx$$

where

$$\omega(x,t)t = \underbrace{\gamma G_x t}_{k} \, x = kx$$

The NMR signal

$$s(k) = \int_{\Omega} m_{xy}(x) e^{-ikx} dx$$

where

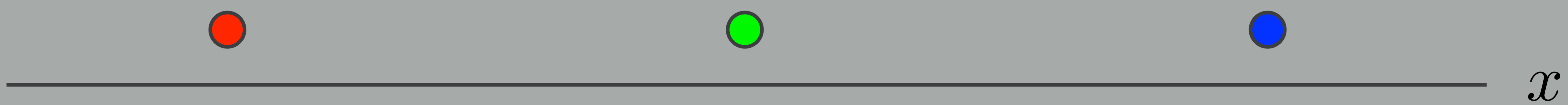
$$k = \gamma G_x t$$

$$k \propto 1/x$$

spatial frequency

What does this mean?

The NMR signal
(lab frame)



$$s(k) = m_{xy}(x_-)e^{-ikx_-} + m_{xy}(x_o)e^{-ikx_0} + m_{xy}(x_+)e^{-ikx_+}$$

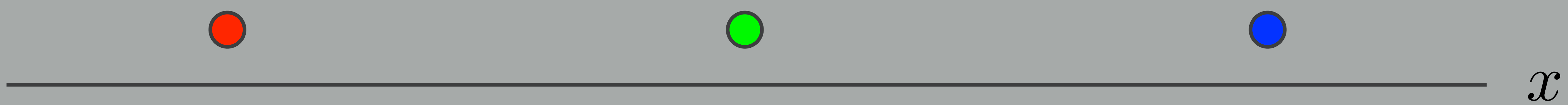
$$k(x_o - \Delta x) \qquad kx_o \qquad k(x_o + \Delta x)$$

$$s(k) = \sum_l m_{xy}(x_o + x_l)e^{-ik(x_o+x_l)} = \sum_l m_{xy}(x_o + l\Delta x)e^{-ik(x_o+l\Delta x)}$$

$$k = \gamma G_x t$$

$$x_\pm \equiv x_o \pm \Delta x$$

The NMR signal
(rotating frame)



$$s(k) = m_{xy}(x_-)e^{-ikx_-} + m_{xy}(x_o)e^{-ikx_0} + m_{xy}(x_+)e^{-ikx_+}$$

$$-k\Delta x \qquad\qquad 0 \qquad\qquad k\Delta x$$

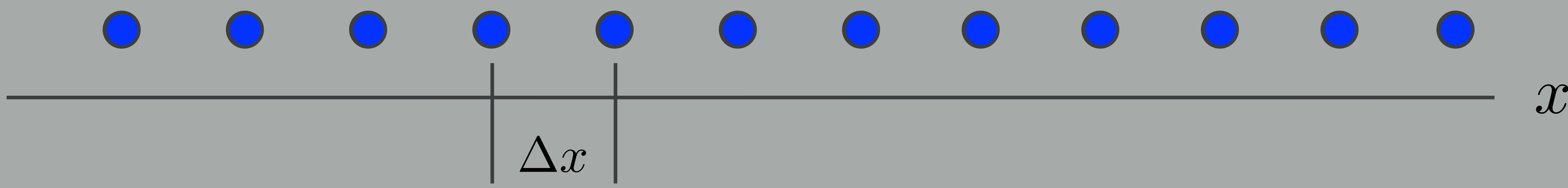
$$s(k) = \sum_{l=-1,0,1} m_{xy}(x_l)e^{-ikx_l} = \sum_{l=-1,0,1} m_{xy}(l\Delta x)e^{-ikl\Delta x}$$

$$k = \gamma G_x t$$

$$x_\pm \equiv \pm\Delta x$$

Signal adds coherently when $k = 2\pi/\Delta x$, i.e., when the k value matches the spatial frequency of the spin distribution

The NMR signal

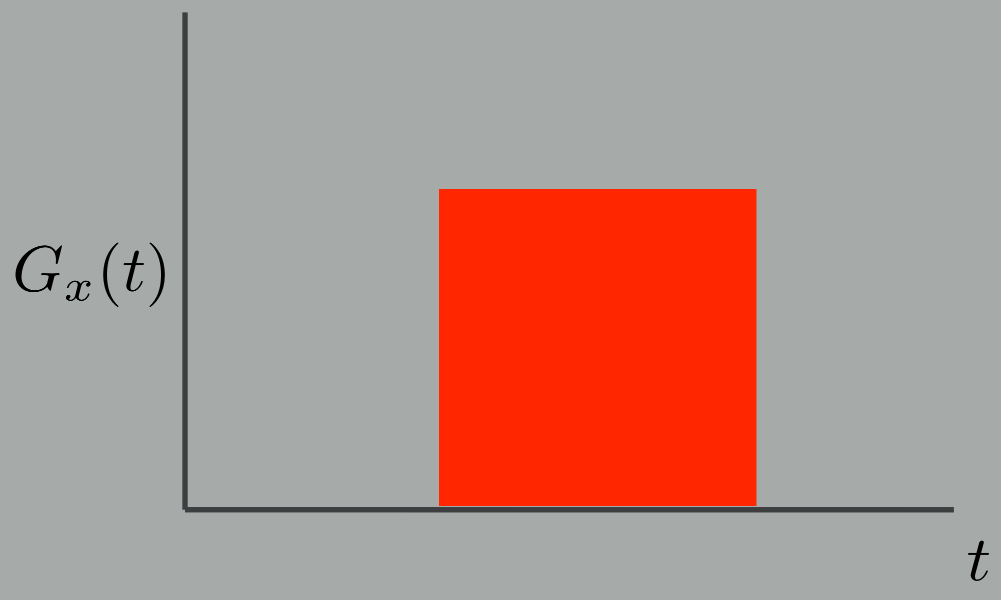


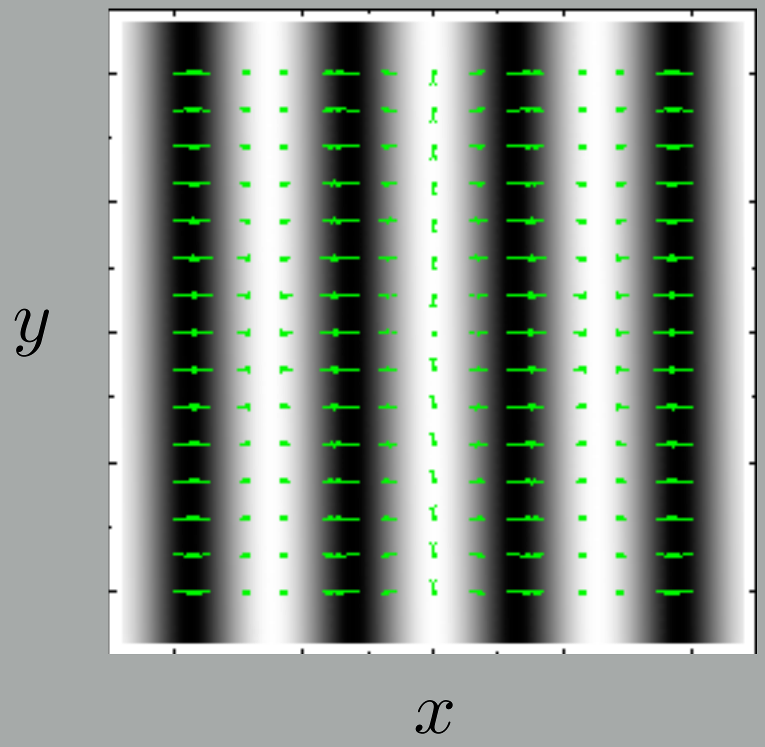
$$s(k) = \sum_{l=-n}^{n} m_{xy}(l\Delta x) e^{-ikl\Delta x} \qquad k = \gamma G_x t$$

$$s(k) = \int_{\Omega} m_{\perp}(x) e^{-ikx}\, dx$$

Signal adds coherently when $k = 2\pi/\Delta x$, i.e., when the k value matches the spatial frequency of the spin distribution

Spatial modulation of the phase





phase modulated along gradient direction

Vectorize phase

Vector form (arbitrary direction)

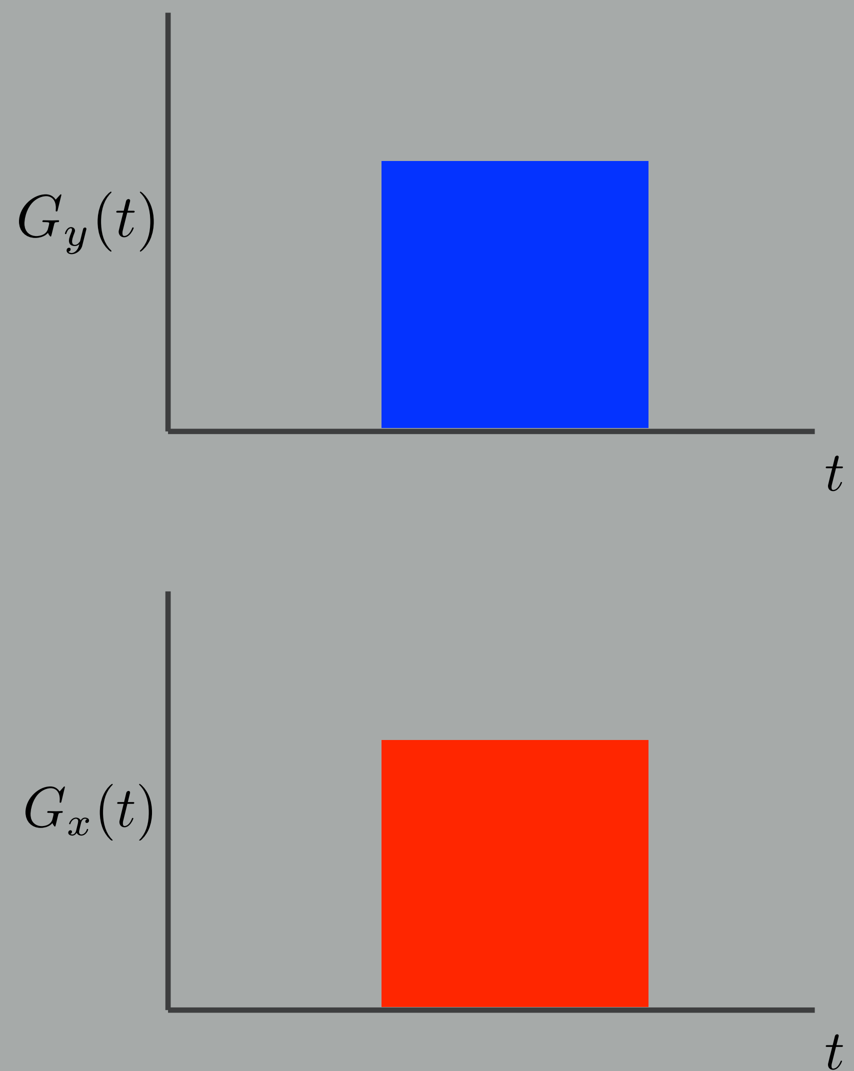
$$\boldsymbol{k} = \begin{pmatrix} k_x \\ k_y \end{pmatrix} \qquad \boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$

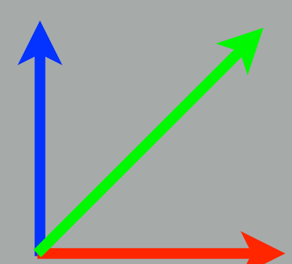
$$\boldsymbol{k} = \begin{pmatrix} k_x \\ k_y \end{pmatrix} = \gamma \begin{pmatrix} G_x \\ G_y \end{pmatrix} t$$

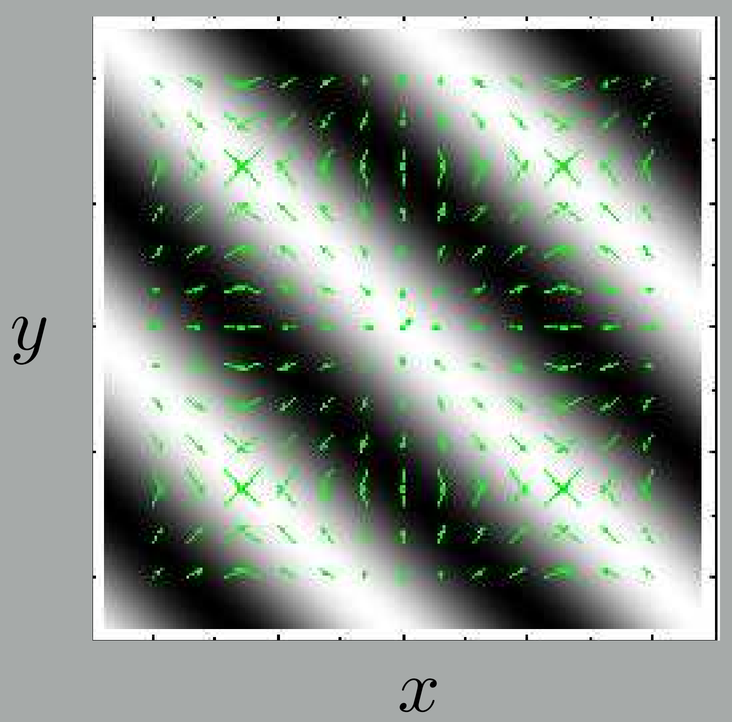
$$\boldsymbol{k} \cdot \boldsymbol{x} = k_x x + k_y y = \gamma G_x t x + \gamma G_y t y$$

Spatial modulation of the phase

$$\boldsymbol{k} \cdot \boldsymbol{x} = k_x x + k_y y = \gamma G_x t x + \gamma G_y t y$$

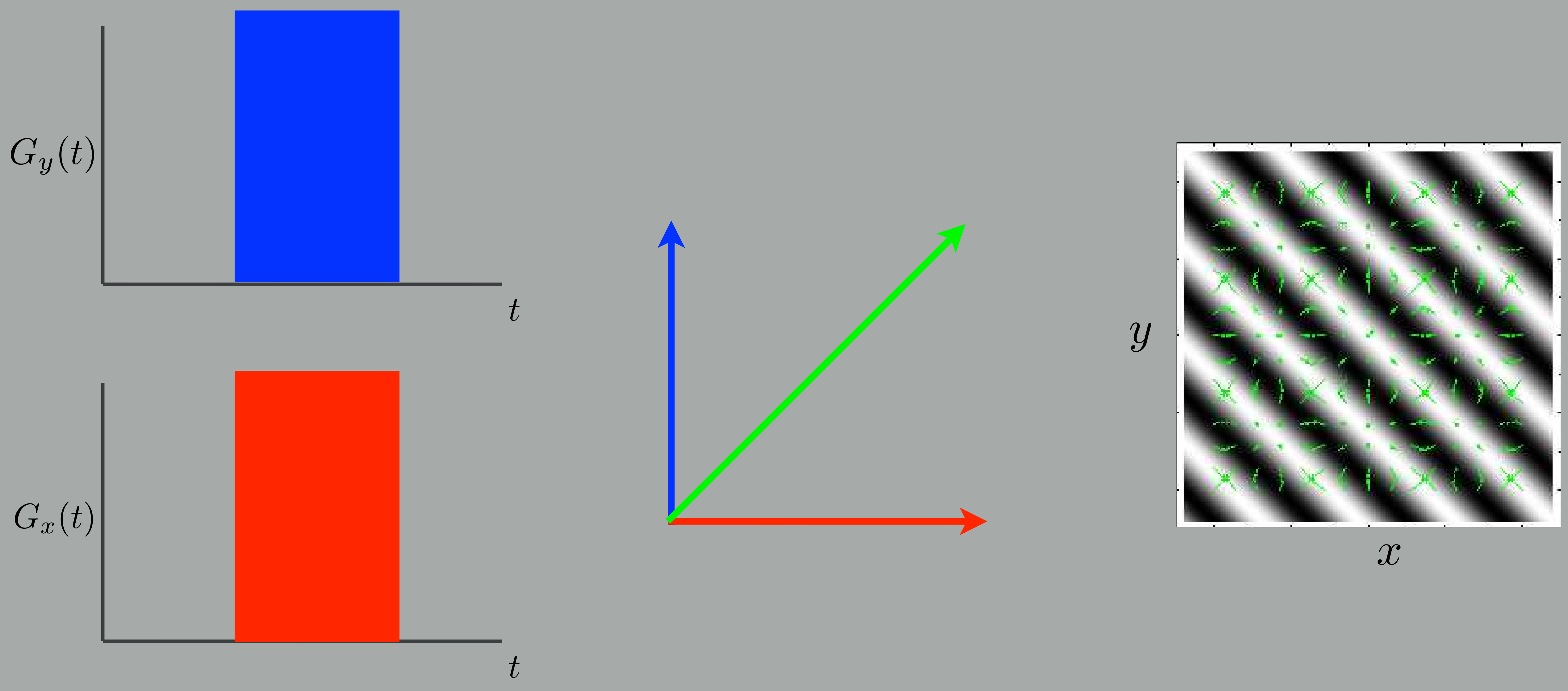






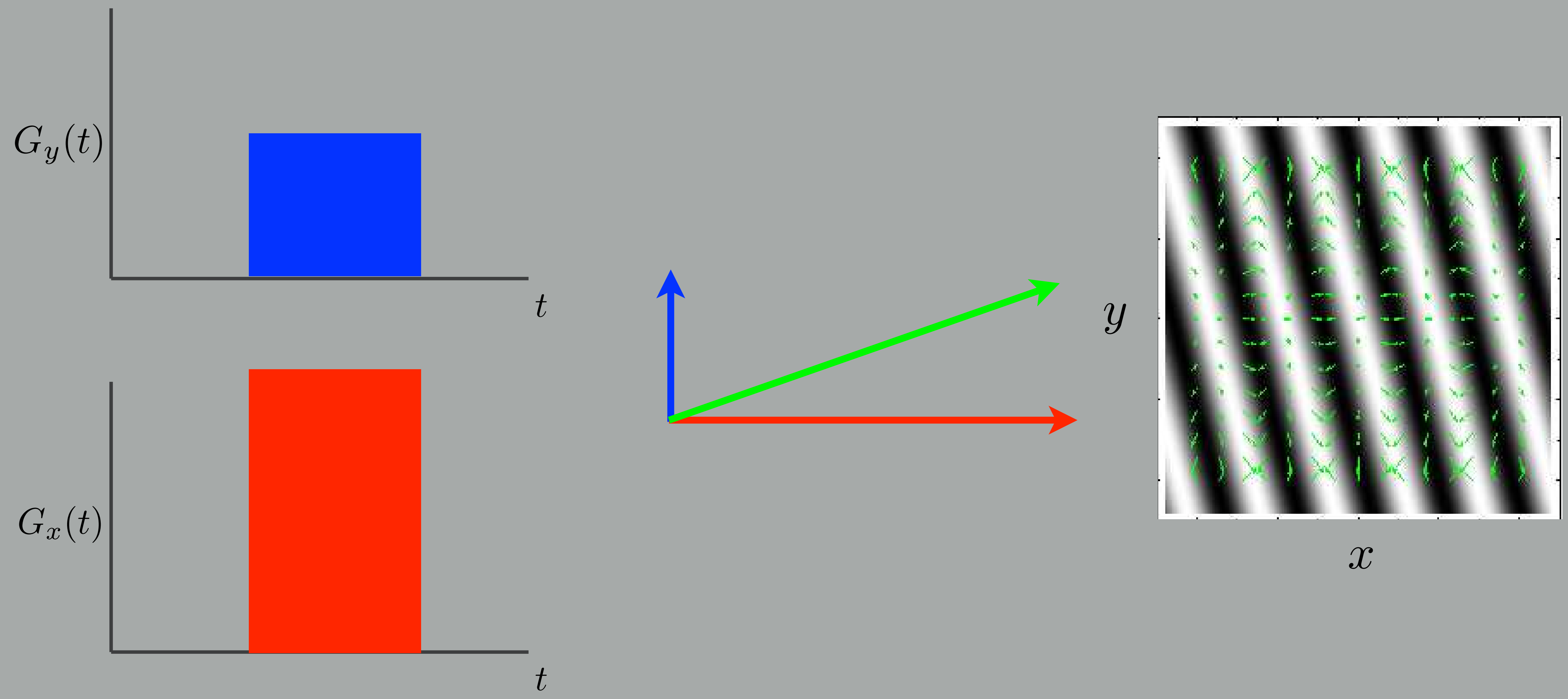
gradients add like vectors

Spatial modulation of the phase



scaling gradients changes degree of modulation

Spatial modulation of the phase



relative amplitude of gradient changes modulation direction

The NMR signal

$$s(\varphi) = \int_\Omega m_\perp(\boldsymbol{x}, t) e^{-i\varphi(\boldsymbol{x}, t)}\, d\boldsymbol{x}$$

where $$\varphi(\boldsymbol{x}, \tau) = \int_0^\tau \boldsymbol{G}(t) \cdot \boldsymbol{x}\, dt$$

$$\boldsymbol{x}(t) = \boldsymbol{x} \Rightarrow \varphi(\boldsymbol{x}, \tau) = \boldsymbol{x} \cdot \underbrace{\int_0^\tau \boldsymbol{G}(t)\, dt}_{\boldsymbol{k}}$$

The NMR signal

$$s(\boldsymbol{k}) = \int_\Omega m_\perp(\boldsymbol{x}, t) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{x}$$

The signal is the *Fourier Transform*
of the transverse magnetization

For static tissue (and perfect scanner)

$$m_\perp(\boldsymbol{x}, t) = m_\perp(\boldsymbol{x})$$

The Image

signal

$$s(\boldsymbol{k}) = \int_{\Omega} m_{\perp}(\boldsymbol{x}) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{x}$$

↓

Inverse Fourier Transform

↓

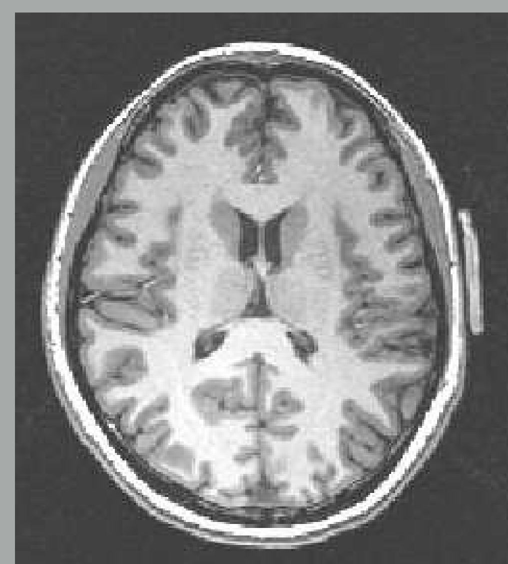
image

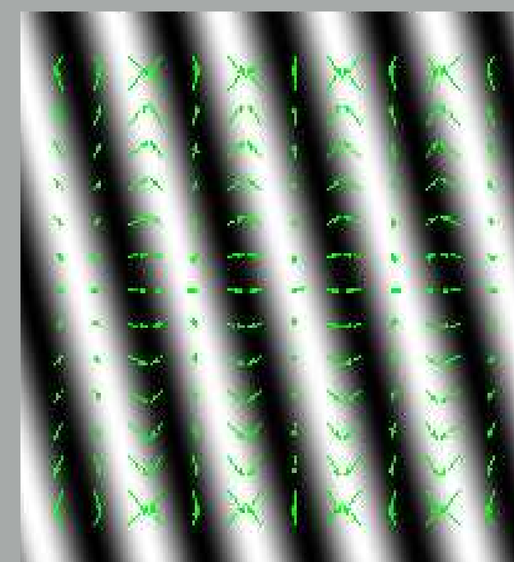
$$m_{\perp}(\boldsymbol{x}) = \int s(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\, d\boldsymbol{k}$$

The NMR signal

$$s(\varphi) = \int_\Omega m_\perp(\boldsymbol{x},t)e^{-i\varphi(\boldsymbol{x},t)}\,d\boldsymbol{x}$$

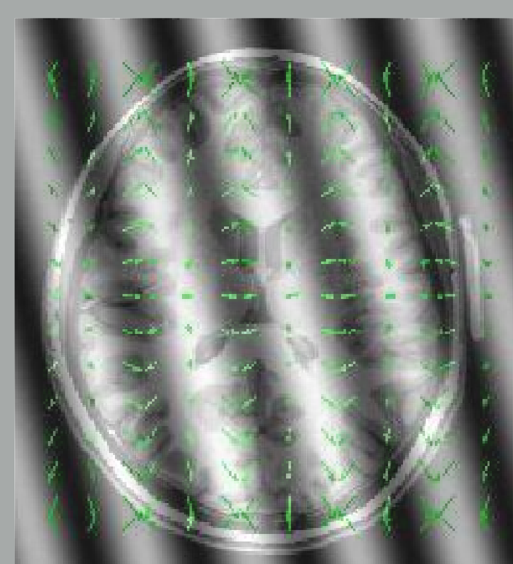
$$= \int_\Omega$$





$$d\boldsymbol{x}$$

$$= \int_\Omega$$



$$d\boldsymbol{x}$$