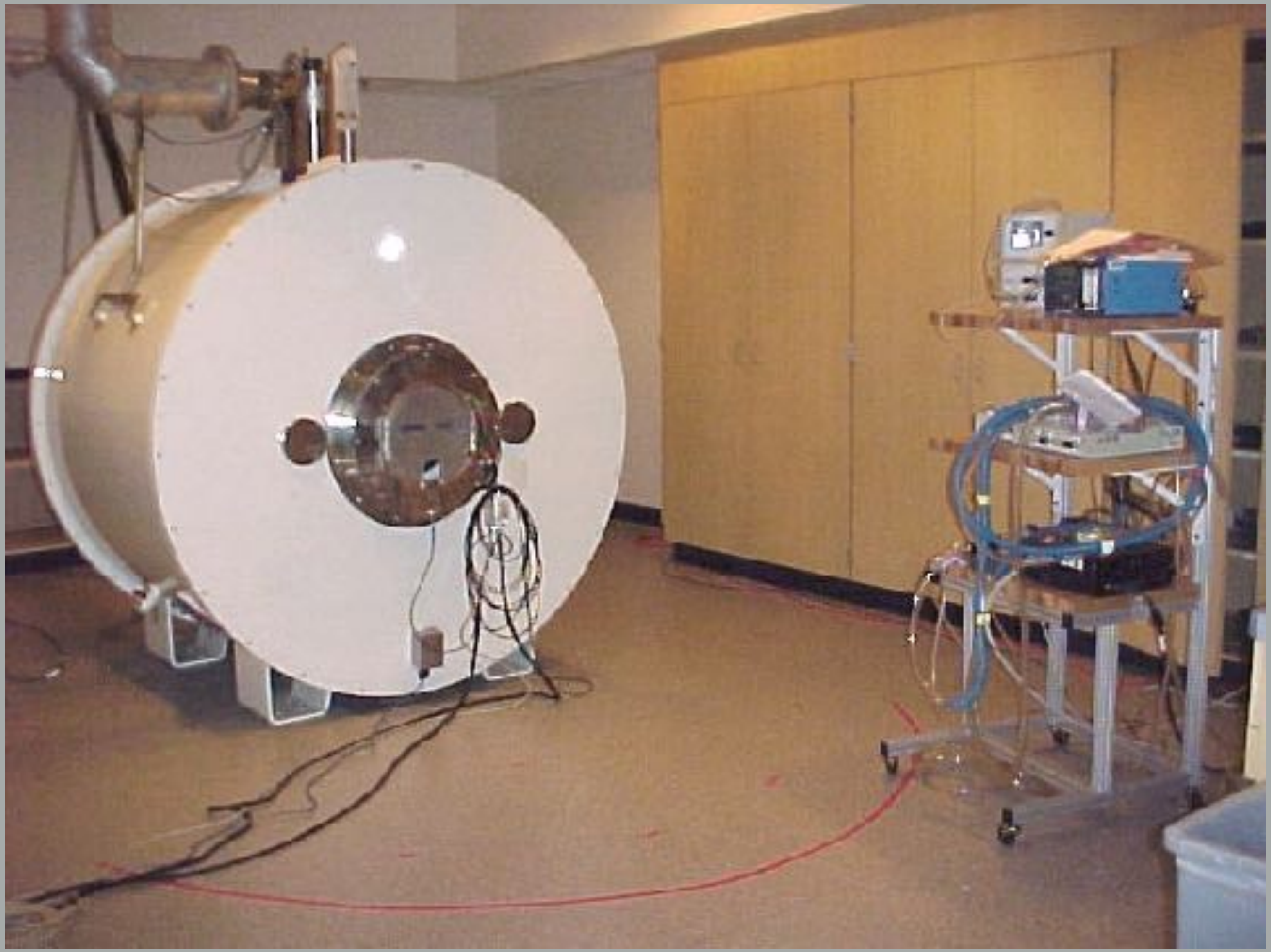
Lecture 4
Magnetic Resonance Imaging:
Physical Principles

Human magnet

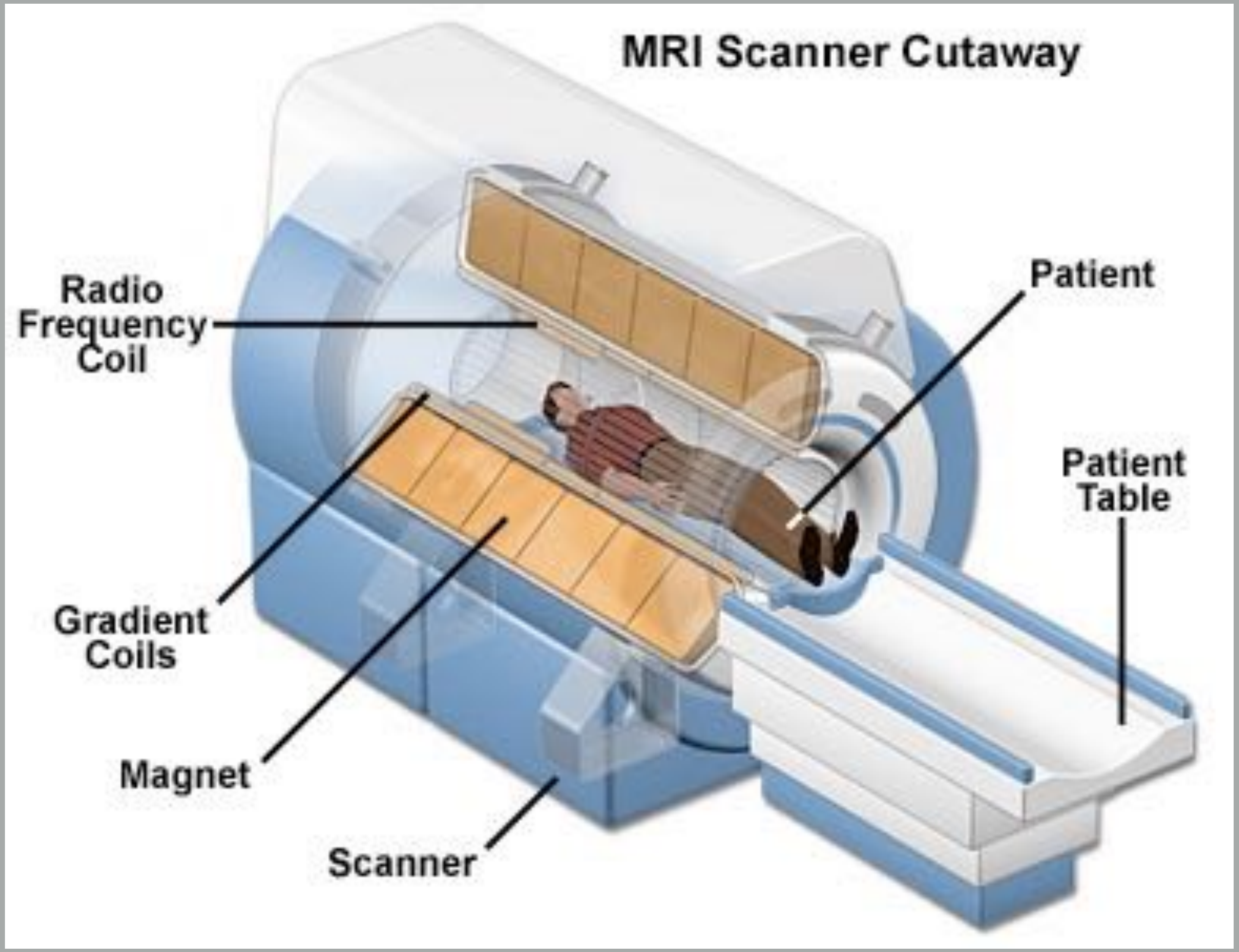
3 Tesla Clinical Magnet

The magnet



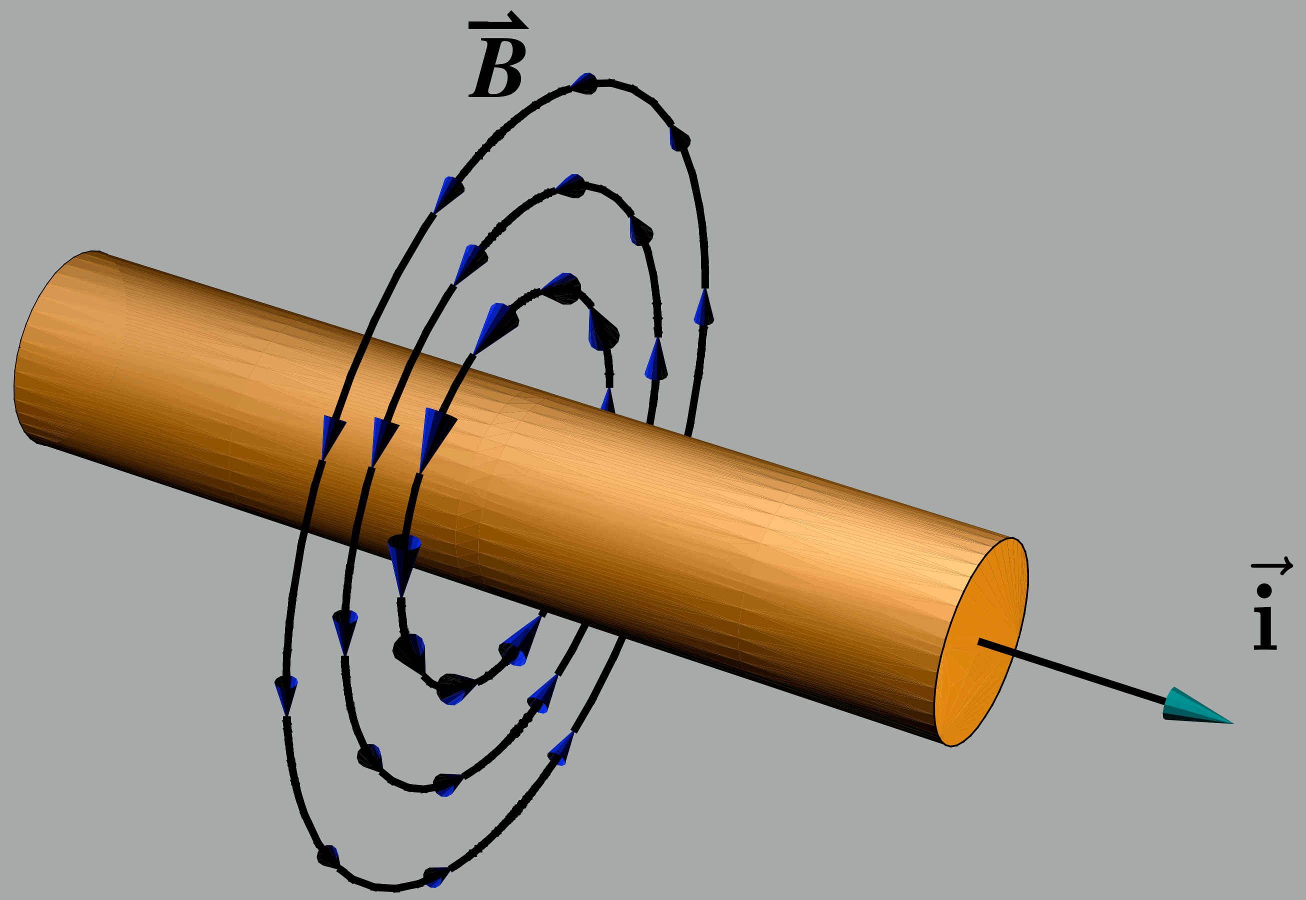
7 Tesla animal system

Clinical magnet



http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/

The Law of Biot-Savart



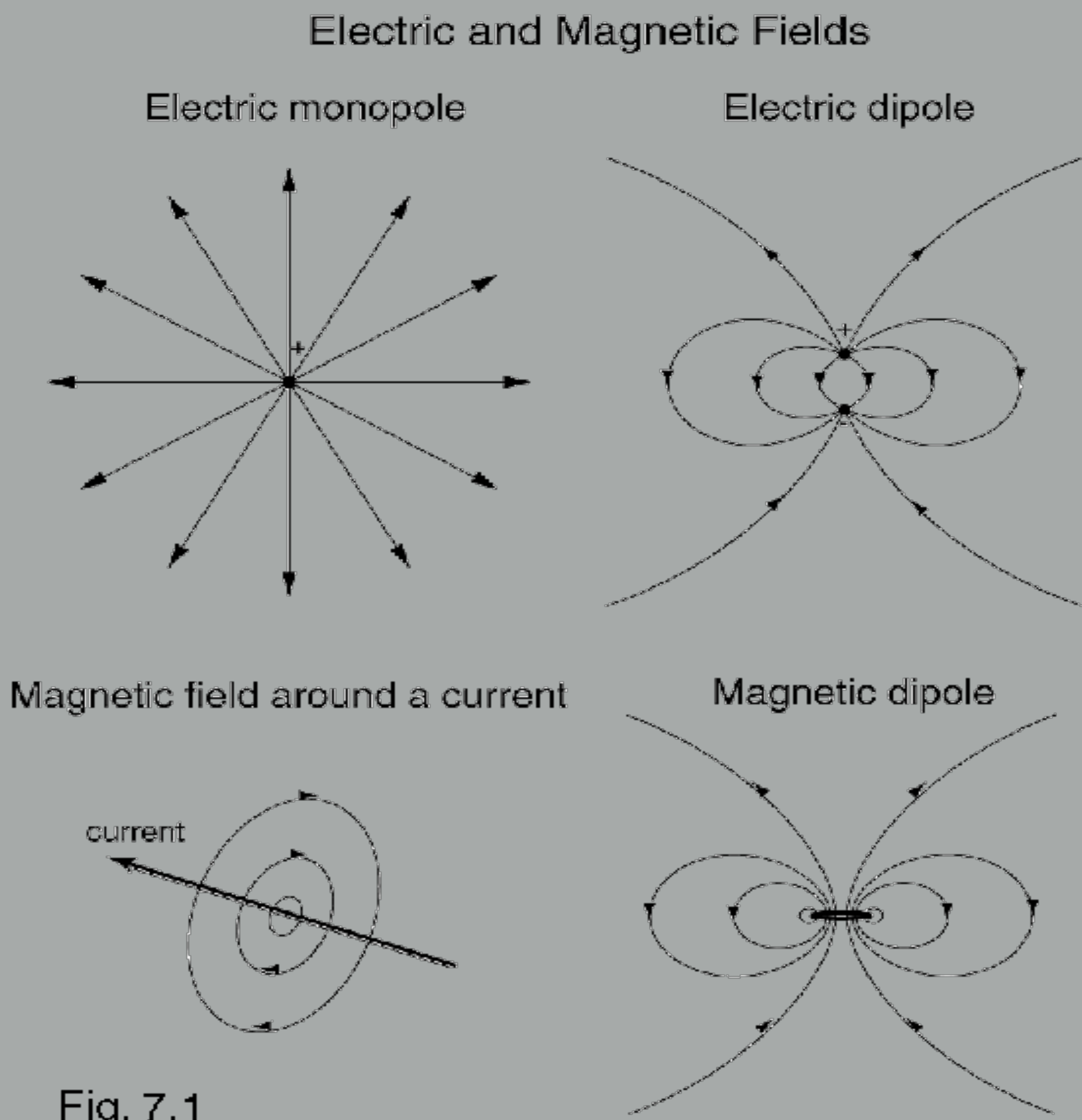
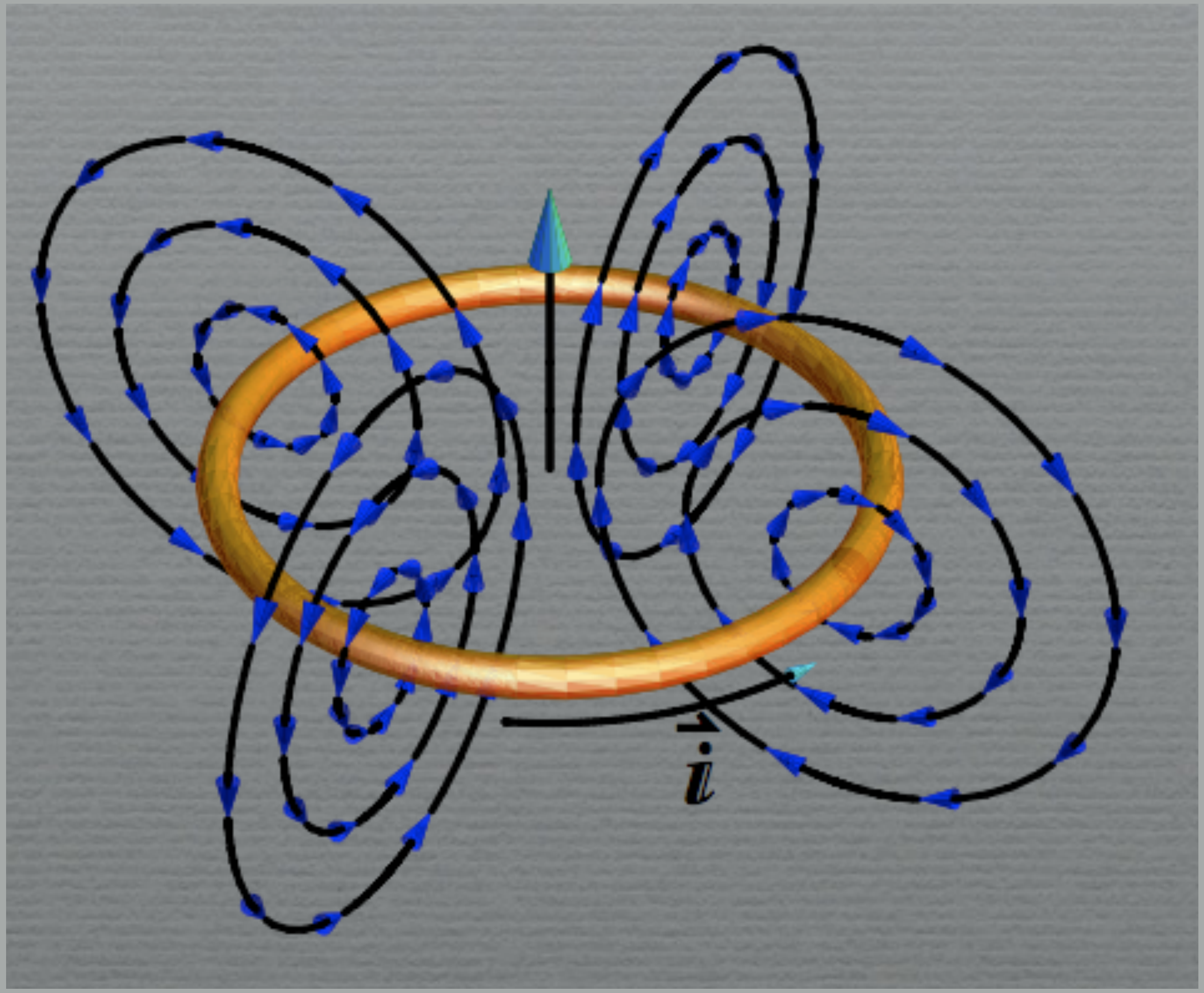
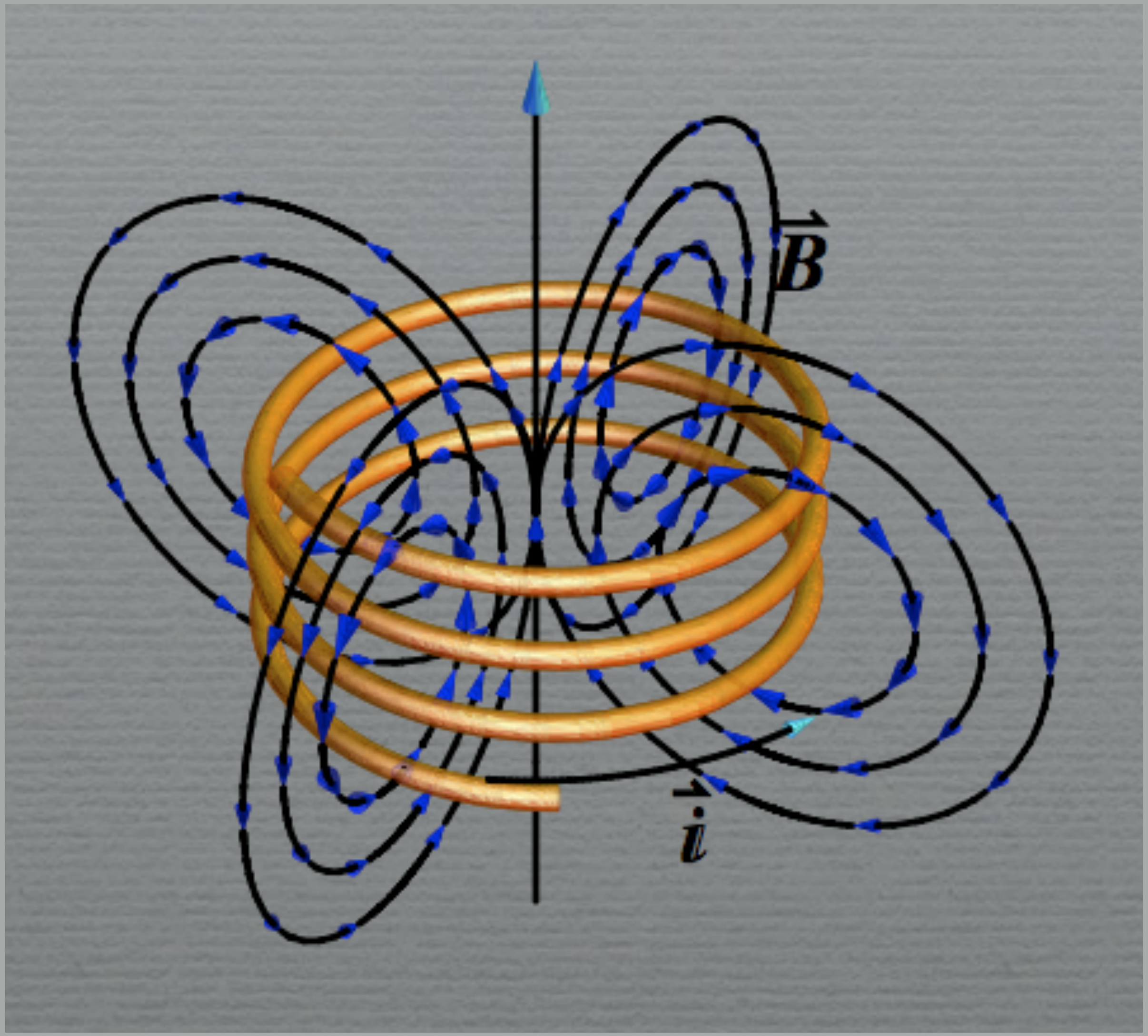

Electric and Magnetic Fields
Electric monopole
Electric dipole
+
Magnetic field around a current
current
Magnetic dipole
Fig. 7.1

Fig. 7.1

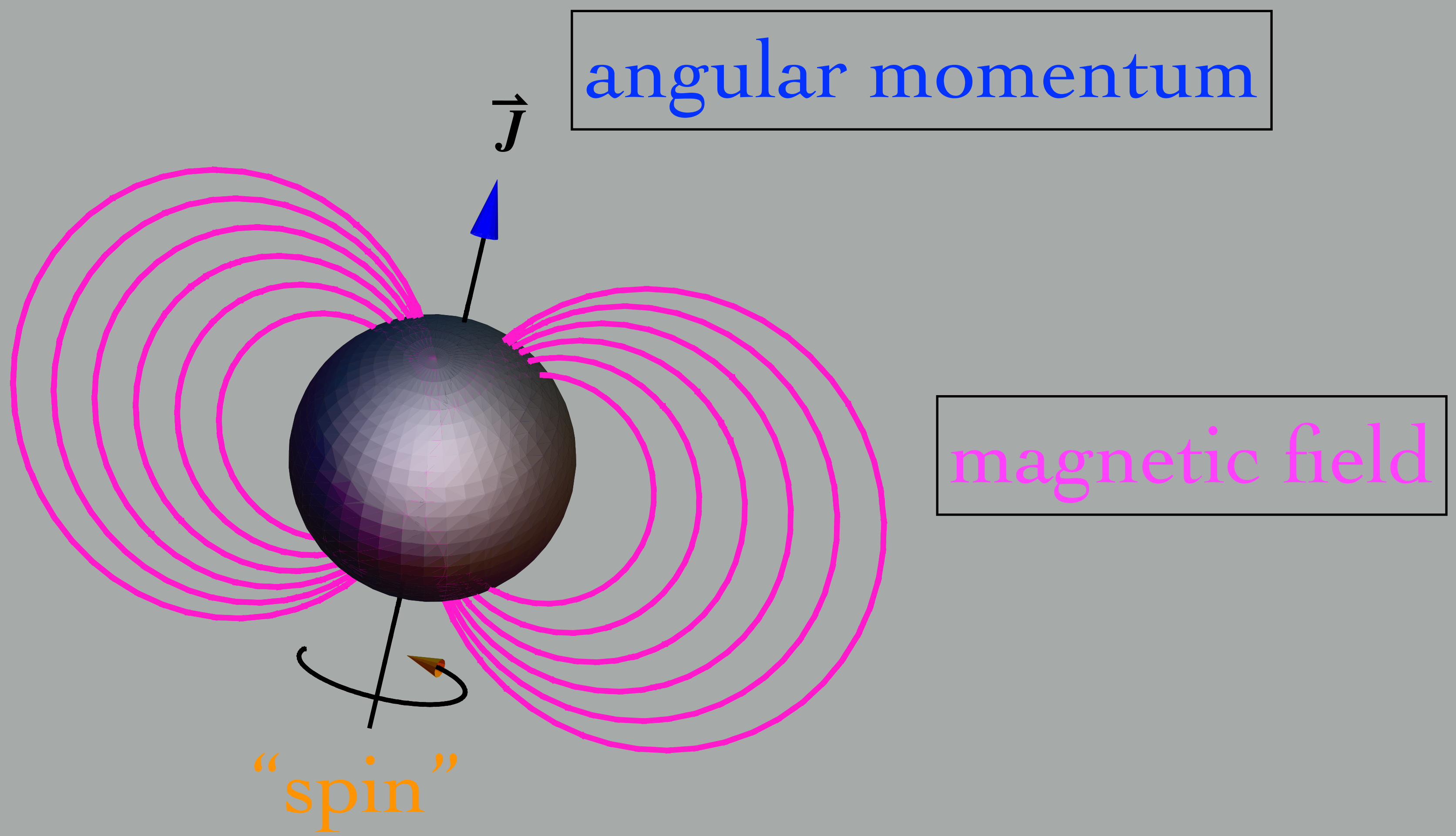
A loop



A solenoid

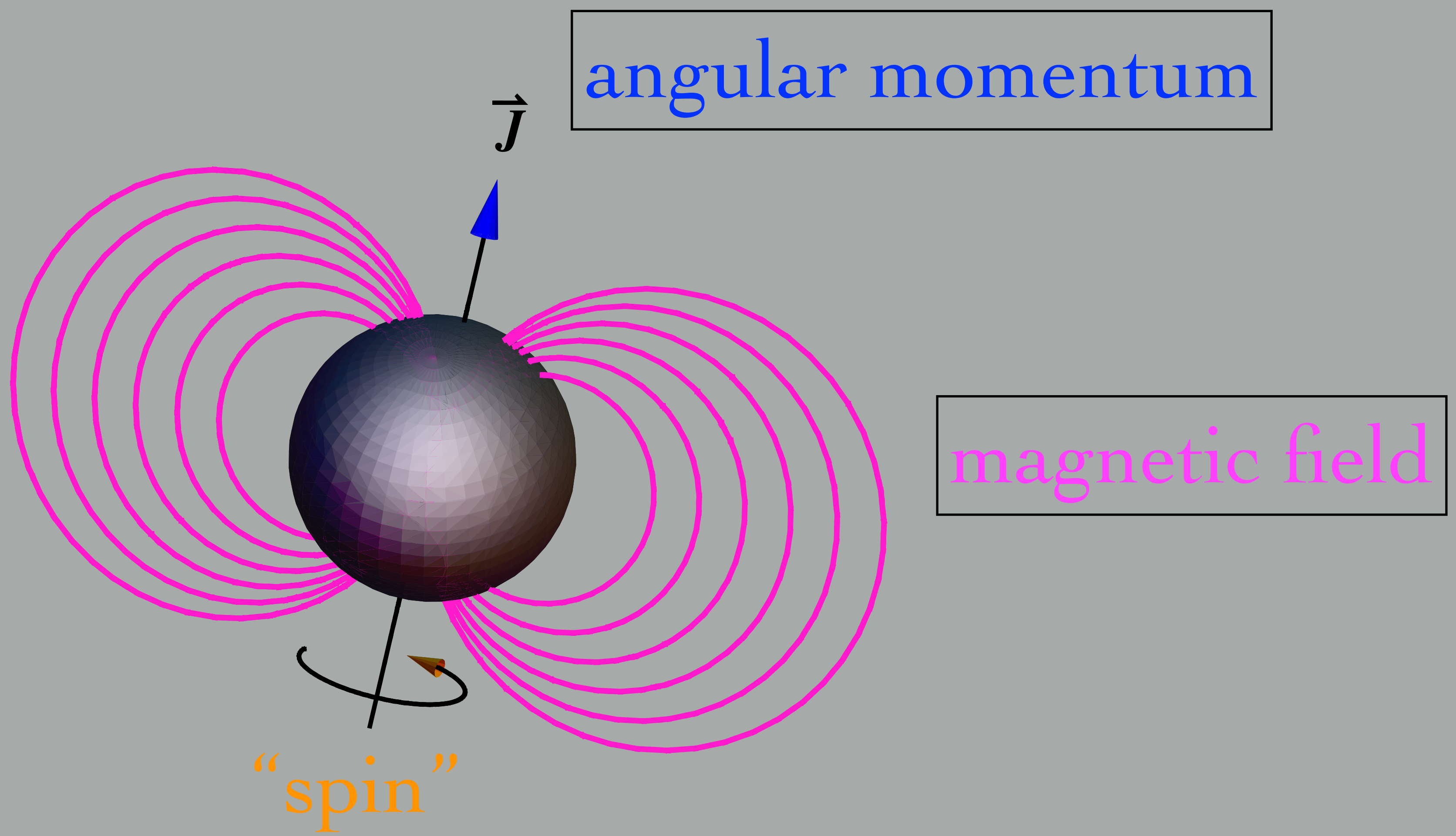


Nuclear spin



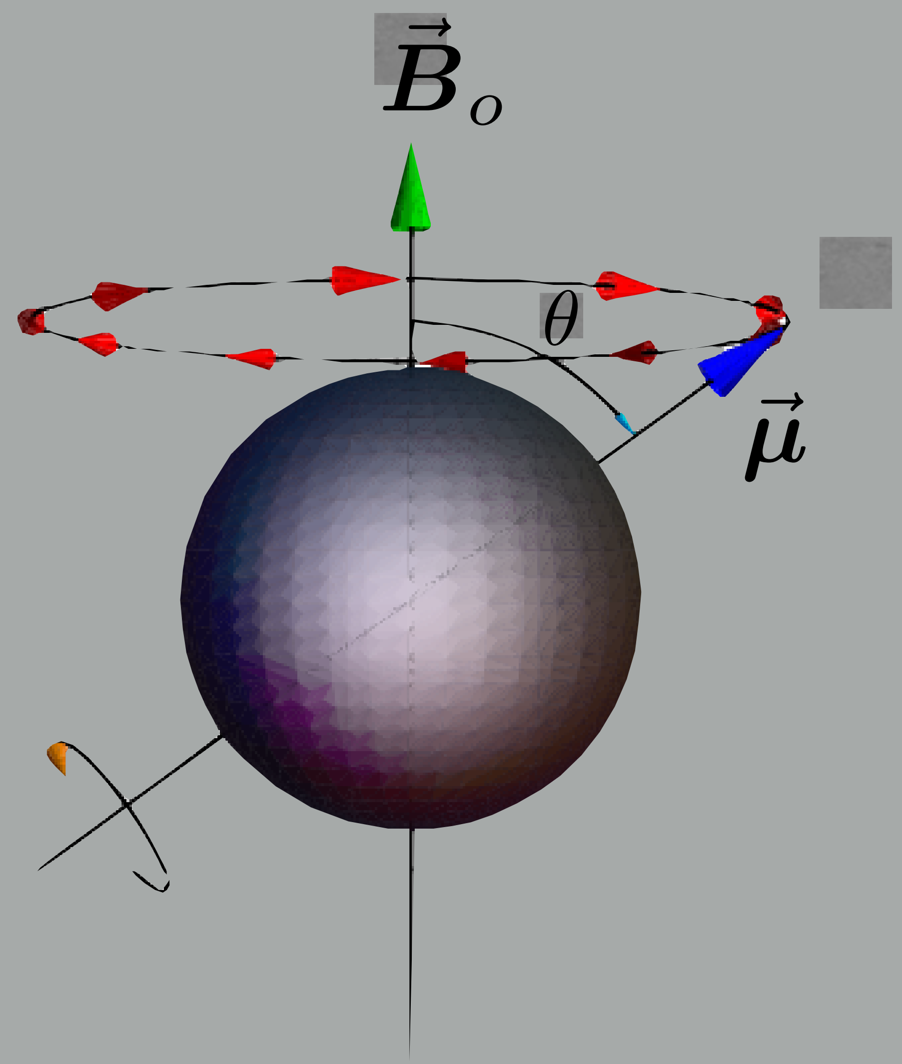
spin is a quantum mechanical phenomenon

Nuclear spin



spinning charge produces magnetic field and thus a *magnetic moment* $\vec{\mu}$ in the direction of $\vec{J}$

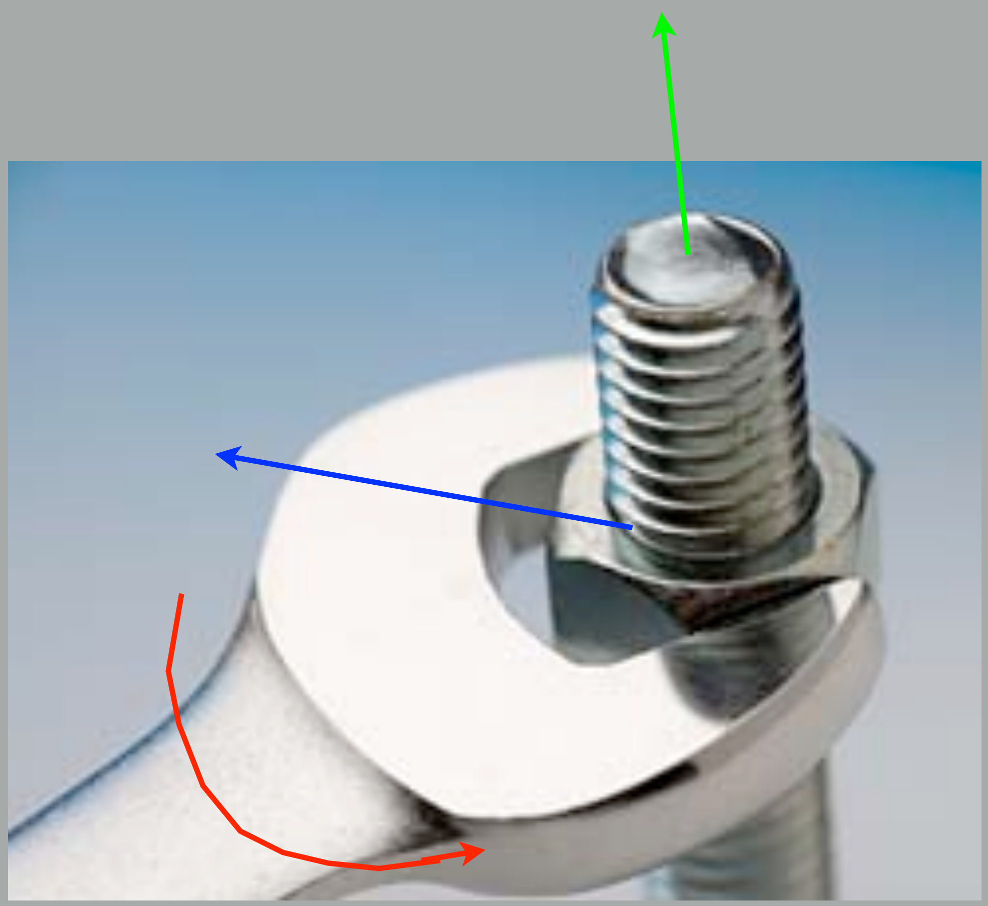
Nuclear spin

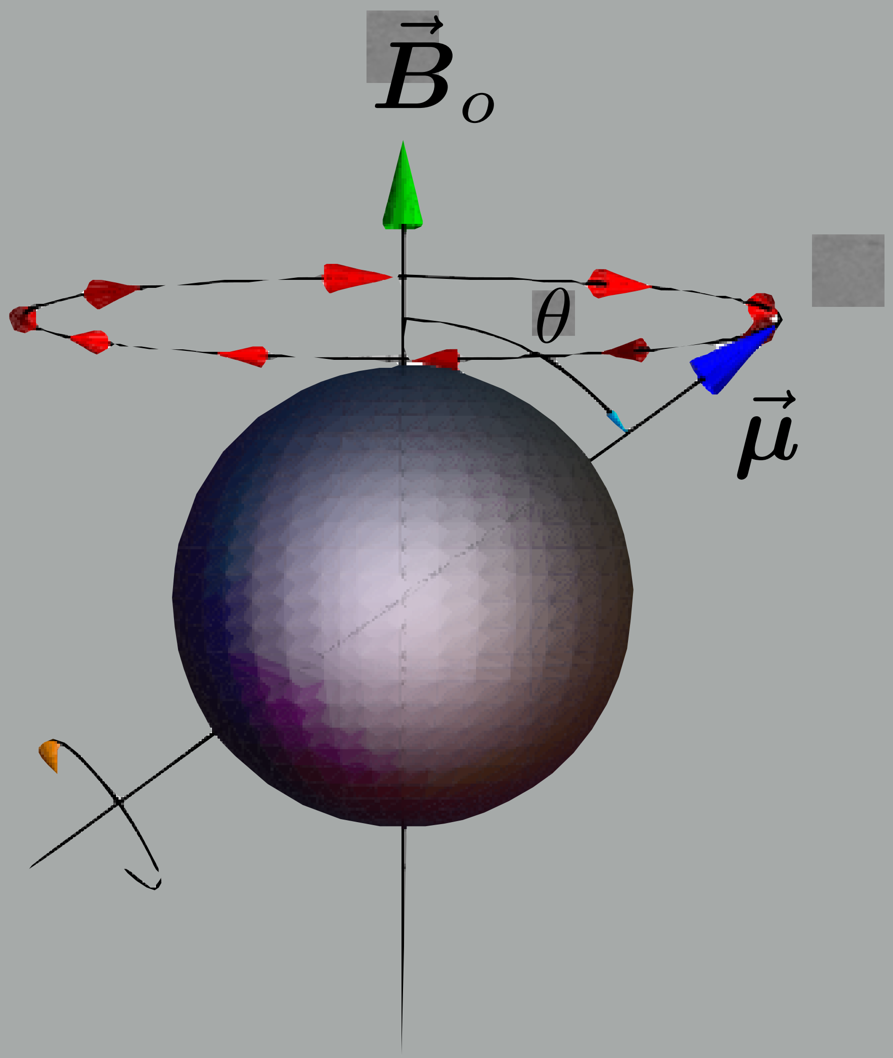


$$\tau = \mu \times B$$

Interaction of magnetic moment $\vec{\mu}$ with magnetic field causes $\vec{\mu}$ to precess about $\vec{B}_o$

Nuclear spin





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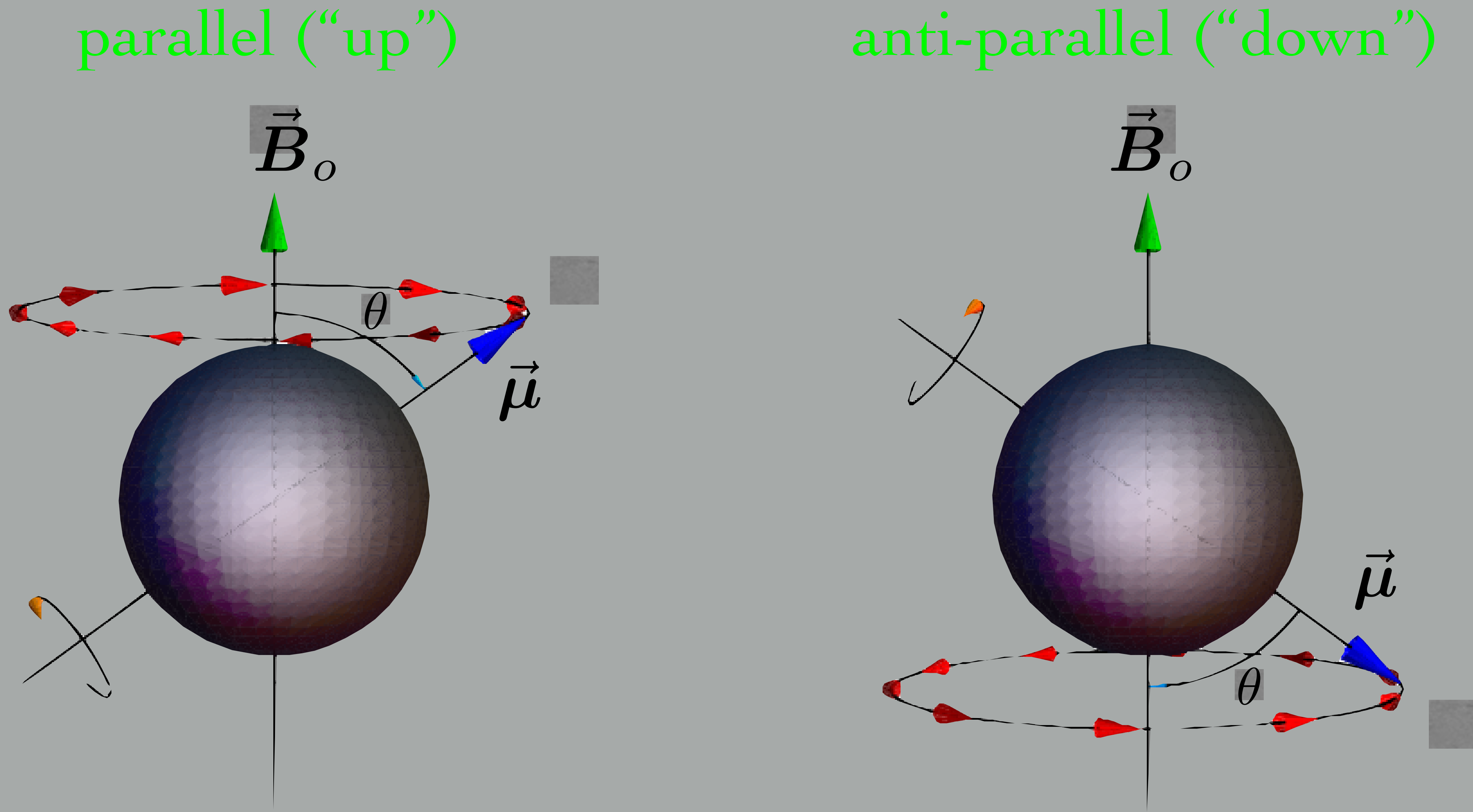
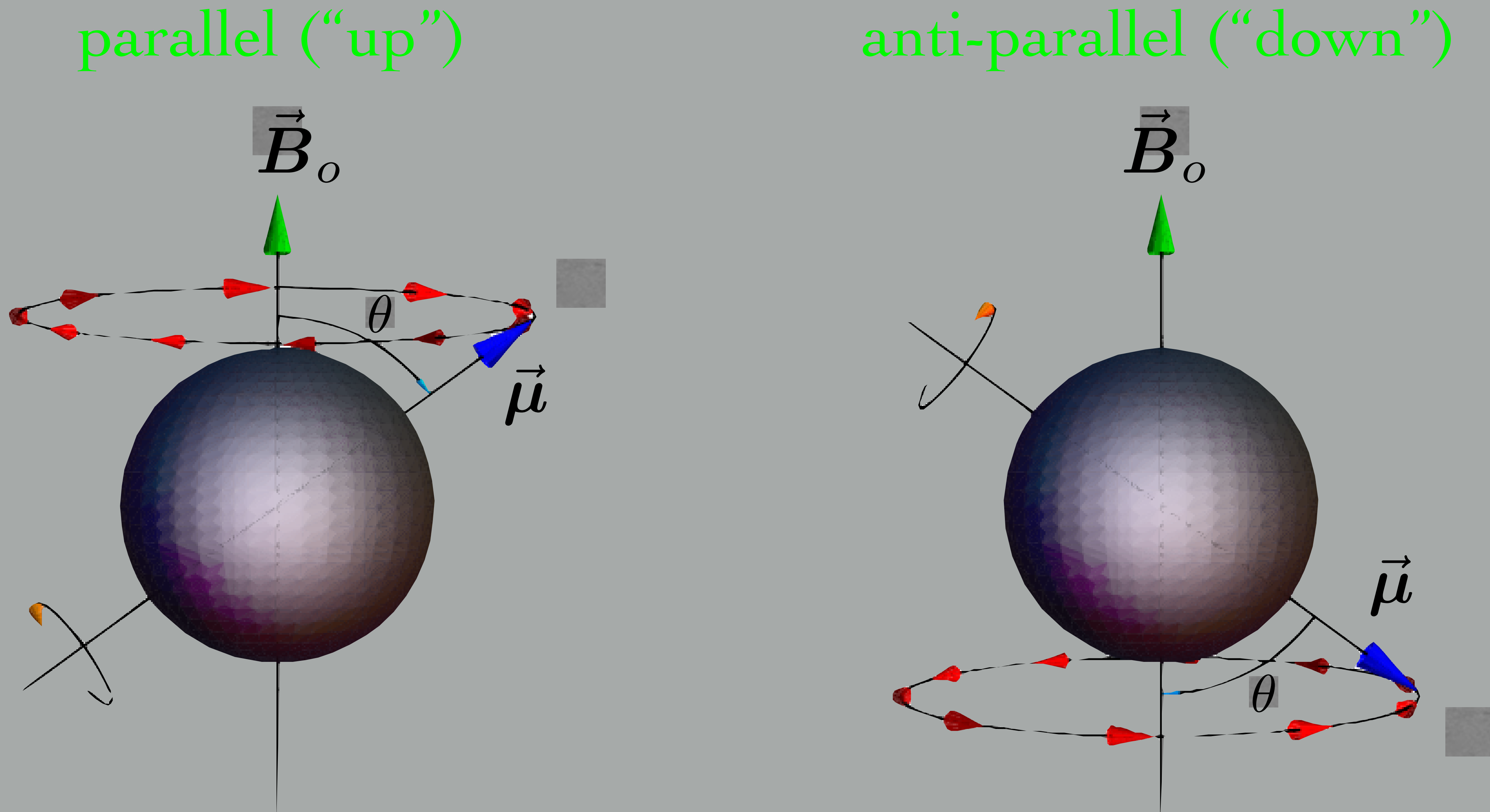
Nuclear spin



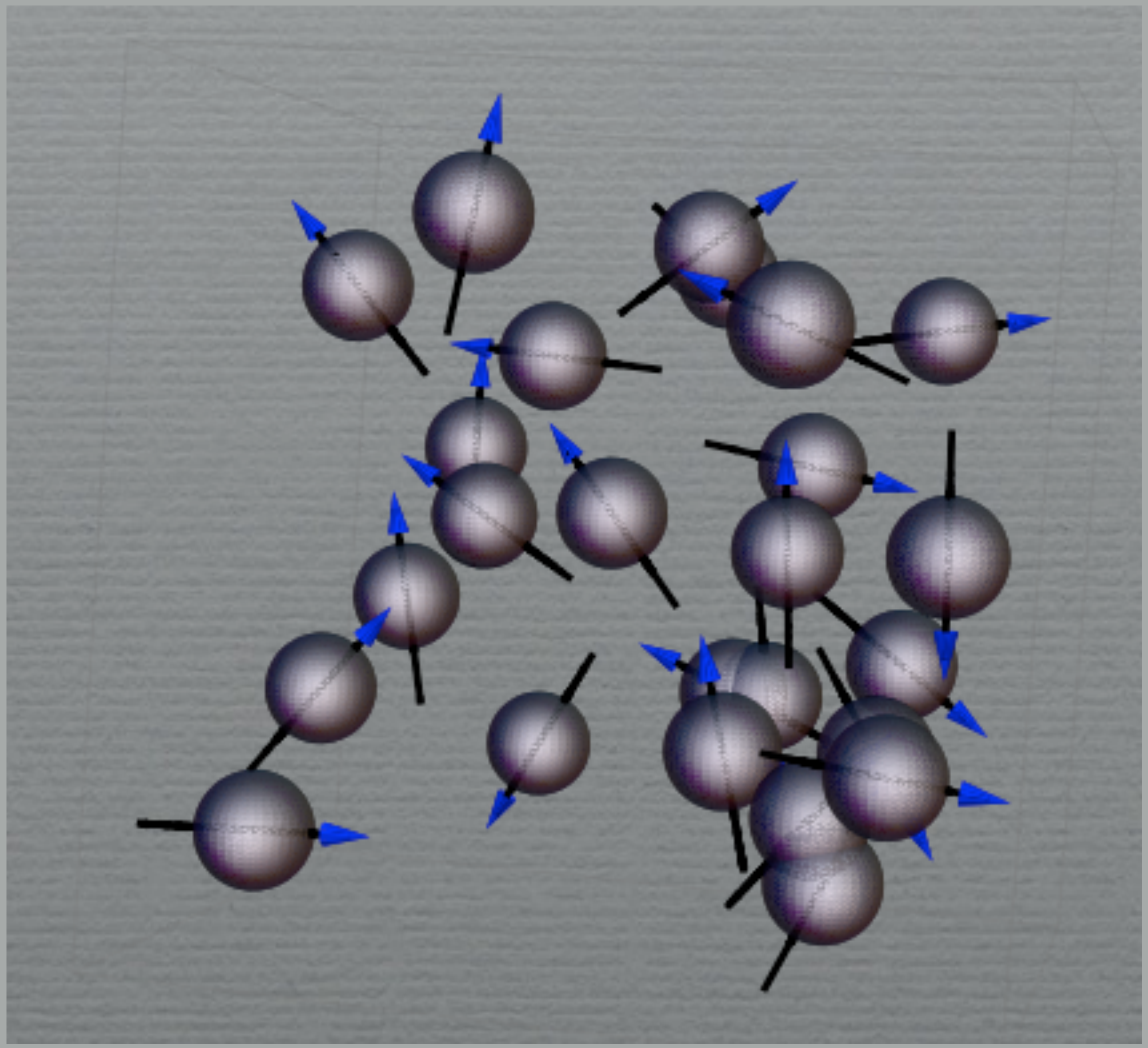
The measurement of a *single* spin produces one of two energy states and two orientations

Nuclear spin



The measurement of a *single* spin produces one of two energy states and two orientations. The energy is *quantized*.

Nuclear spins

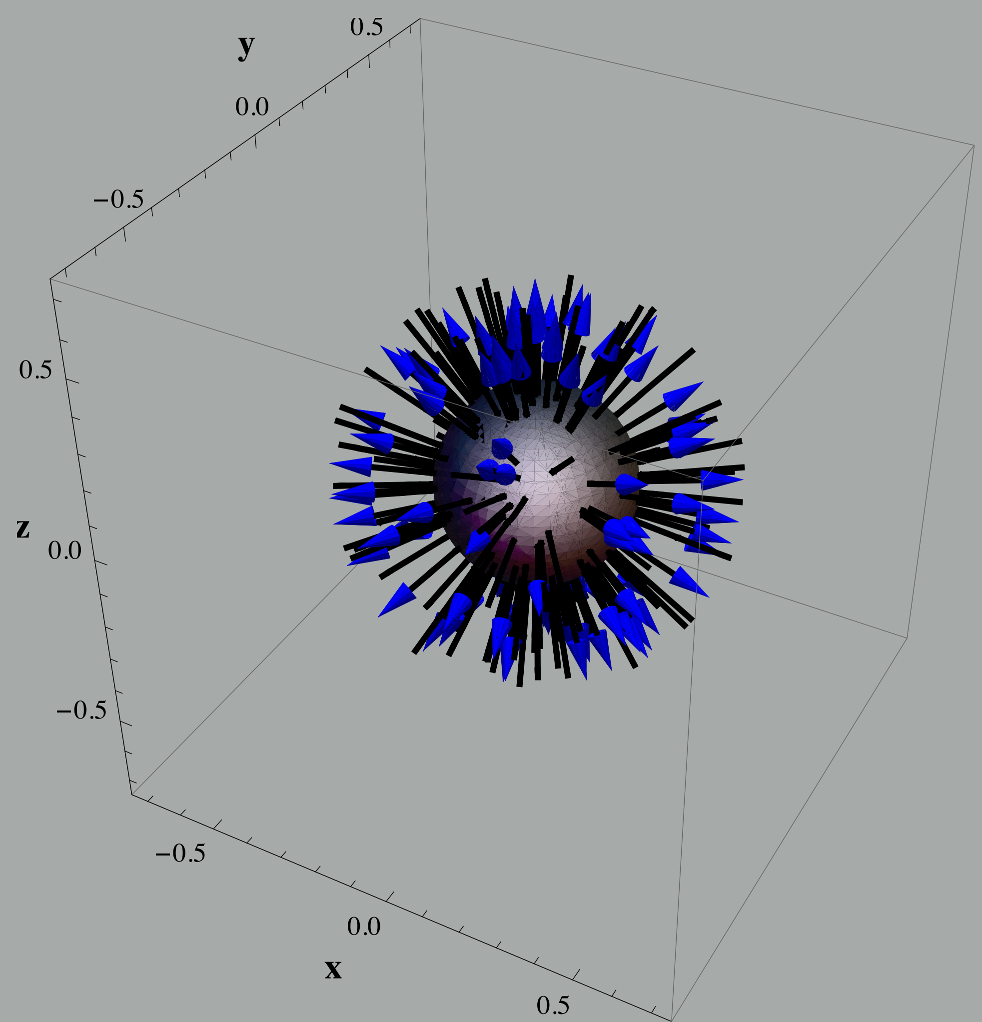


But a collections of spins interact,
producing mixtures of the two energy states

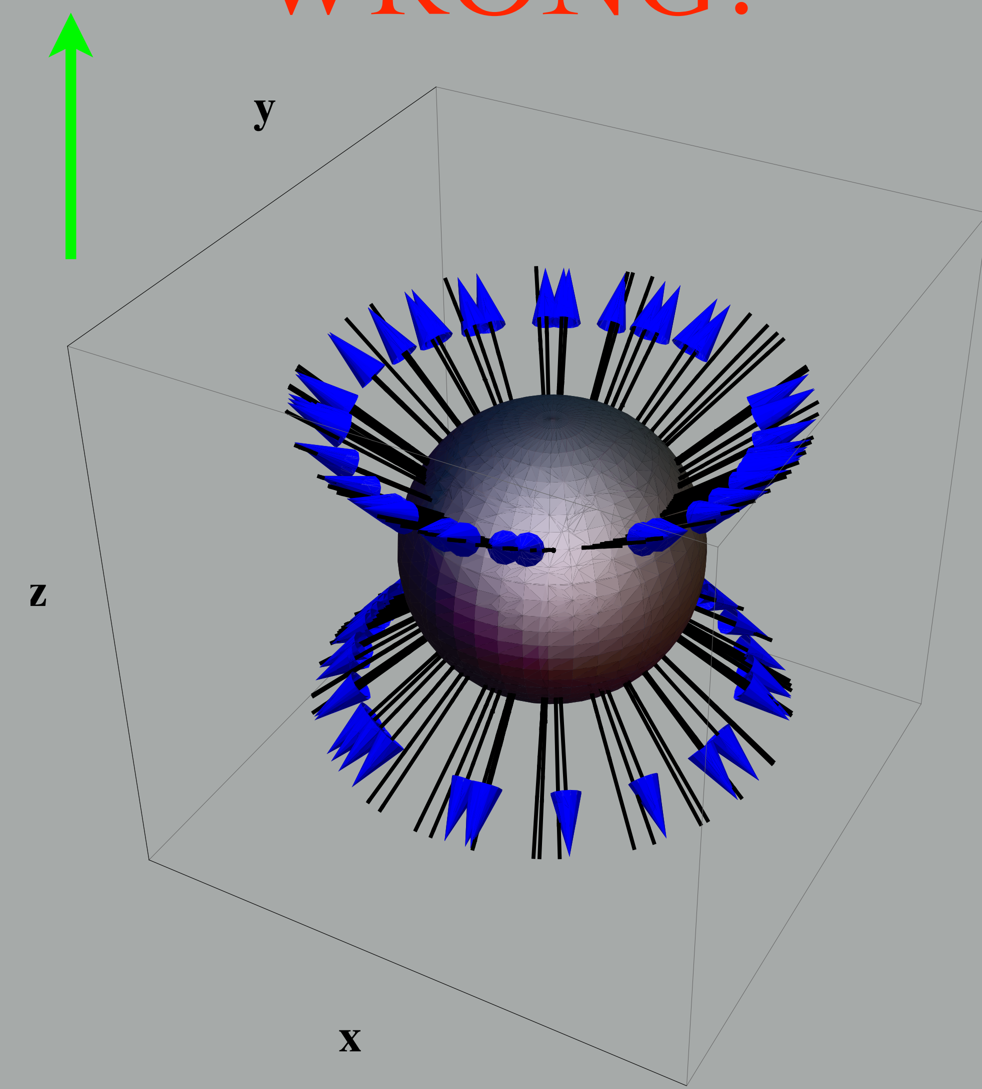
Polarization

$\vec{B}_o$

WRONG!

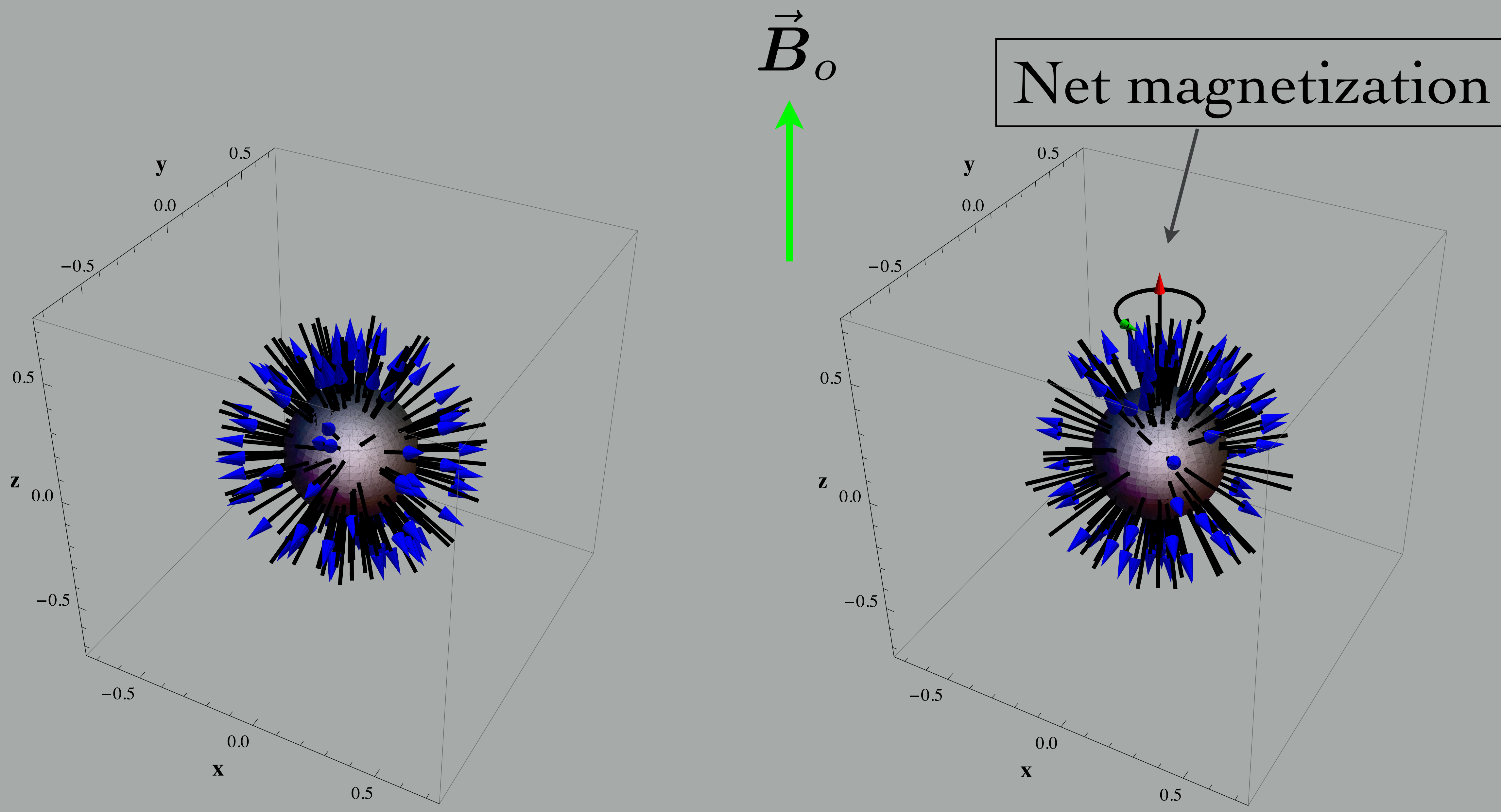


Magnetic field off



Magnetic field on

Polarization

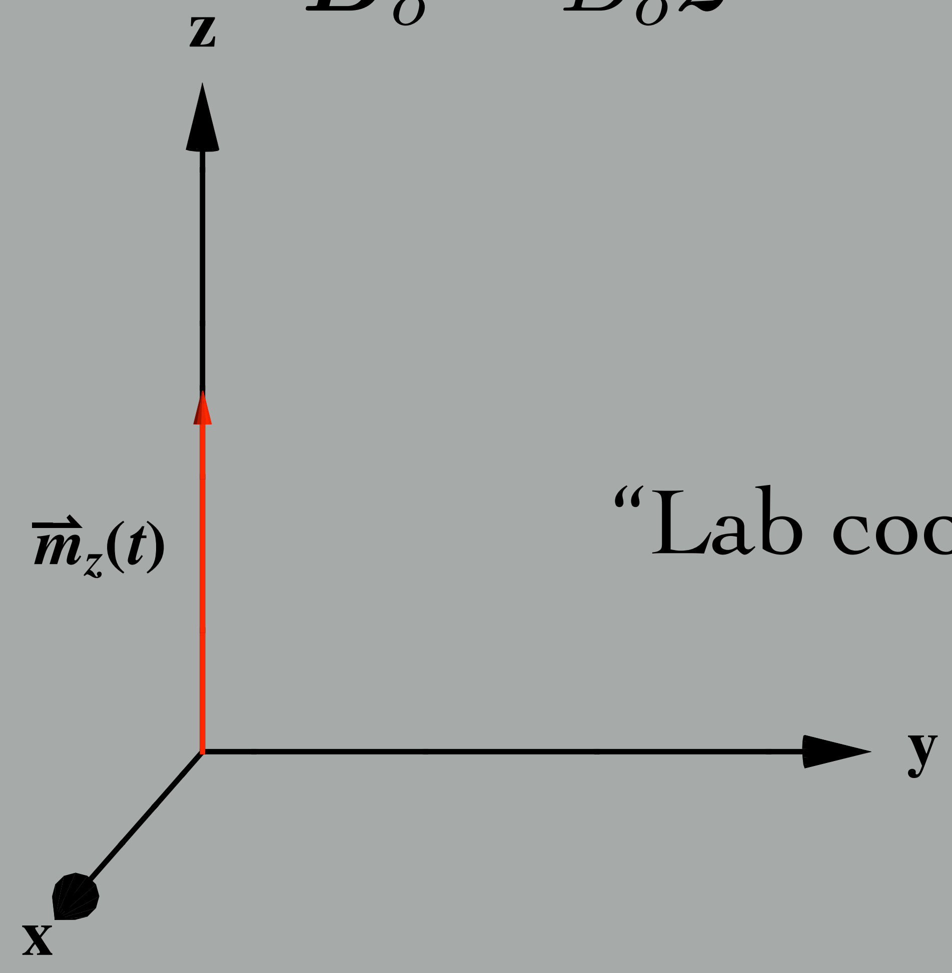


Magnetic field off

Magnetic field on

The Net Magnetization

$$\vec{\boldsymbol{B}}_o = B_o \hat{\boldsymbol{z}}$$

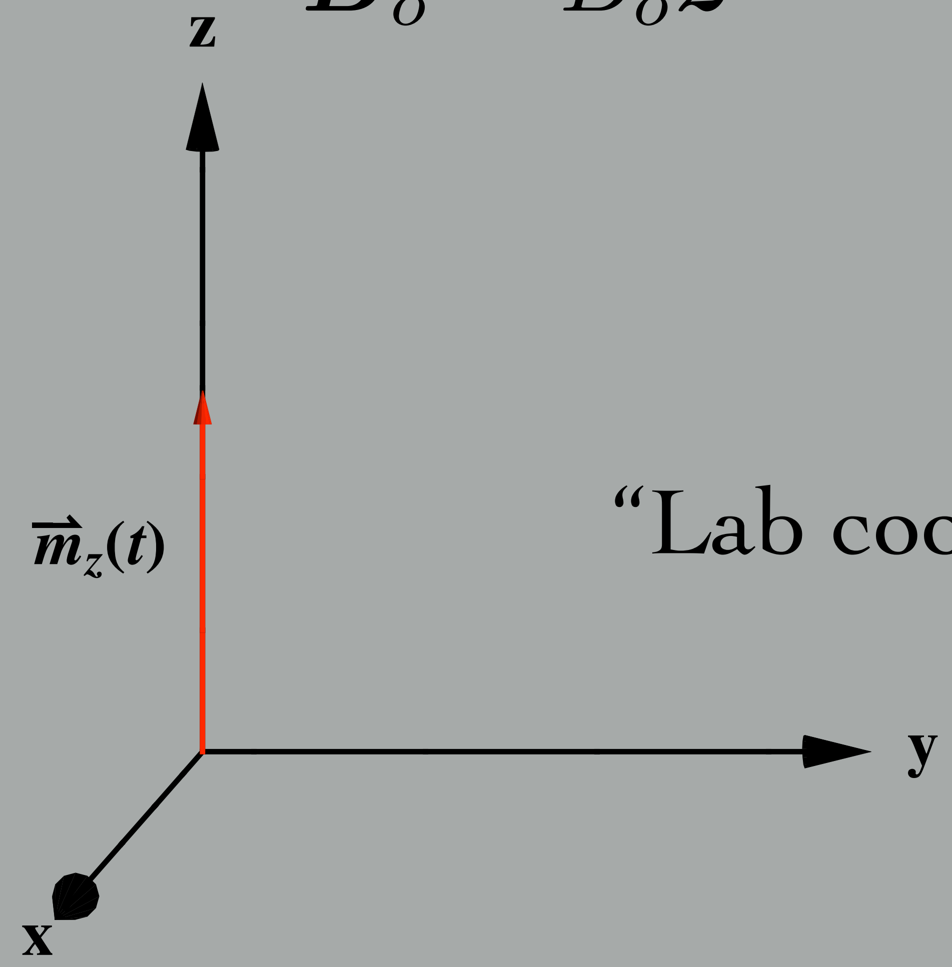


"Lab coordinates"

The net magnetization is a *vector* that can be treated *classically*

The Net Magnetization

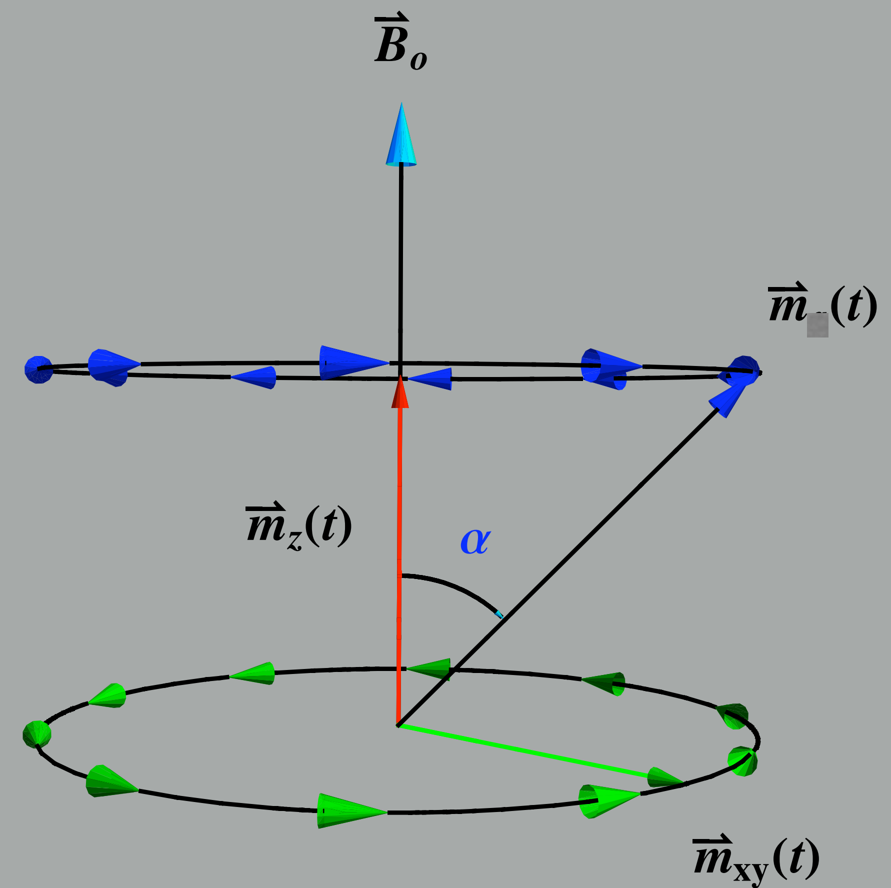
$$\vec{\boldsymbol{B}}_o = B_o \hat{\boldsymbol{z}}$$



"Lab coordinates"

The net magnetization is a *vector* that
can be treated *classically,*
meaning it can have any orientation

Precession of the magnetization



The net magnetization $\vec{\boldsymbol{m}}(t)$ precesses about the magnetic field $\hat{\boldsymbol{B}}_o = B_o\hat{\boldsymbol{z}}$

Equation of motion of the magnetization:

$$\frac{dm(t)}{dt} = \gamma m(t) \times B(t)$$

γ = gyromagnetic ratio

Traditional units

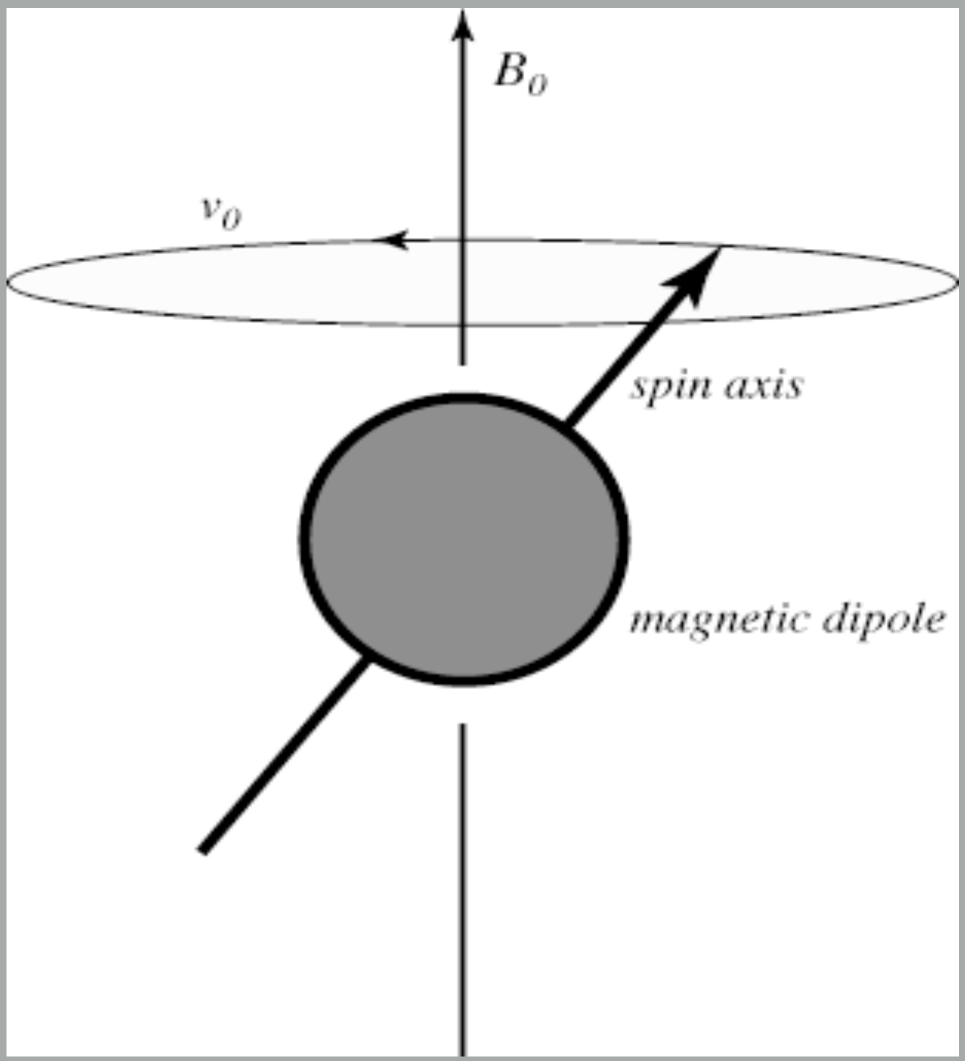
$$\boxed{\omega = \gamma B}$$

$$[\mathrm{Hz}] = \left[\frac{\mathrm{Hz}}{\mathrm{Tesla}}\right] [\mathrm{Tesla}]$$

$$\boxed{\omega = \gamma G x}$$

$$[\mathrm{Hz}] = \left[\frac{\mathrm{Hz}}{\mathrm{Gauss}}\right]\left[\frac{\mathrm{Gauss}}{\mathrm{cm}}\right] [\mathrm{cm}]$$

Precession



Nuclei with an odd number of neutrons or protons possess spin, and precess in a magnetic field

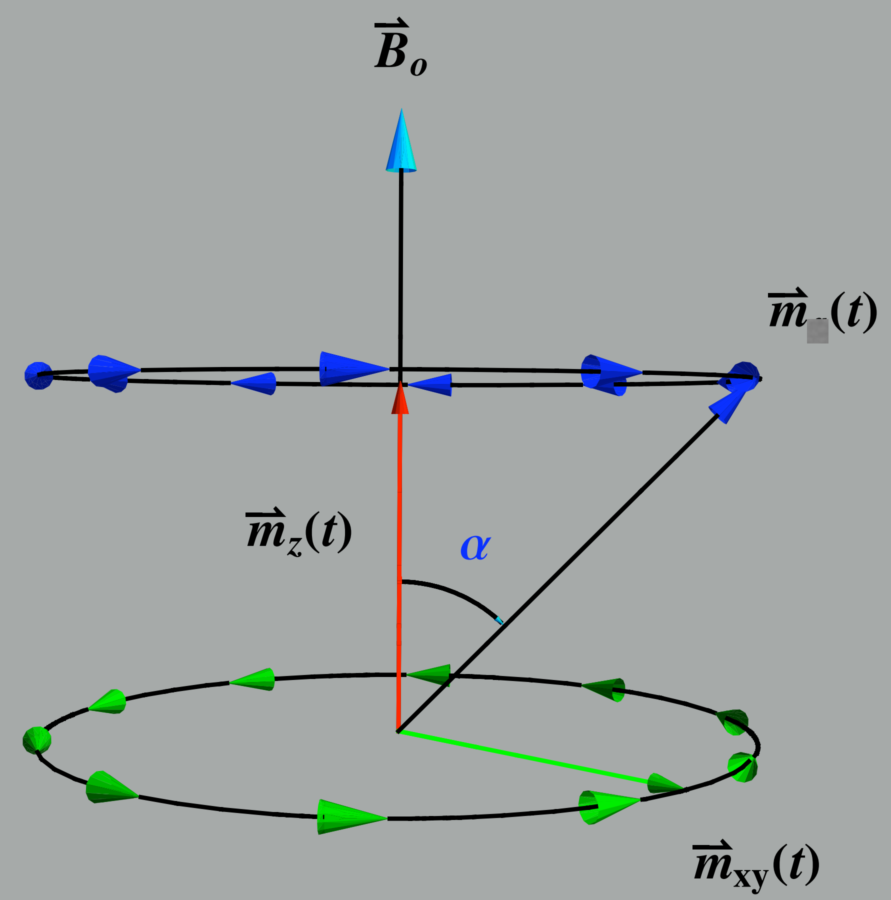
Precession

Resonant Frequency: $\nu_0 = \gamma B_0$
(128 MHz at 3T)

Gyromagnetic Ratio

Nucleus	γ (MHz/T)
^{1}H	42.58
^{13}C	10.71
^{19}F	40.08
^{23}Na	11.27
^{31}P	17.25

Precession of the magnetization

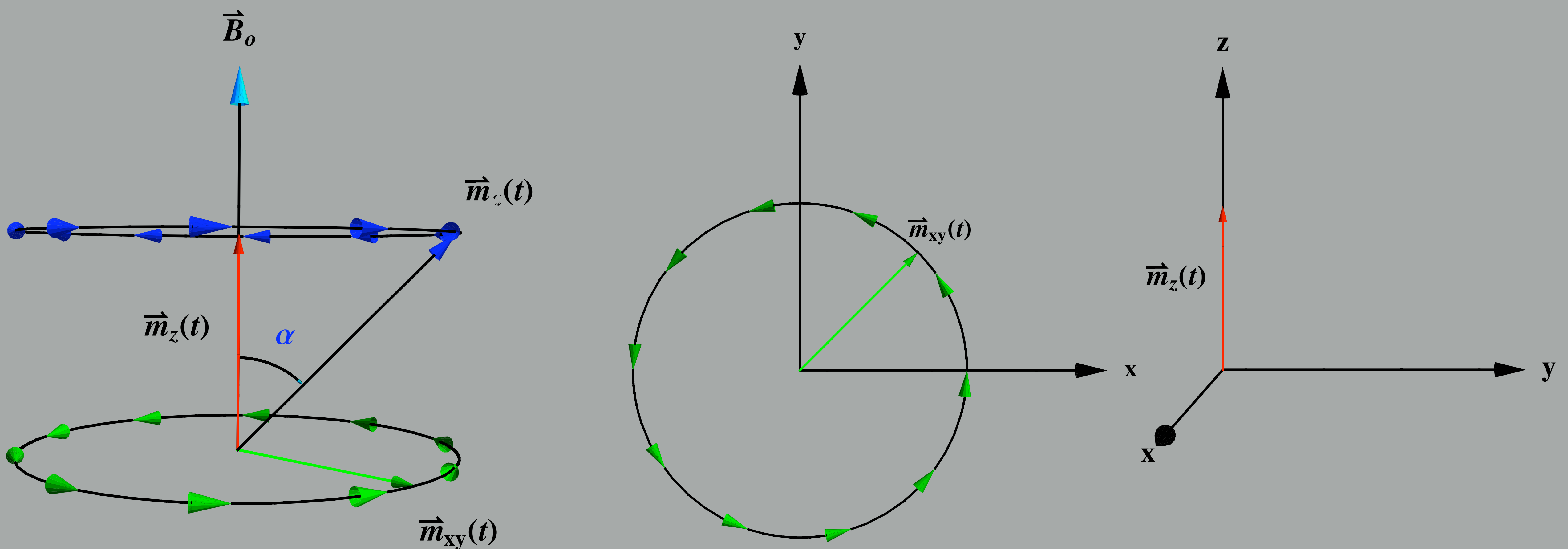


The net magnetization $\vec{m}(t)$ precesses about the magnetic field $\hat{B}_o = B_o\hat{z}$

Frequency of precession: $\boxed{\omega_o = \gamma B_o}$

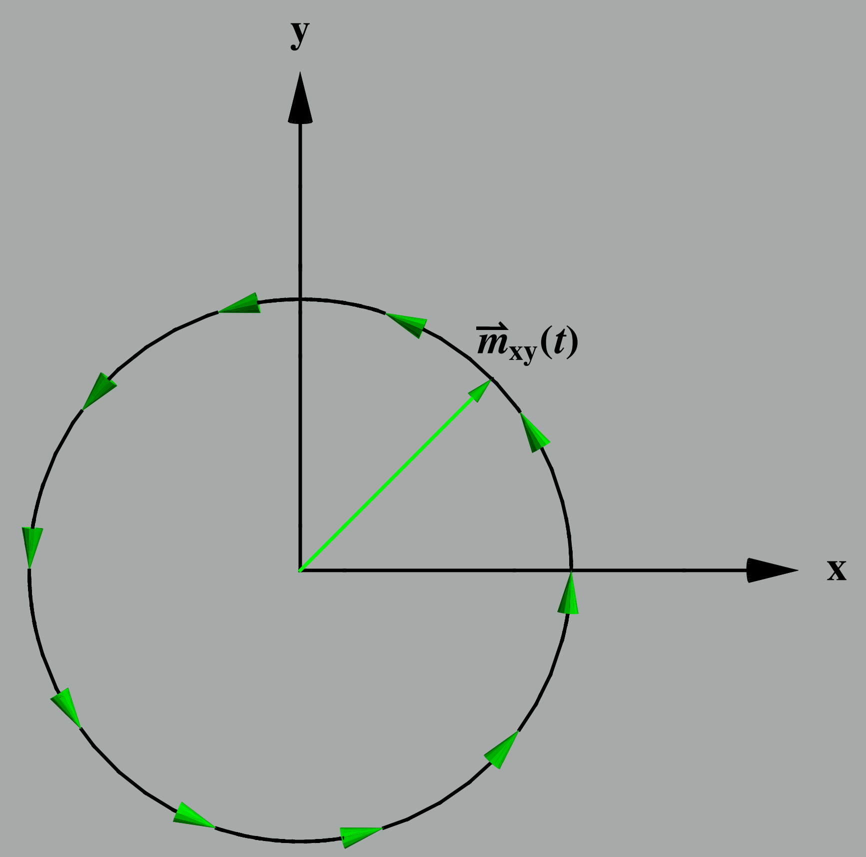
The Larmor Frequency

Precession of the magnetization



If $\vec{\boldsymbol{m}}(t)$ is at an angle α relative to $\hat{\boldsymbol{z}}$,
it has a *longitudinal* component $\vec{\boldsymbol{m}}_z(t)$ and a *transverse* component $\vec{\boldsymbol{m}}_{xy}(t)$

Precession of the magnetization



$$\vec{m}_{xy}(t) = |\vec{m}_{xy}(t)| e^{i\omega_0 t}$$ in main field B_0

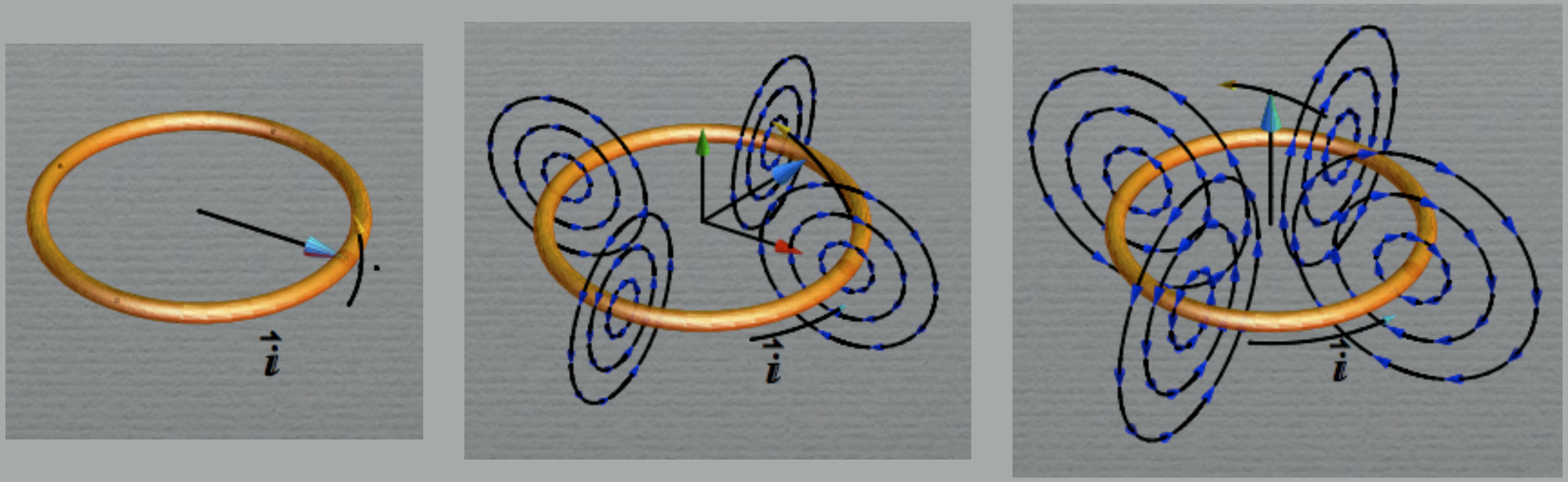
this is why we introduced complex numbers!

The NMR signal phase

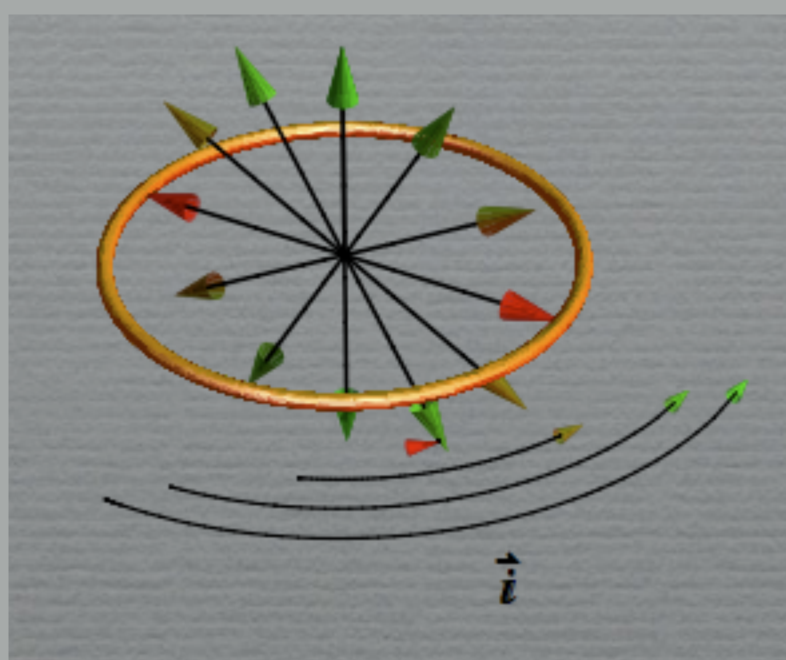
$$\vec{m}_\perp(t) = |\vec{m}_\perp(t)|e^{-i\varphi(t)}$$

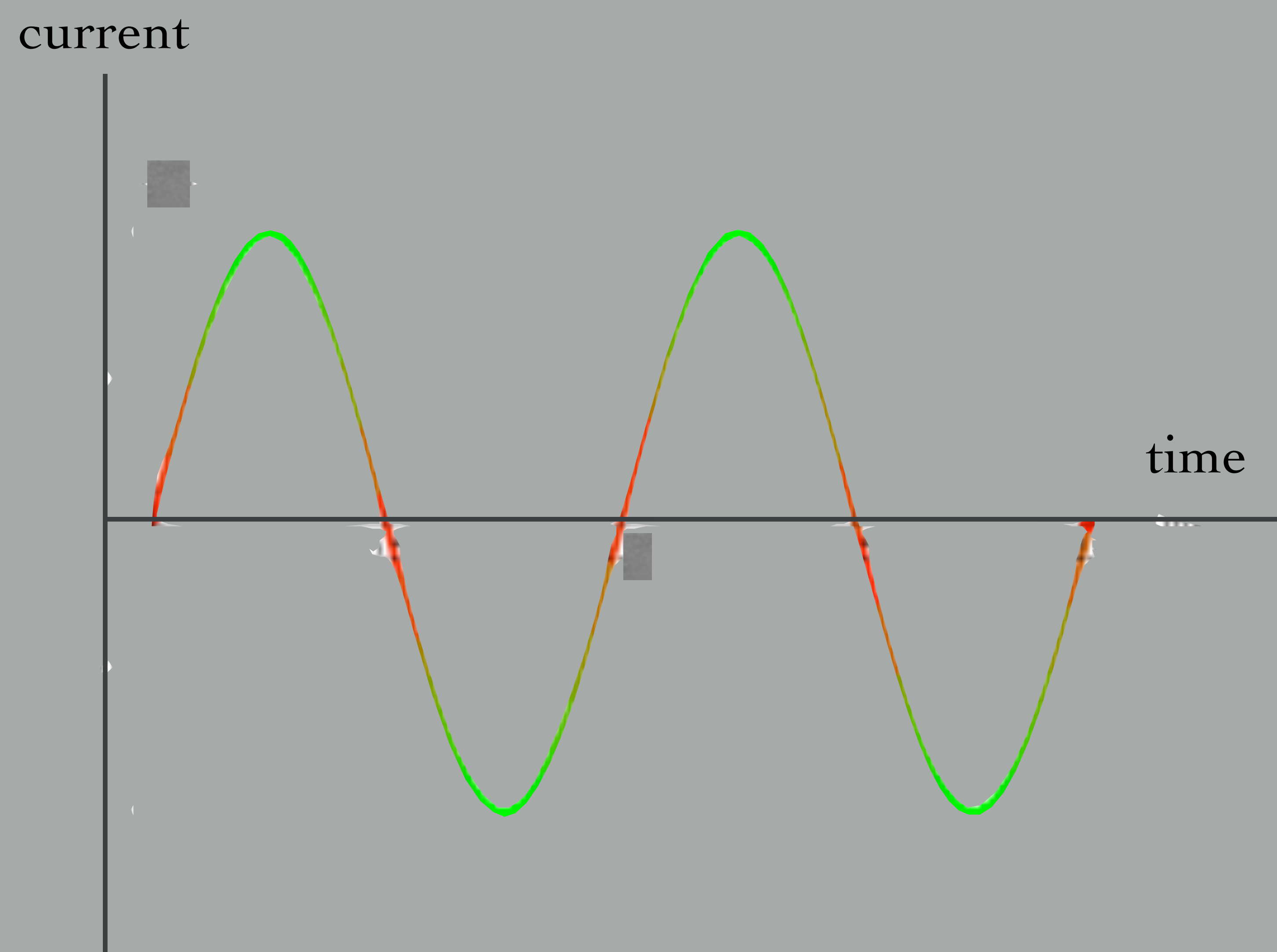
$$\varphi = \omega t \quad \text{where} \quad \omega = \gamma B$$

Precessing magnetization in loop

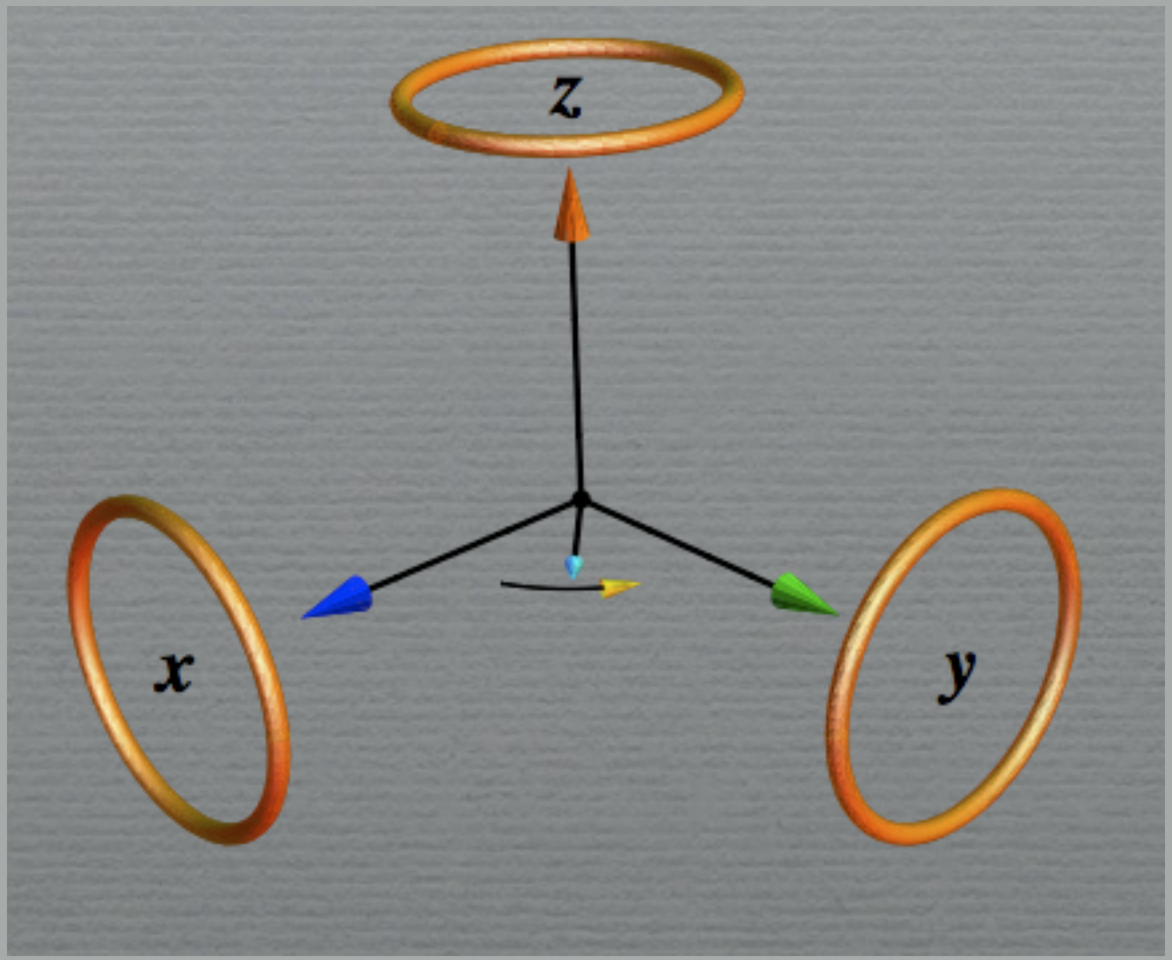


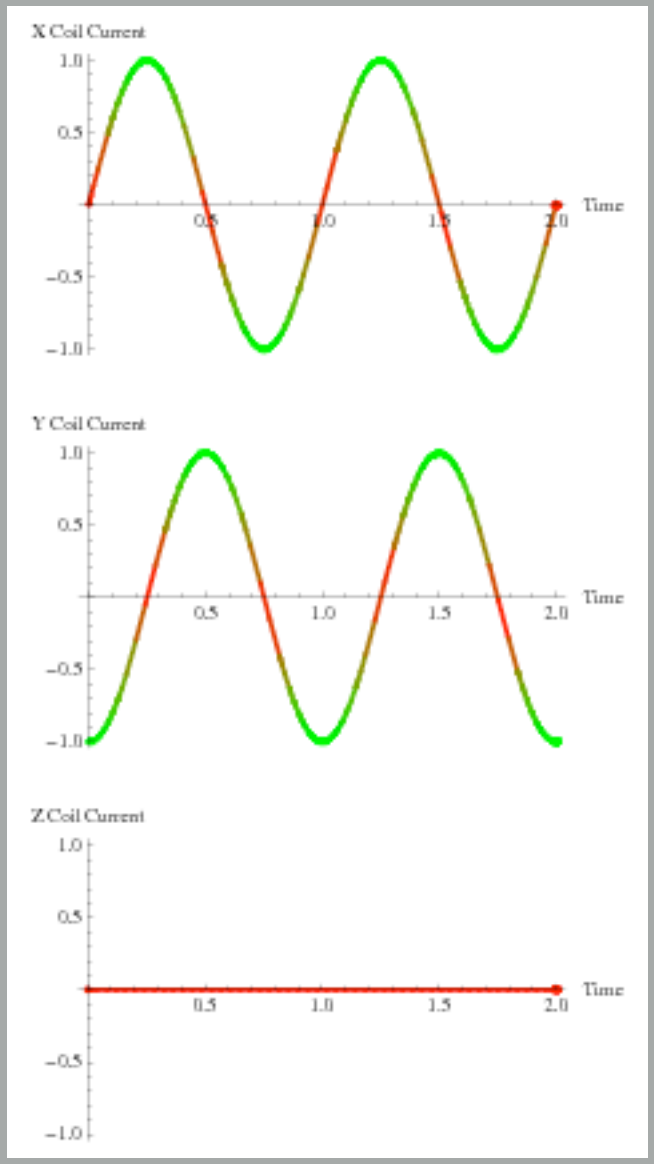
Signal from precessing spin



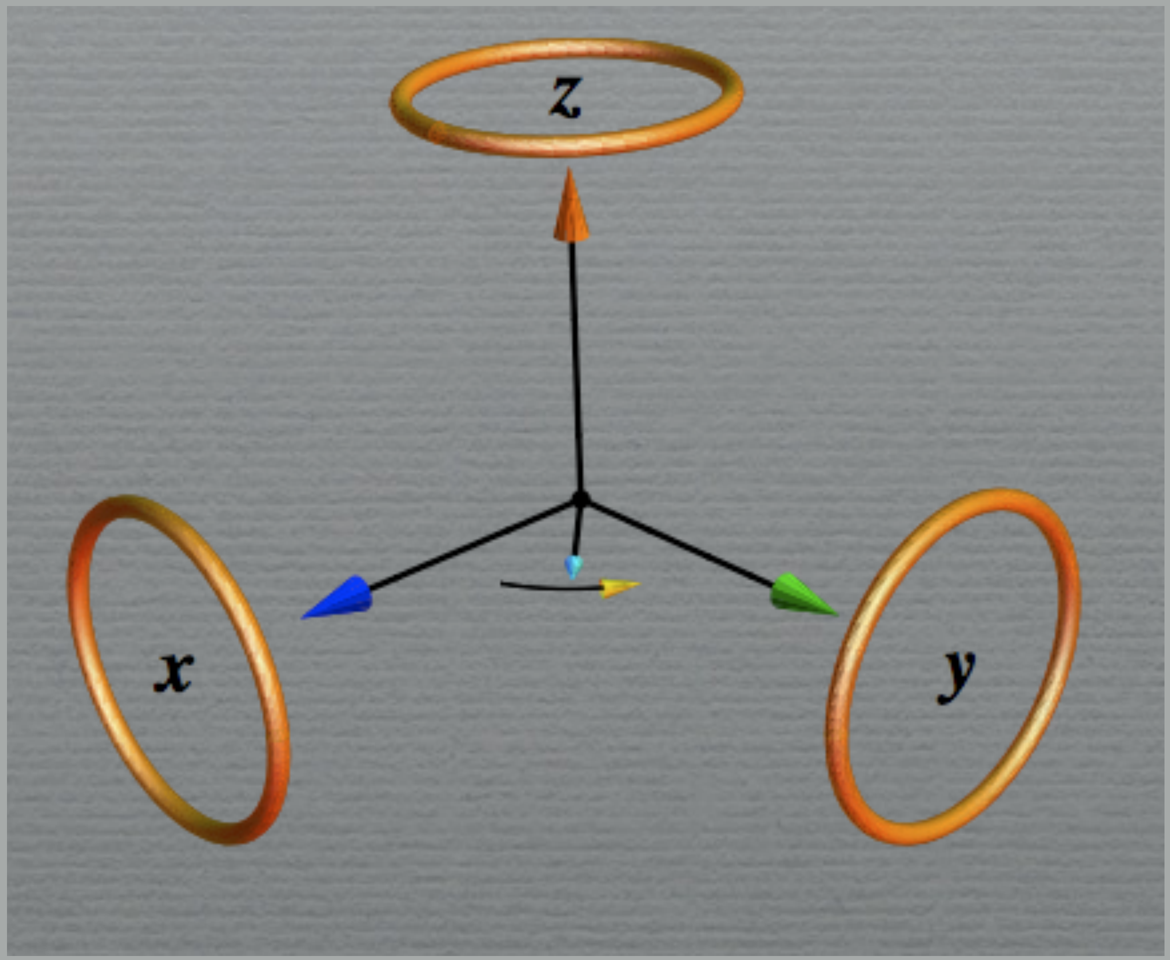


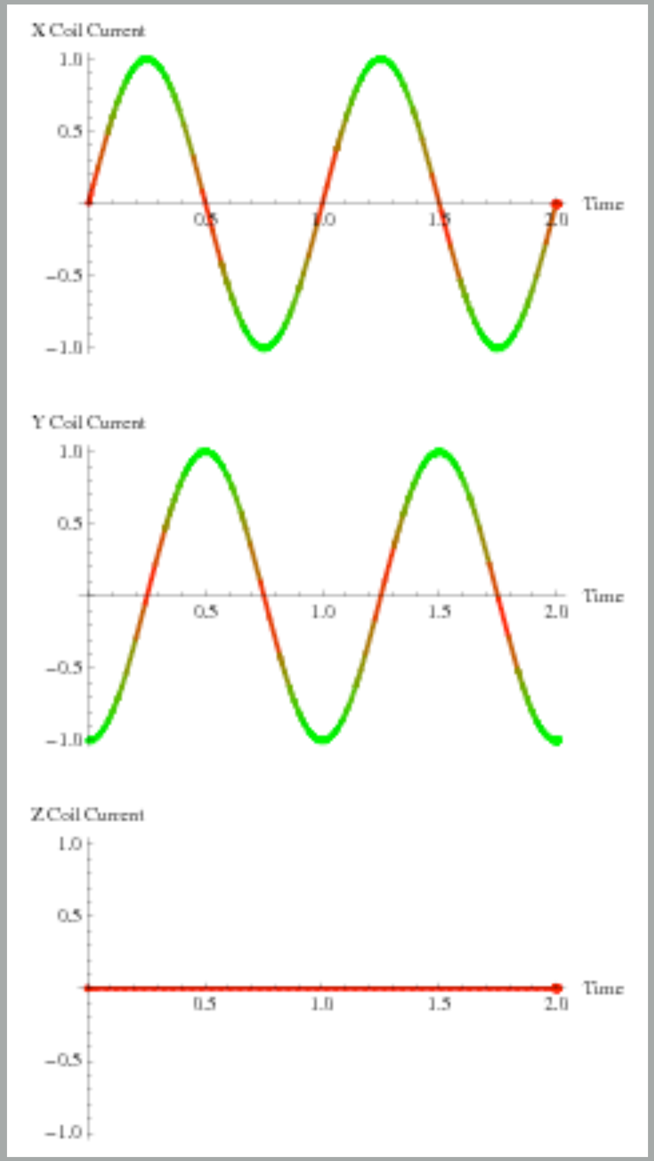
Signal detection





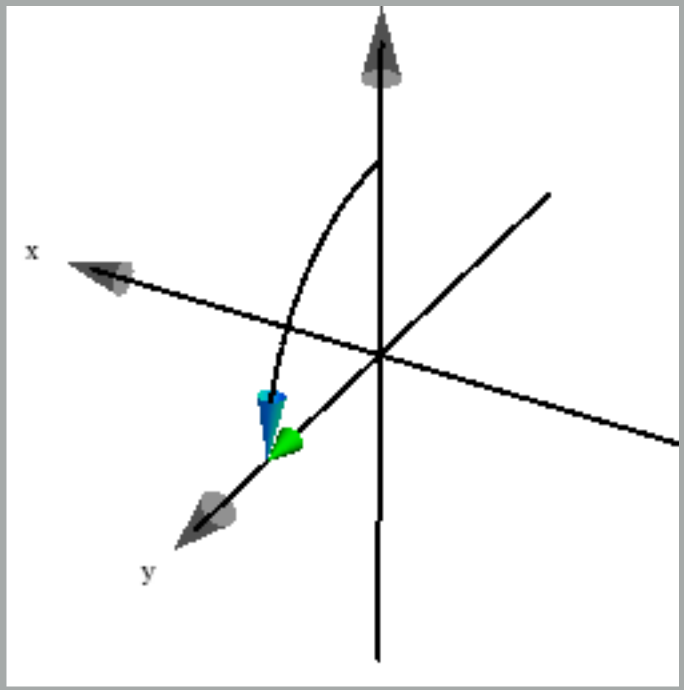
Signal detection





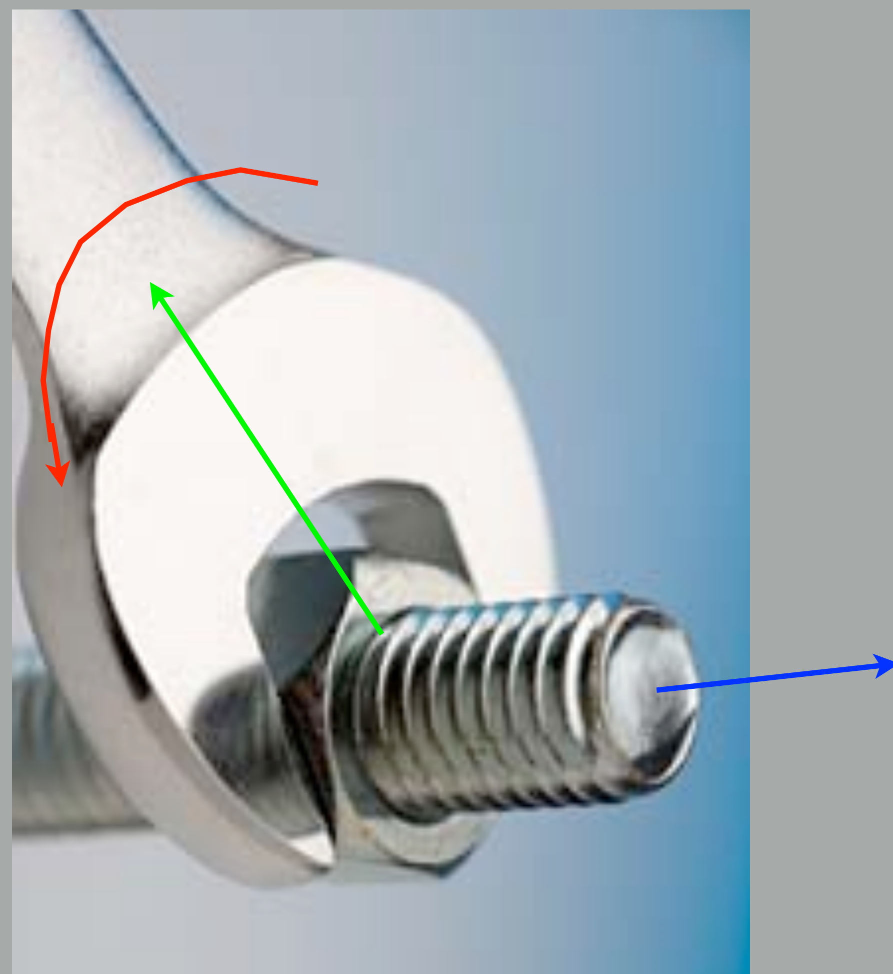
Therefore, we measure m_{xy} only (not m_z)

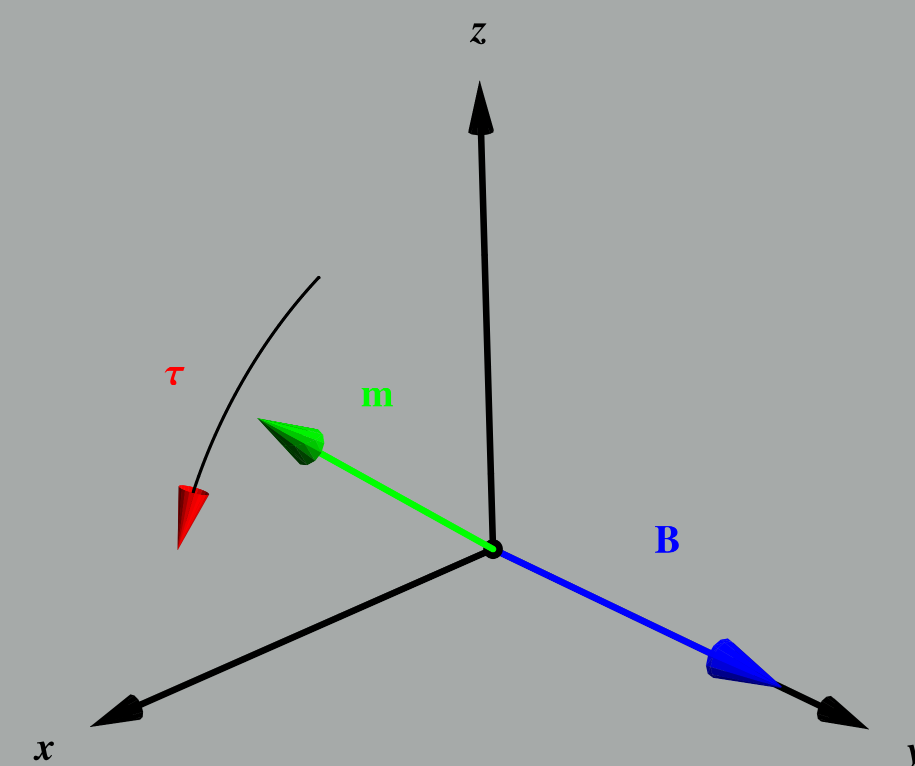
Excitation



Thus, magnetization must be tipped into the transverse plane in order to be detected

Torque



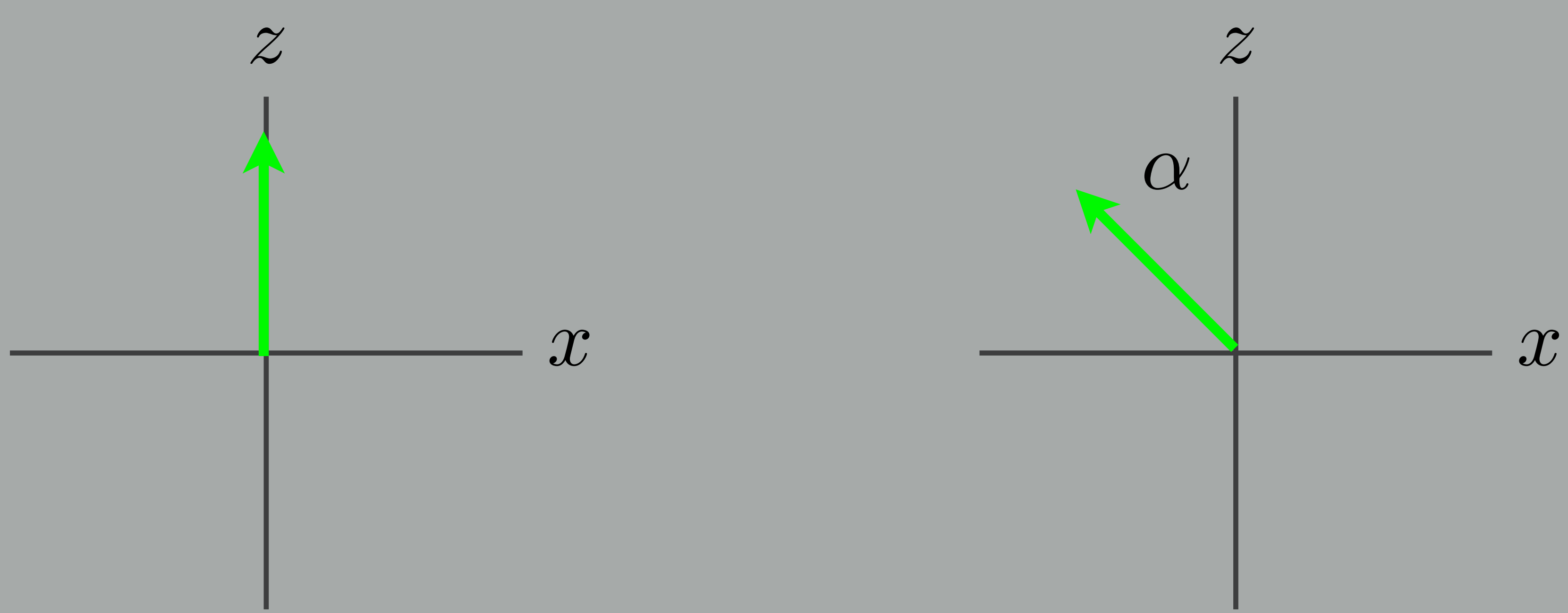


Magnetization is tipped by applying a B-field perpendicular that produces a torque

$$\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}$$

Excitation

$$\omega_1 = \gamma B_1$$



Flip angle

$$\alpha = \omega_1 \tau = \gamma B_1 \tau$$

Example: for protons, $\tau = 0.1ms$ and $B_1 = 0.6G$ gives $\alpha = \pi/2$

Excitation

By extension, we can rotate the magnetization about any axis in the rotating frame using a suitable RF pulse.

Rotating Frame

lab frame

rotating frame

$$\omega_o = \gamma B_o$$

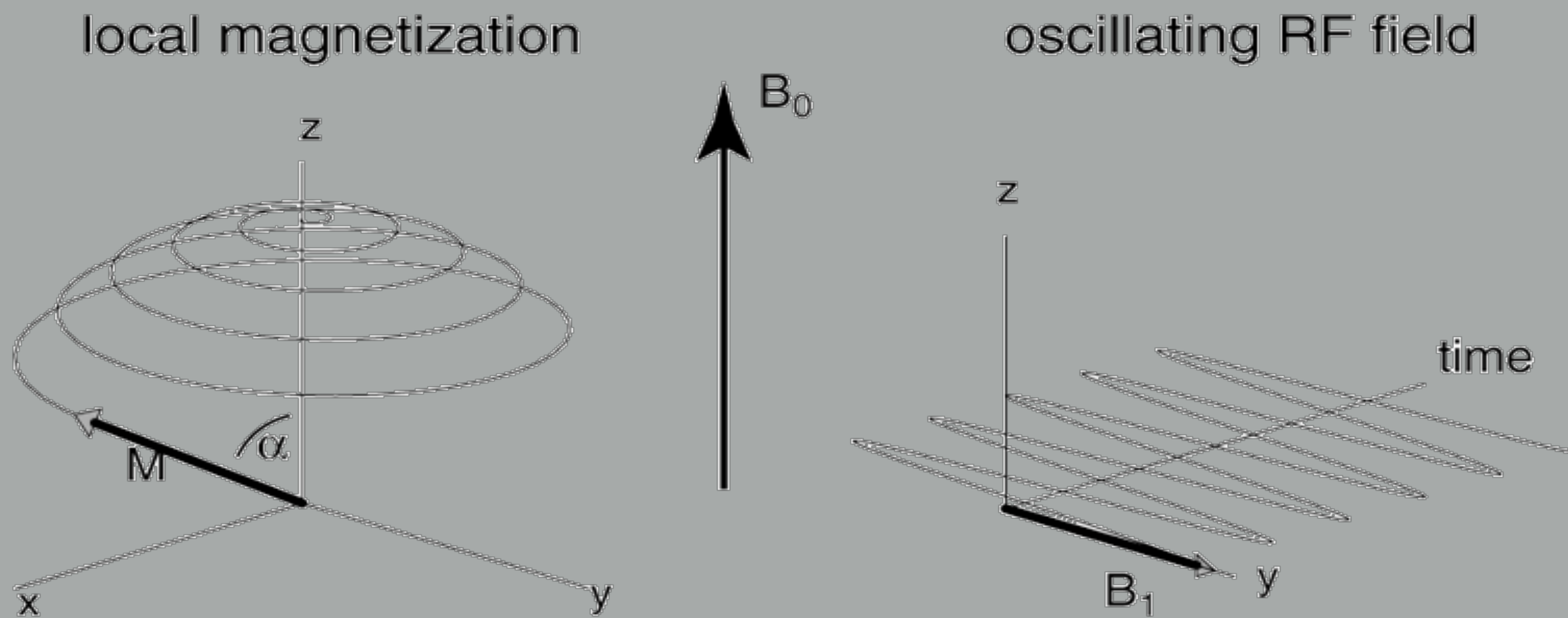
Rotating Frame

lab frame

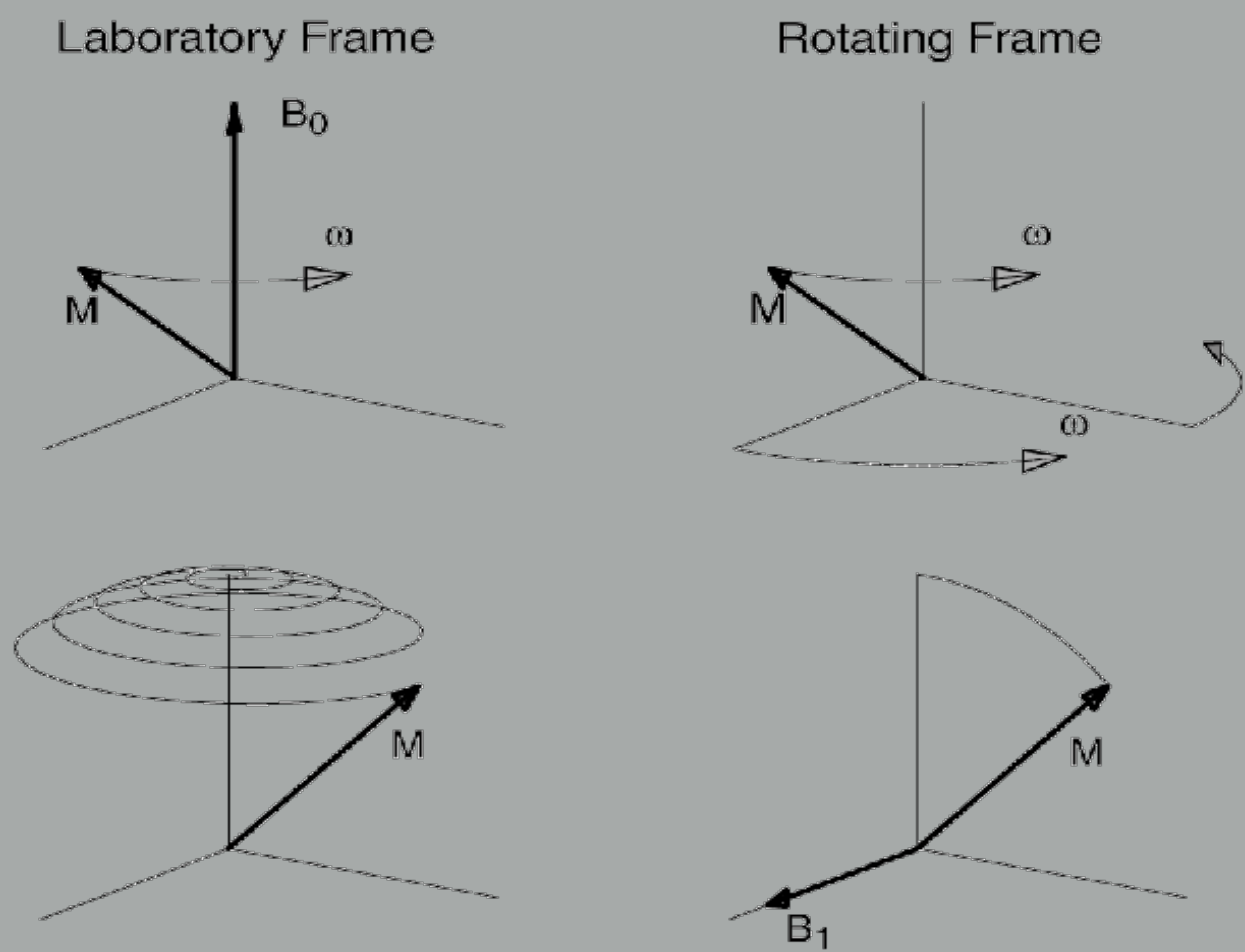
rotating frame

$$\boxed{\omega_o = \gamma B_o}$$

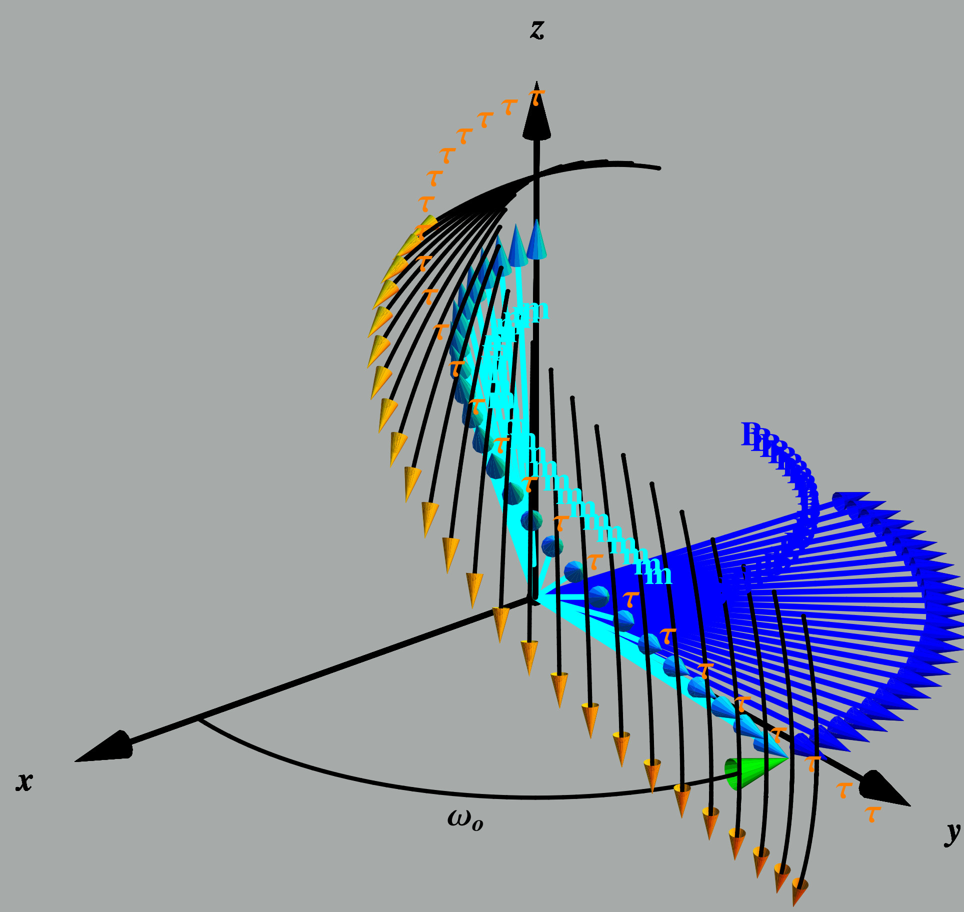
RF Excitation



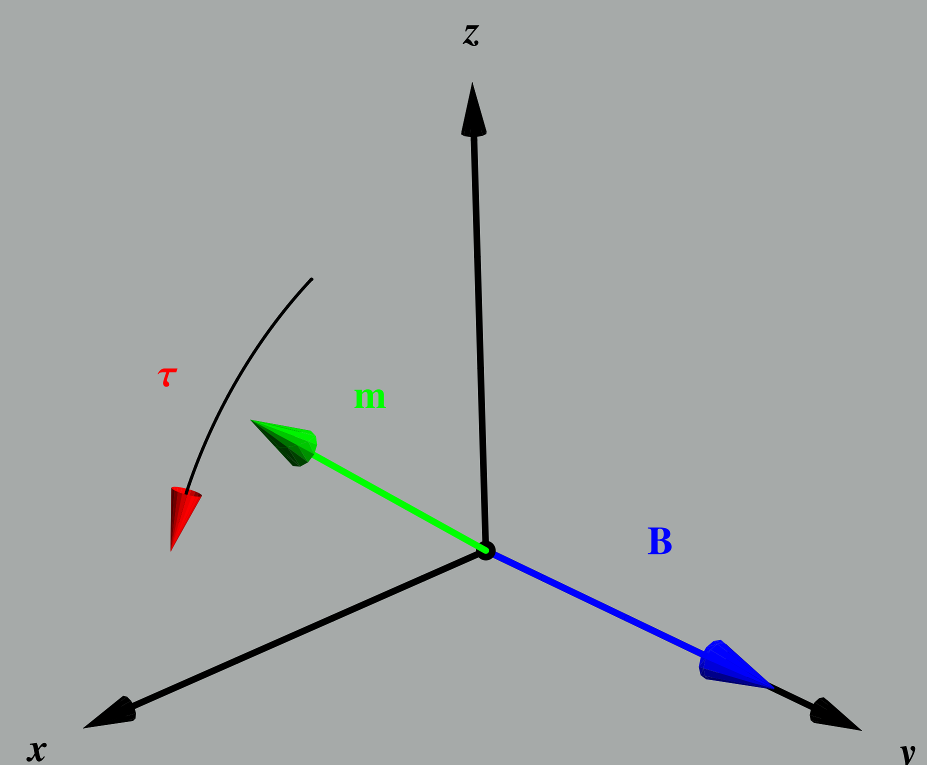
The Rotating Frame of Reference



Excitation



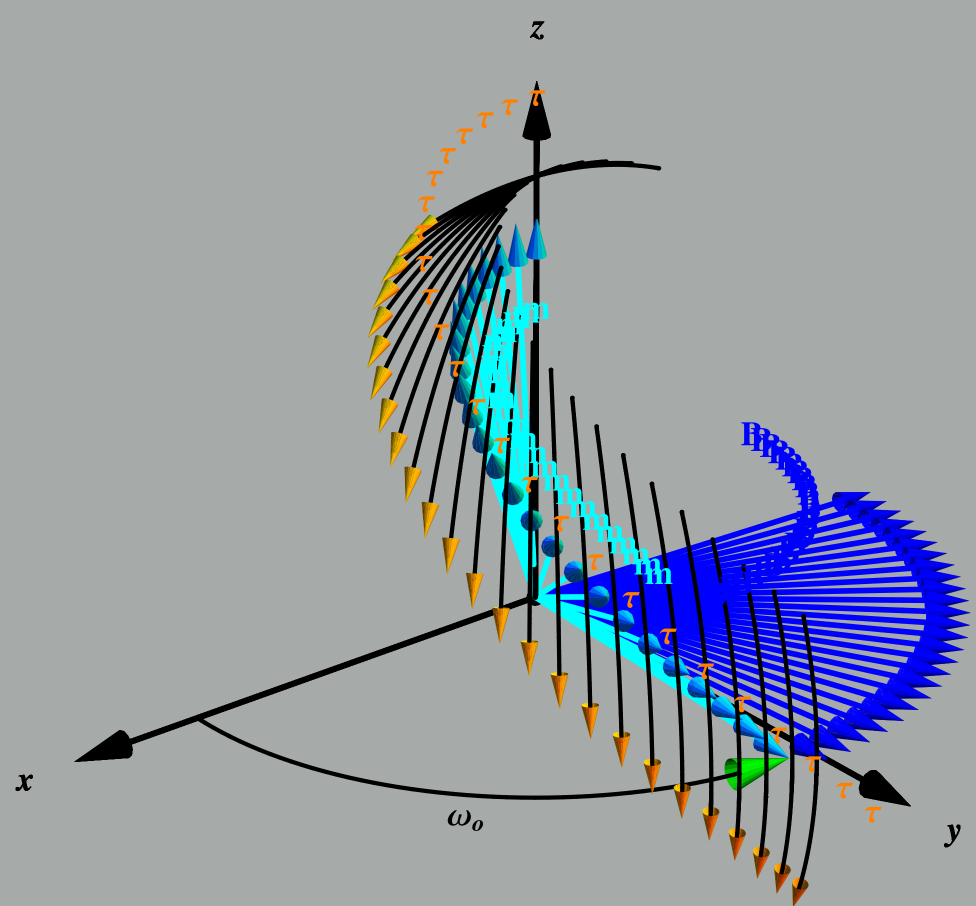
lab frame



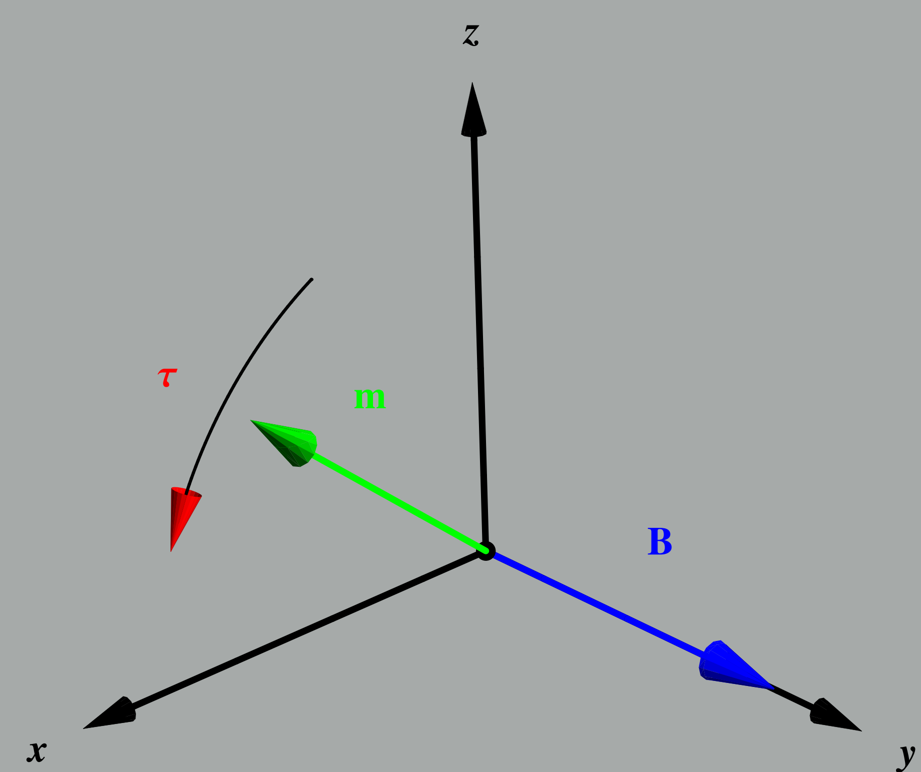
rotating frame

To keep the B-field perpendicular to the precessing magnetization, it must rotate at the same frequency

Excitation



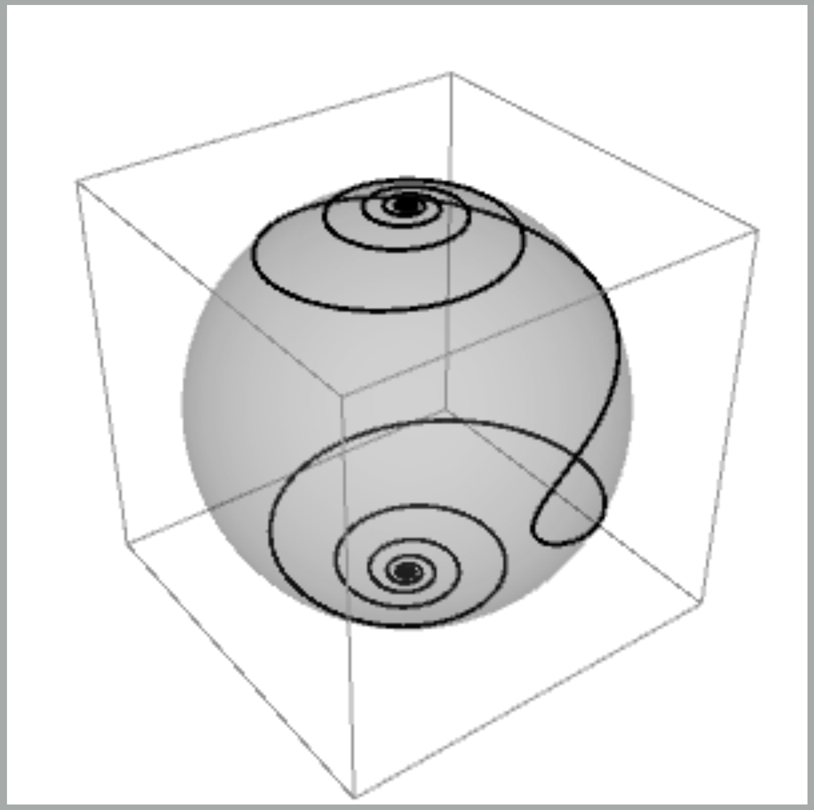
lab frame



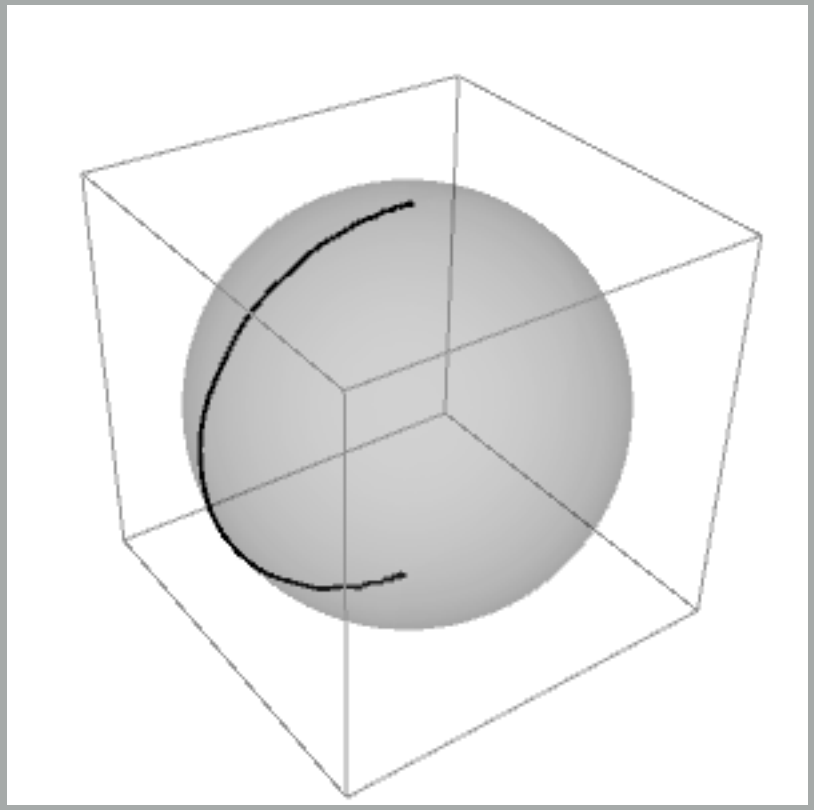
rotating frame

A magnetic field B_1 applied along the y' axis that rotates at frequency $\omega_o = \gamma B_o$ relative to the lab frame is called the *rf excitation pulse*, since ω_o is in the radio-frequency (MHz) range.

Excitation (inversion)



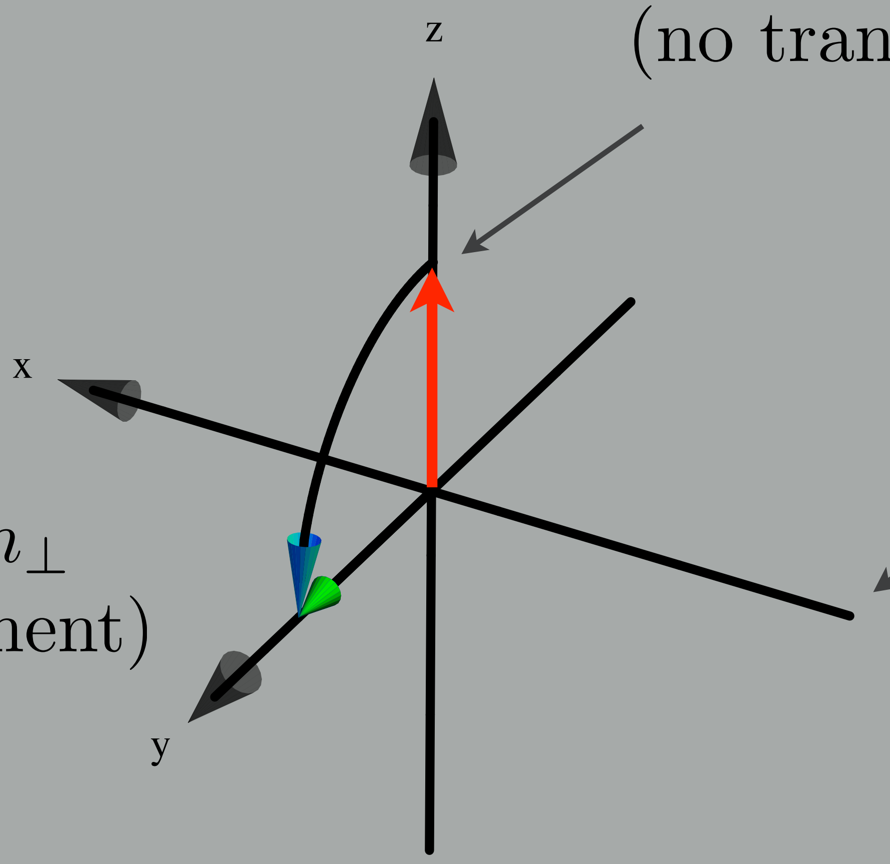
lab frame



rotating frame

Excitation

longitudinal component $m_{\|}$
(no transverse component)

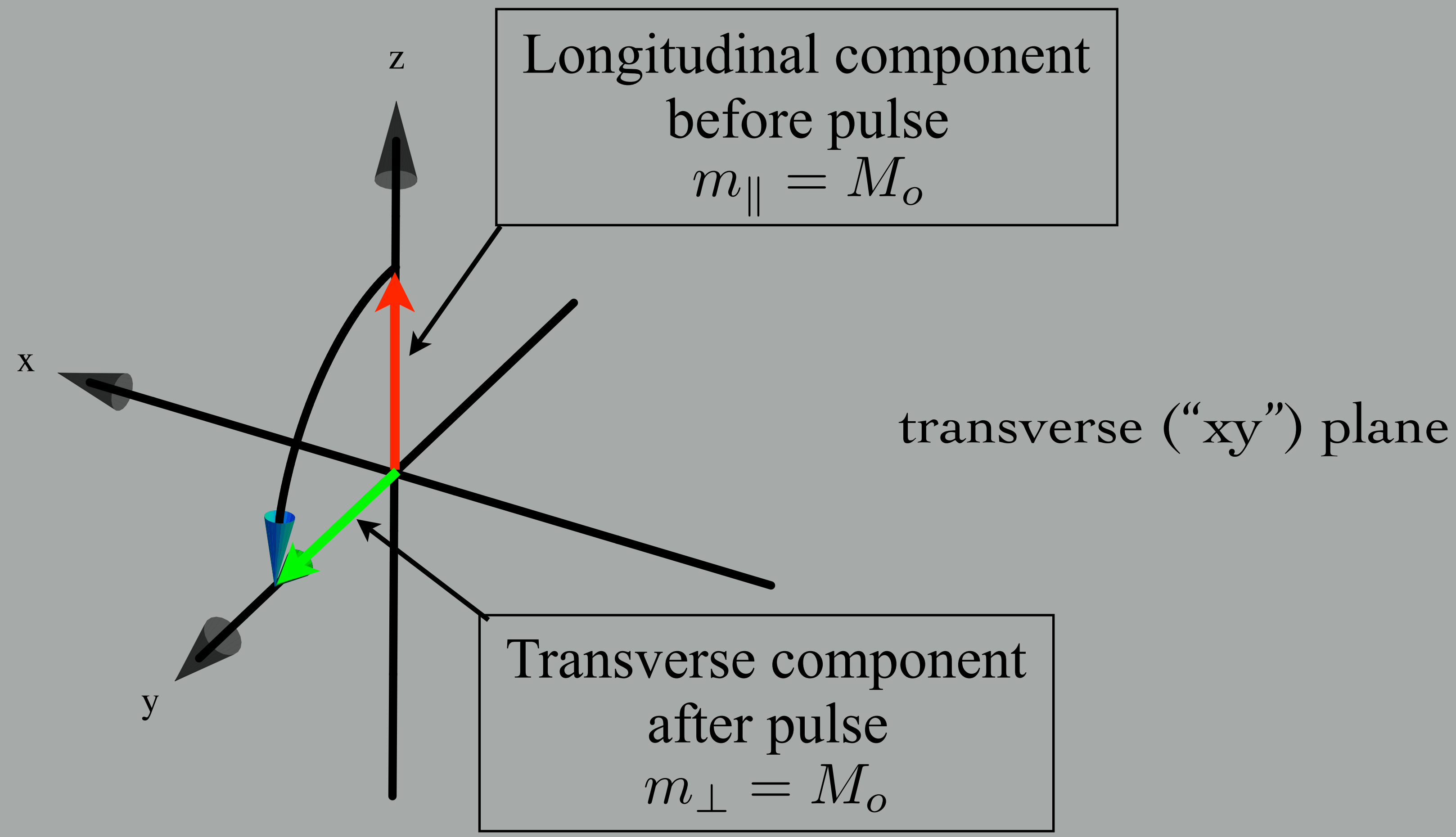


transverse ("xy") plane

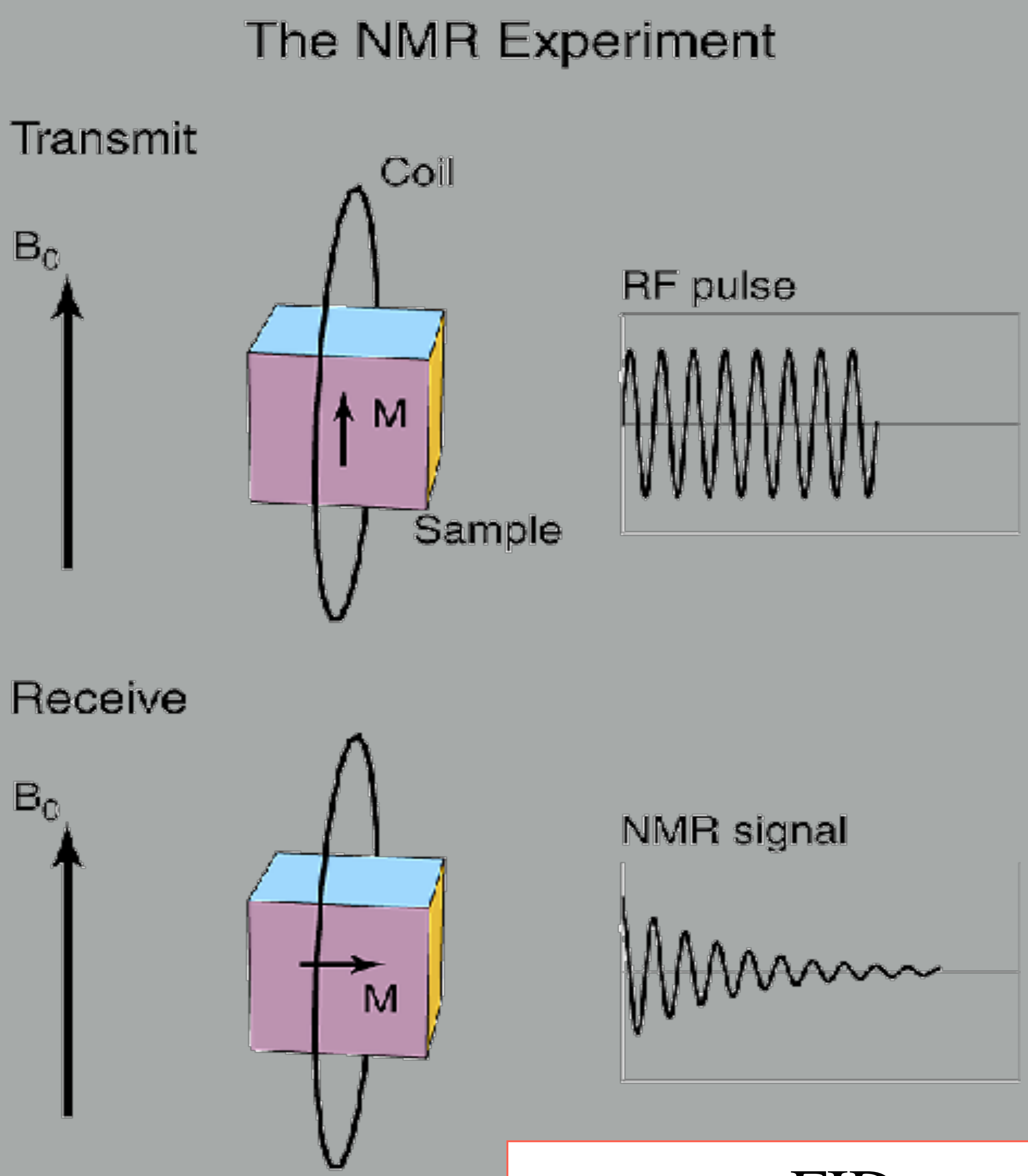
transverse component $m_{\perp}$
(no longitudinal component)

90° RF pulse

Excitation

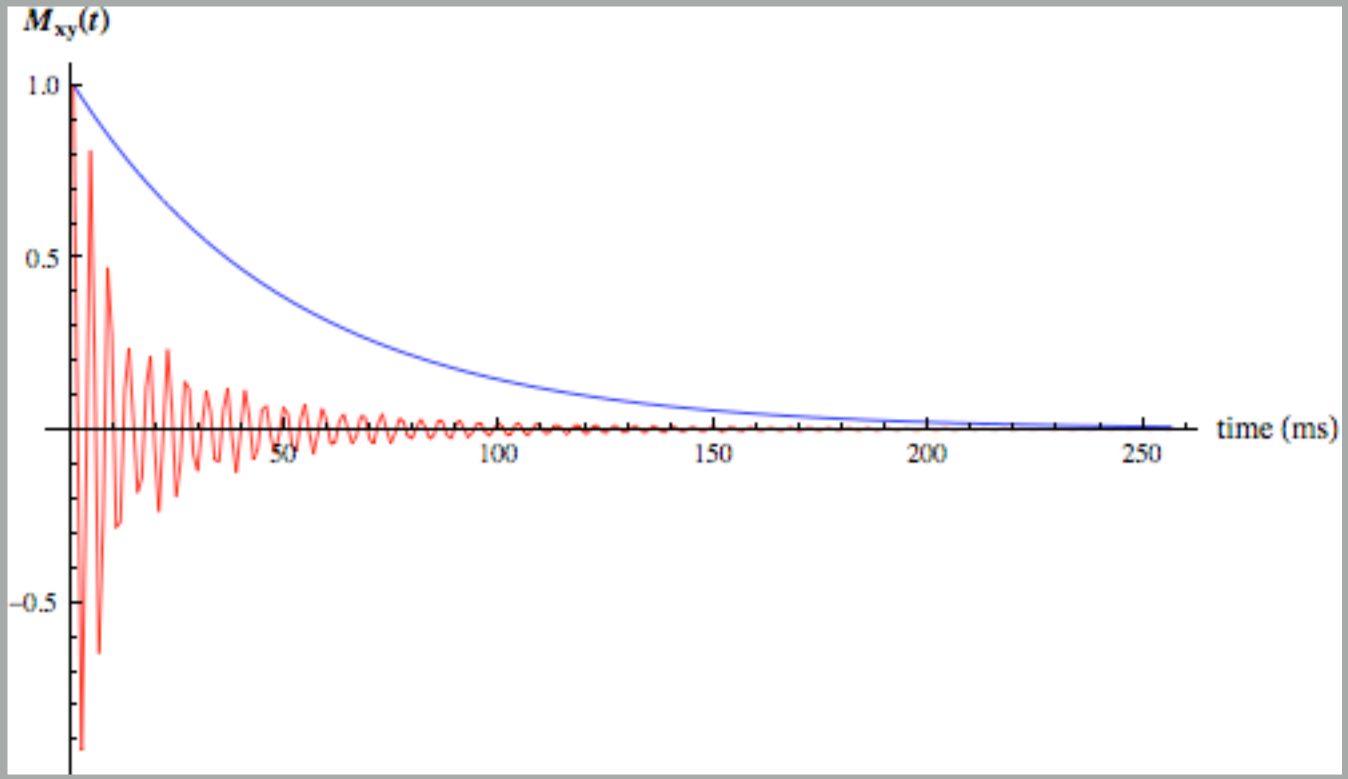


90° RF pulse



FID
Free induction decay

The FID



Free induction decay for a Gaussian distribution of isochromats (blue=on) (red=off) resonance

Free Induction Decay

Immediately following termination of
excitation pulse:

1. Spins precess in main field ***Free*** for excitation pulses
2. There precession generates current in RF coils by the Faraday's Law of ***Induction***
3. This signal diminishes exponentially due to ***Decay*** of the transverse component

This is called
Free Induction Decay

The NMR Experiment

Fig. 4.8

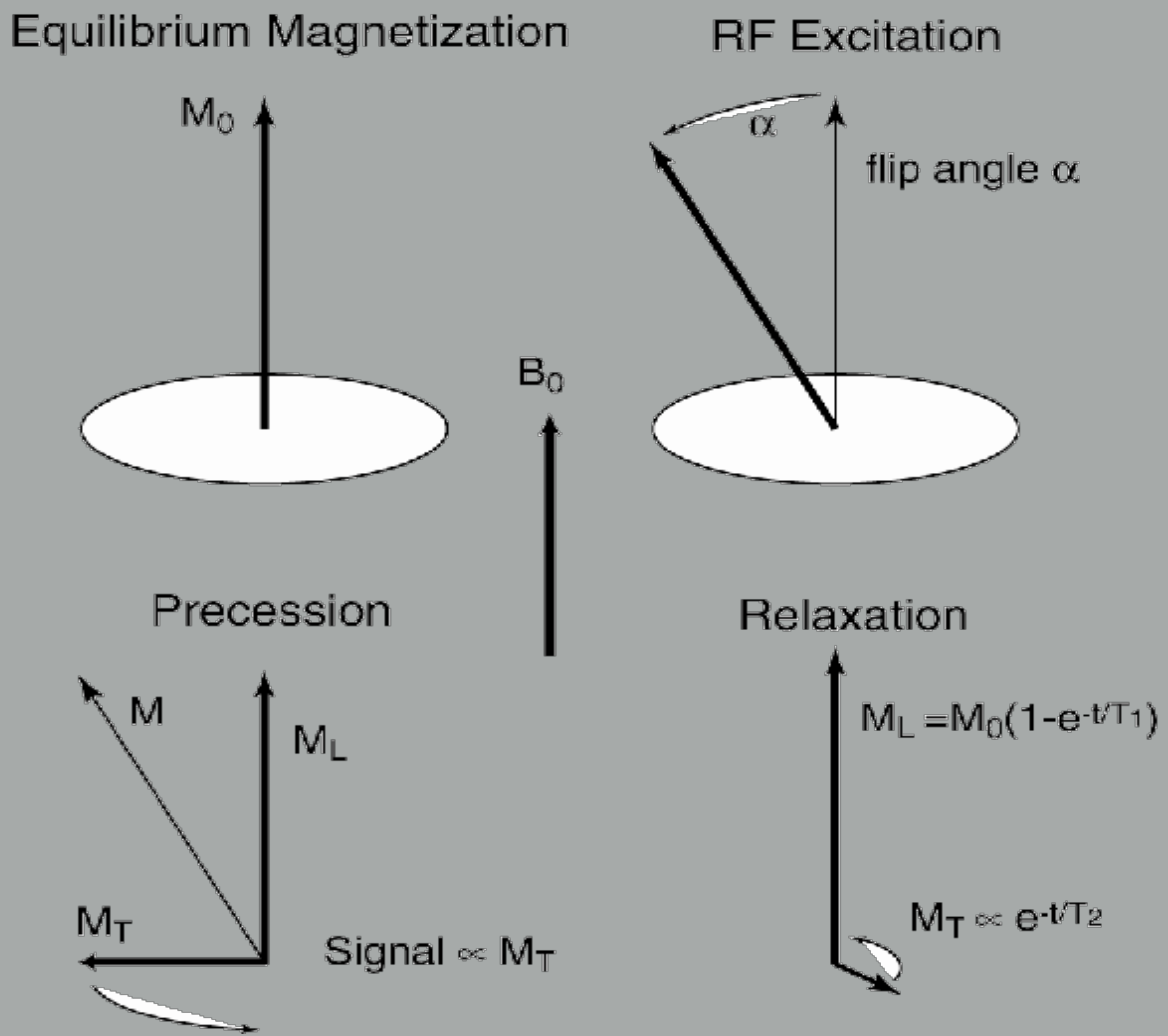
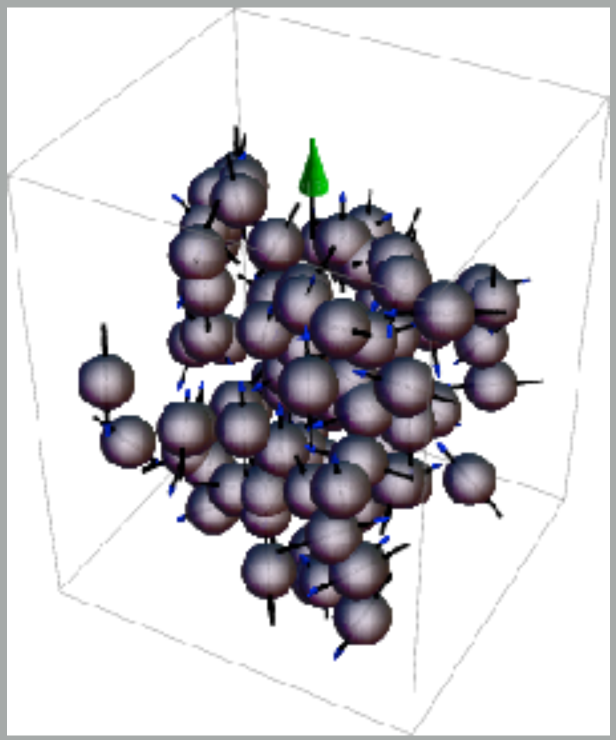
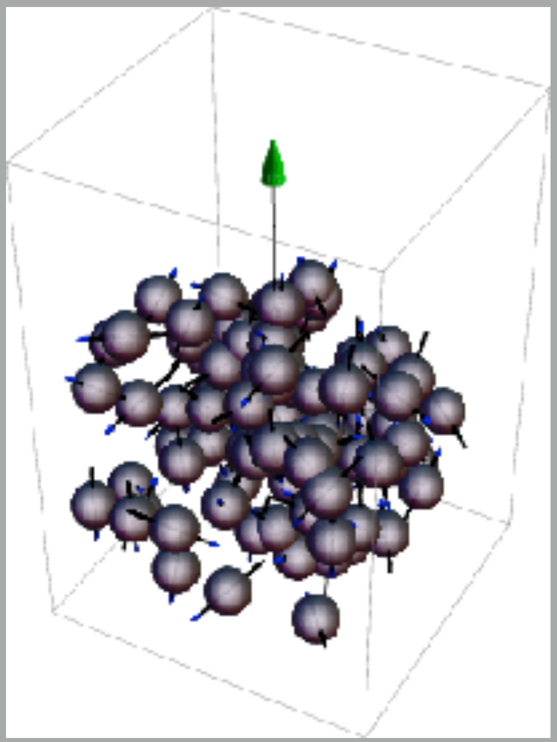
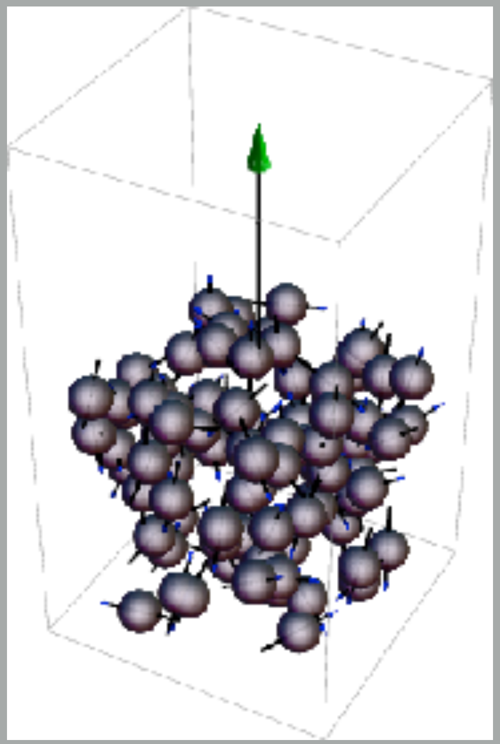


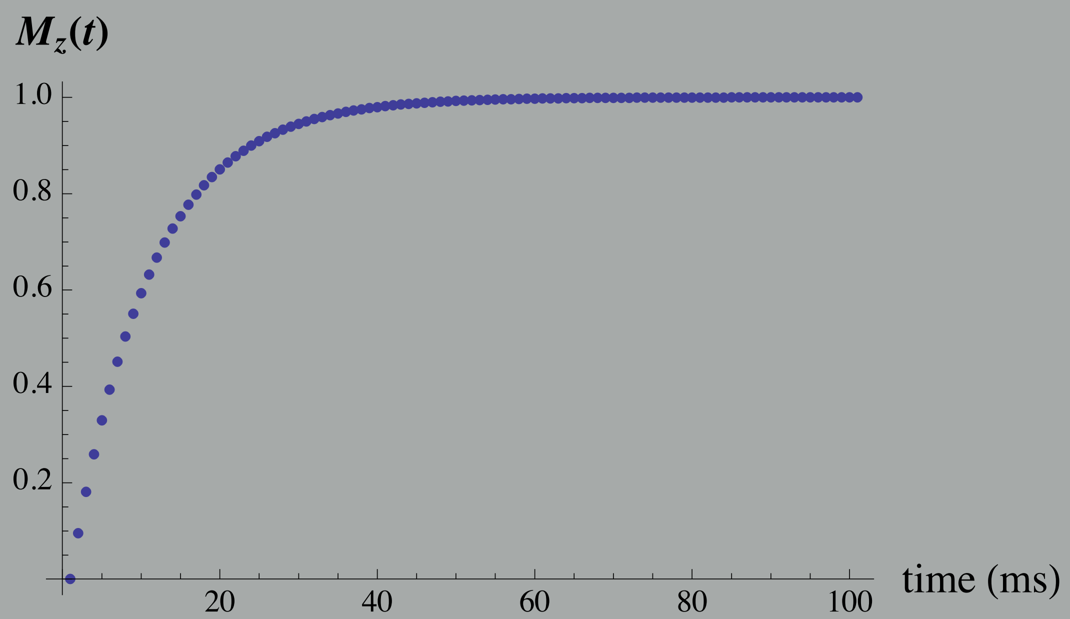
Fig. 4.8

T1 relaxation



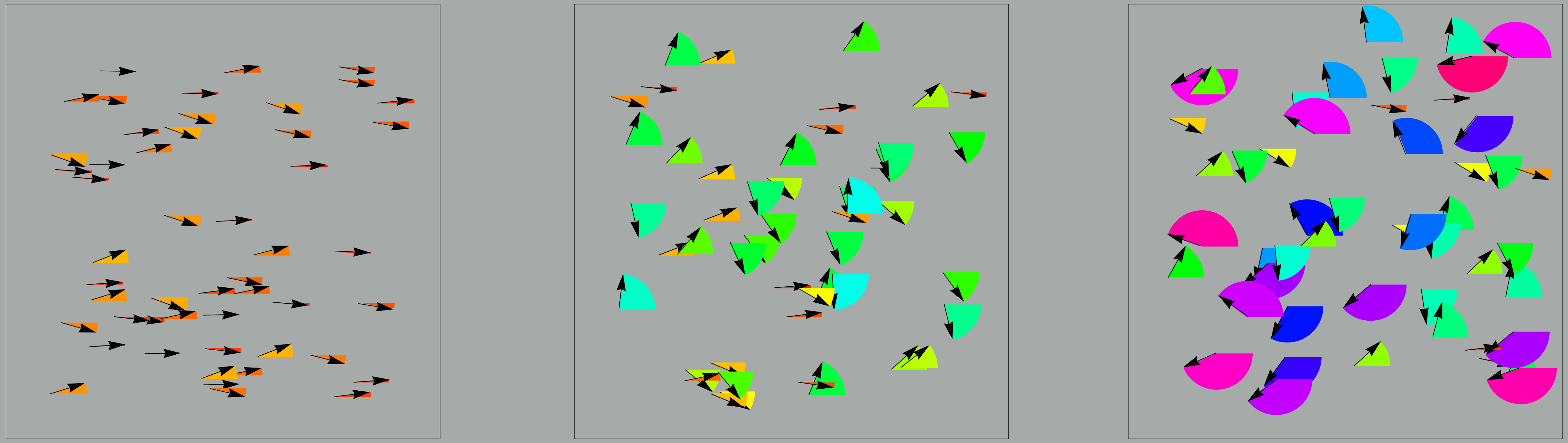






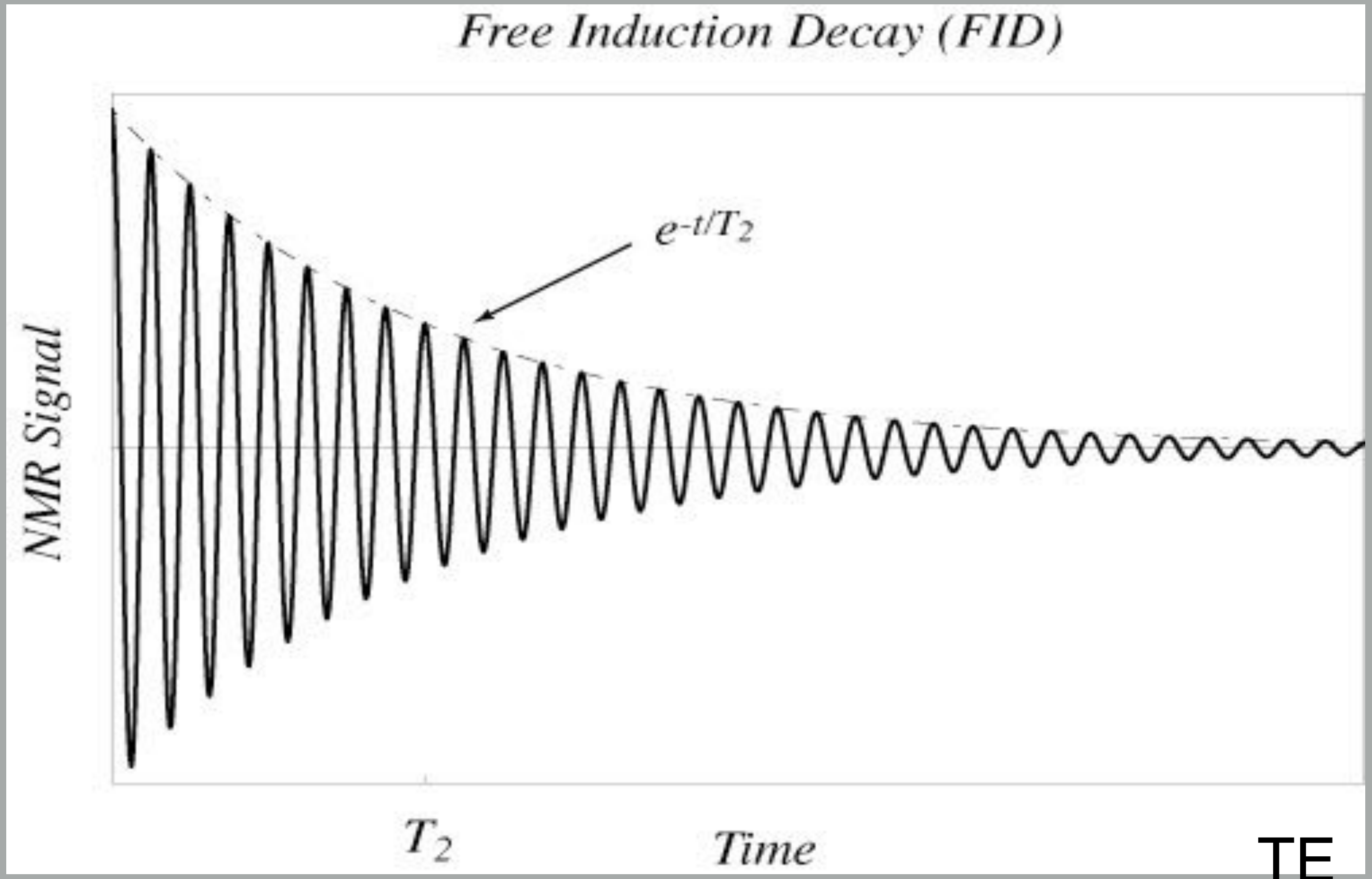
$$M_z(t) = M_o(1 - e^{-t/T_1})$$

T2 decay



Relaxation

The MR Signal



Nuclei tend to align with B_0 with a time constant T1
The NMR signal decays with a time constant T2
The signal is always proportional to the proton density M_0

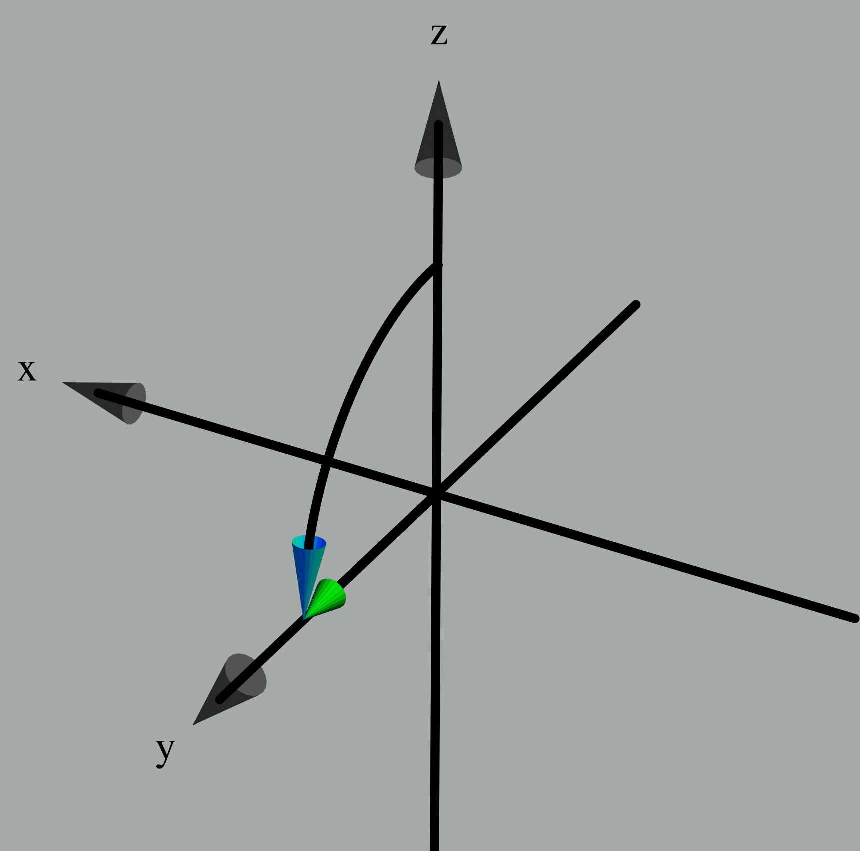
Typical NMR Parameters (1.5T)

	M_0(arb)	T1(ms)	T2(ms)
GM	85	950	95
WM	80	700	80
CSF	100	2500	250

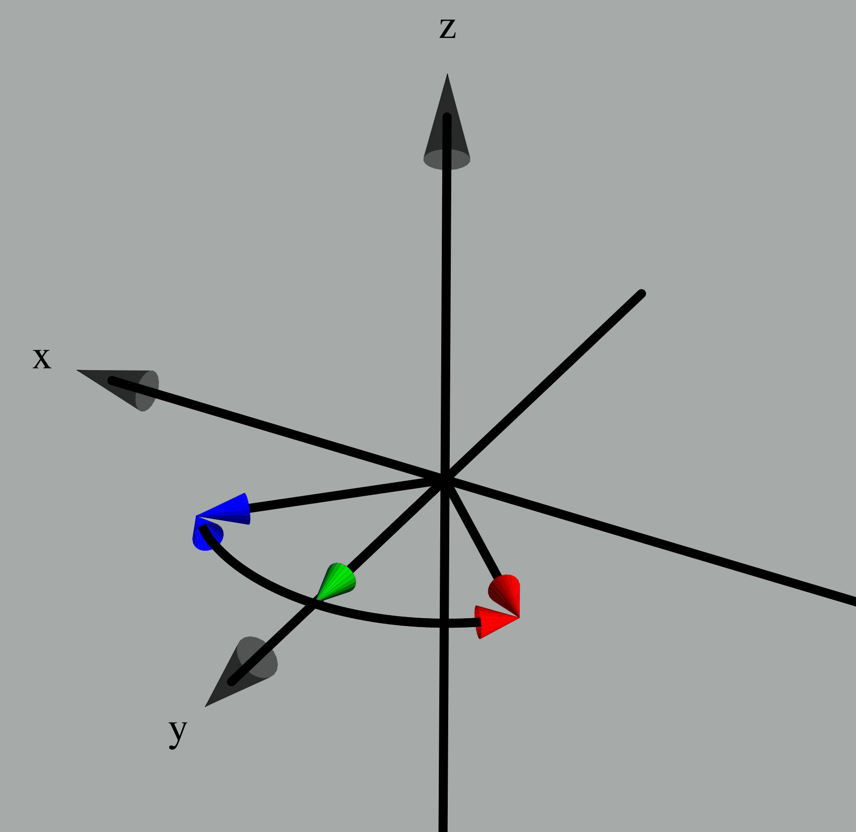
Relaxation

Relaxation Times: T_2, T_2^*
(50 ms at 3T)

Dephasing

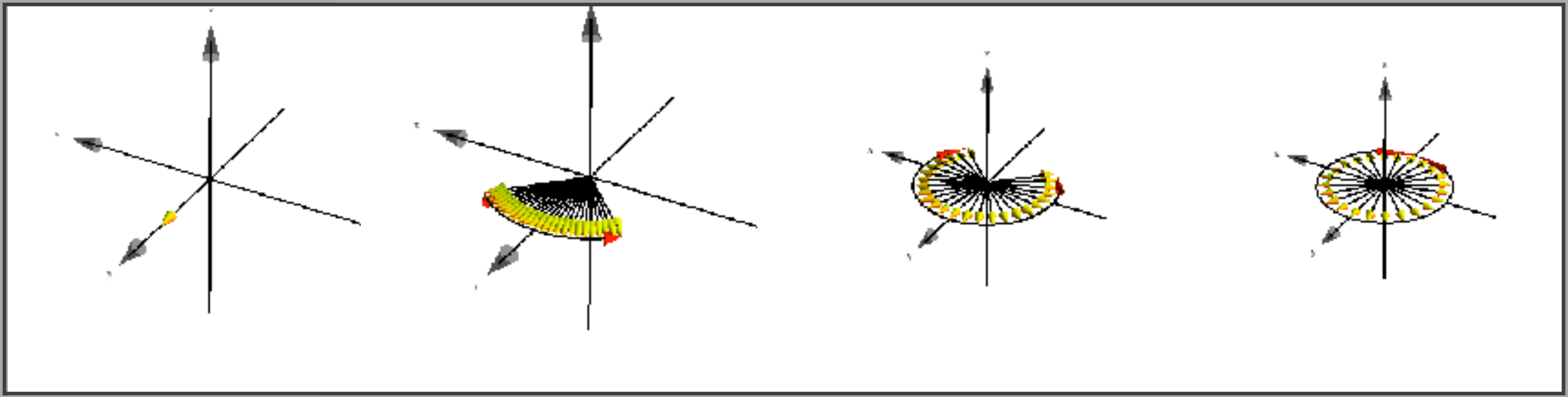


$t = 0$

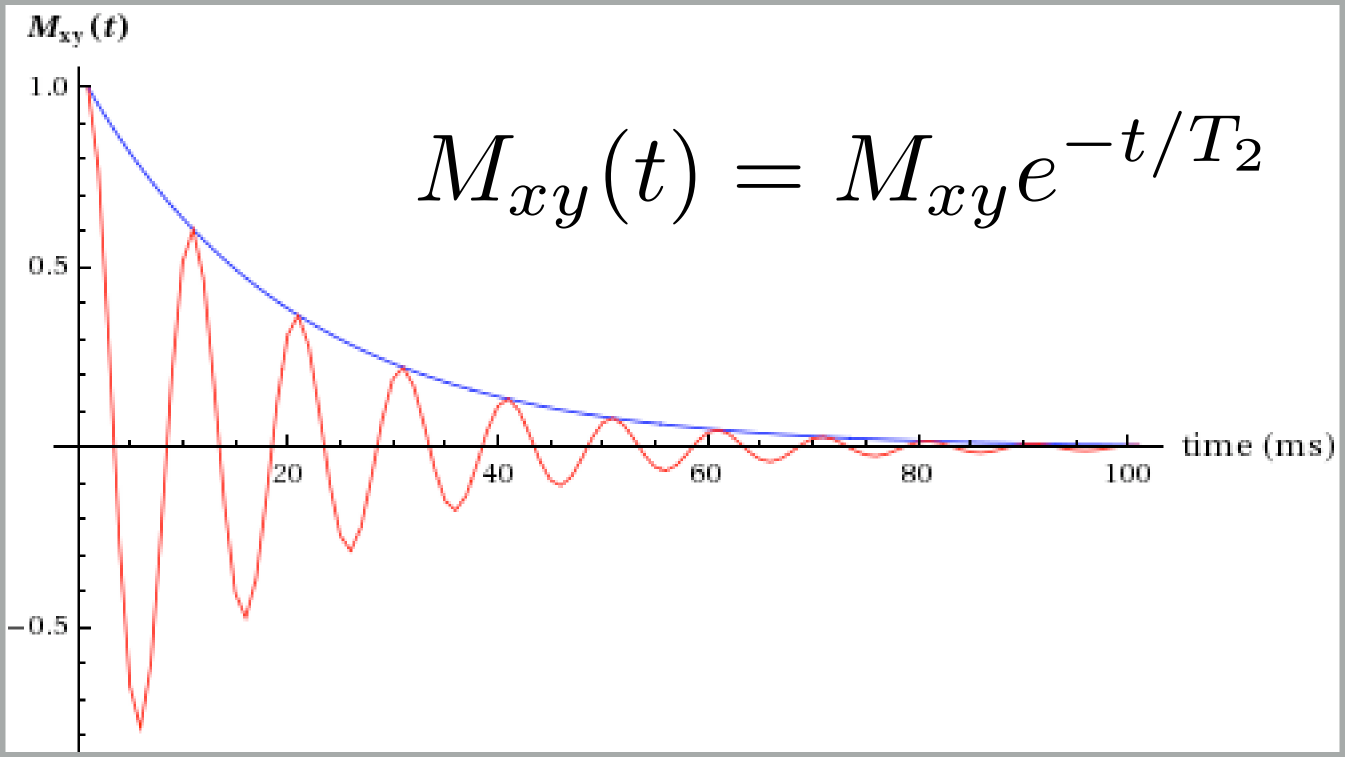


$t = \tau$

Free Induction Decay

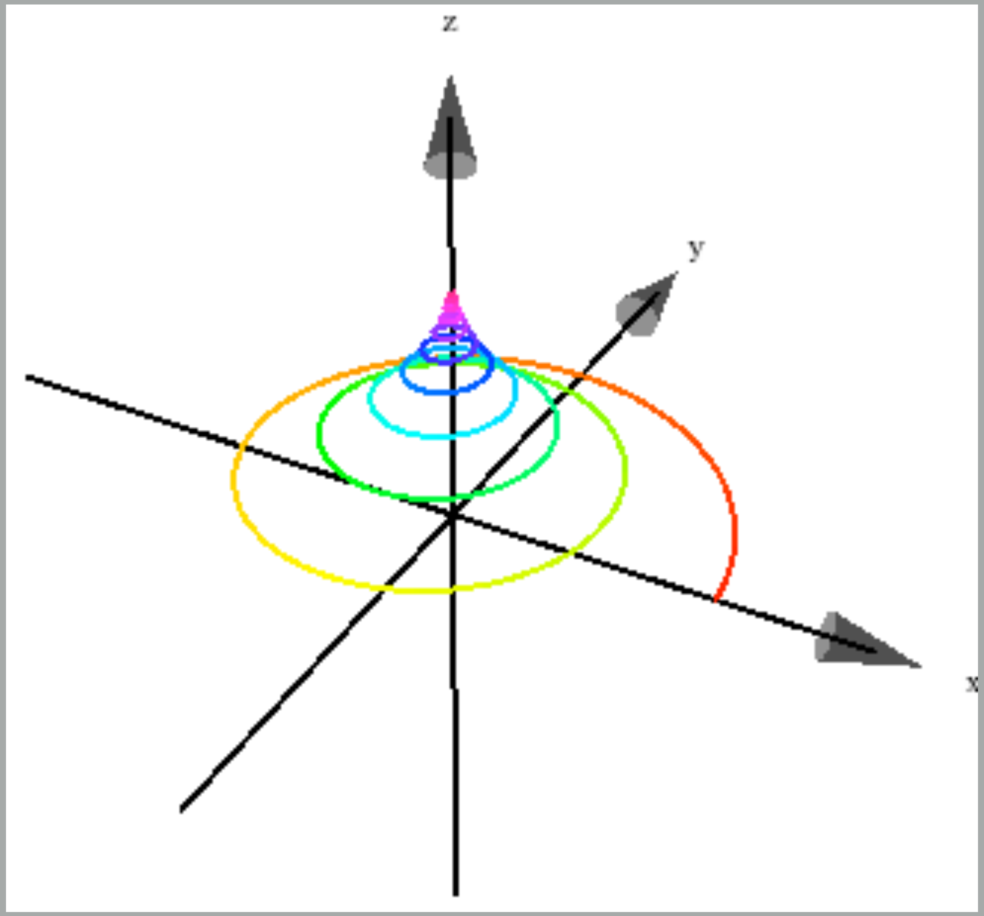


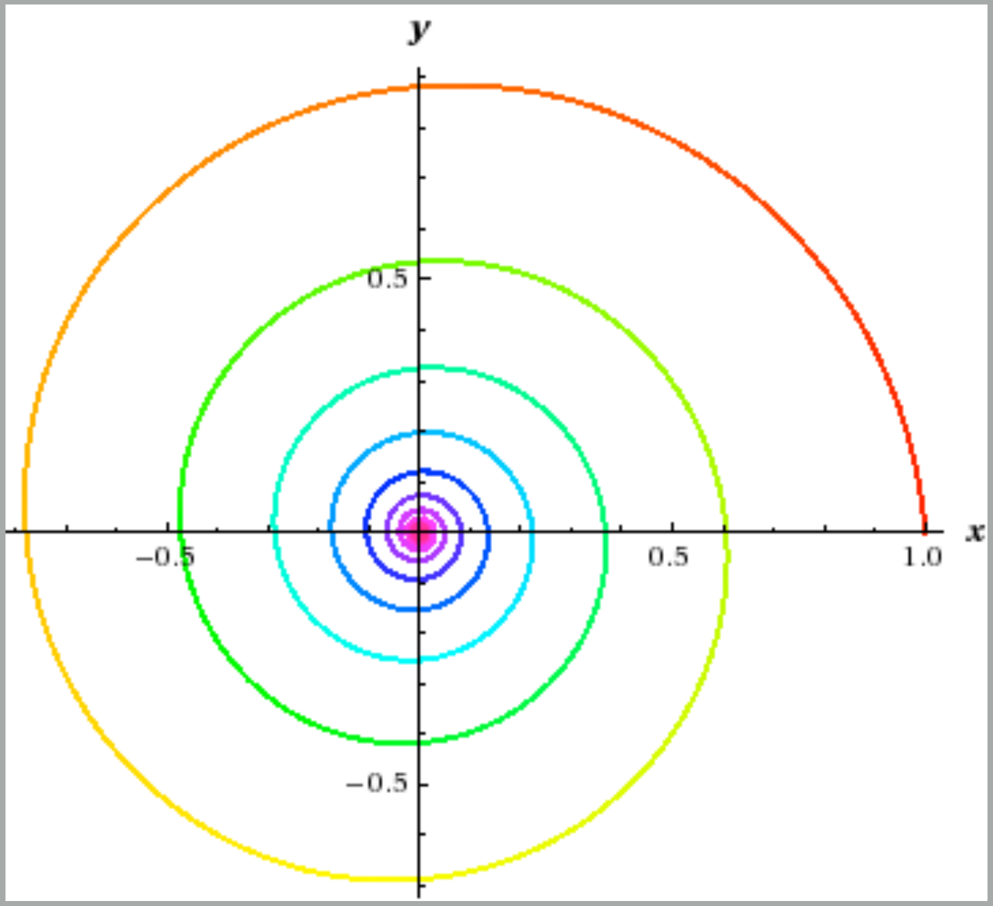
Free Induction Decay



Free induction decay for a single isochromat
blue=on resonance
red=off resonance

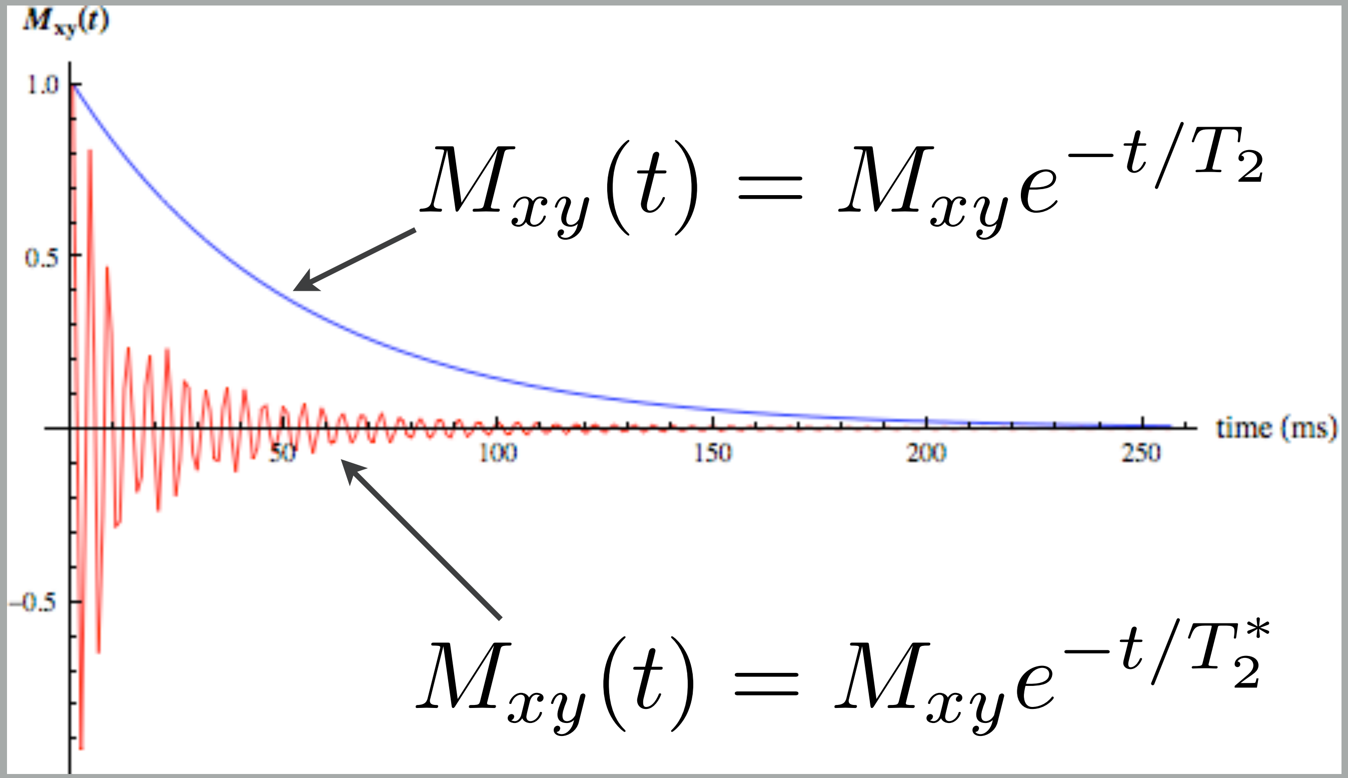
M(x,y,z)





T_2 decay in the xy (transverse plane) and T_1 recovery along z

Free Induction Decay



T_2 is irreversible intrinsic transverse decay

T_2^* includes reversible decay due to field inhomogeneities

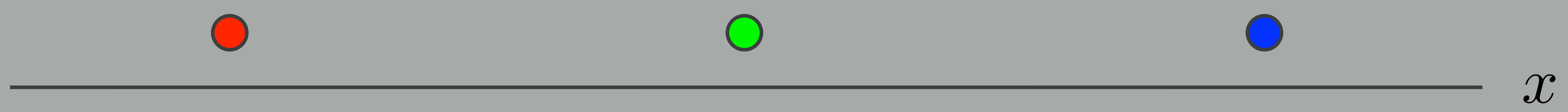
Isochromats

$$\boxed{\omega = \gamma B}$$

All spins precessing at a particular frequency are called an *isochromat*

(same frequency = same "color")

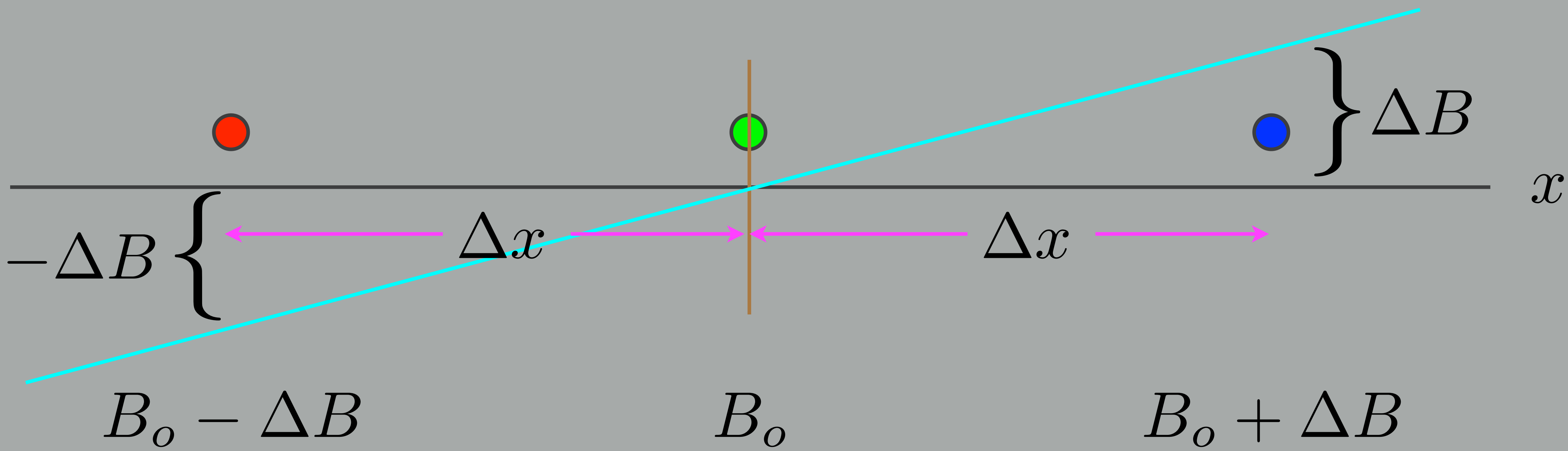
A tale of 3 isochromats



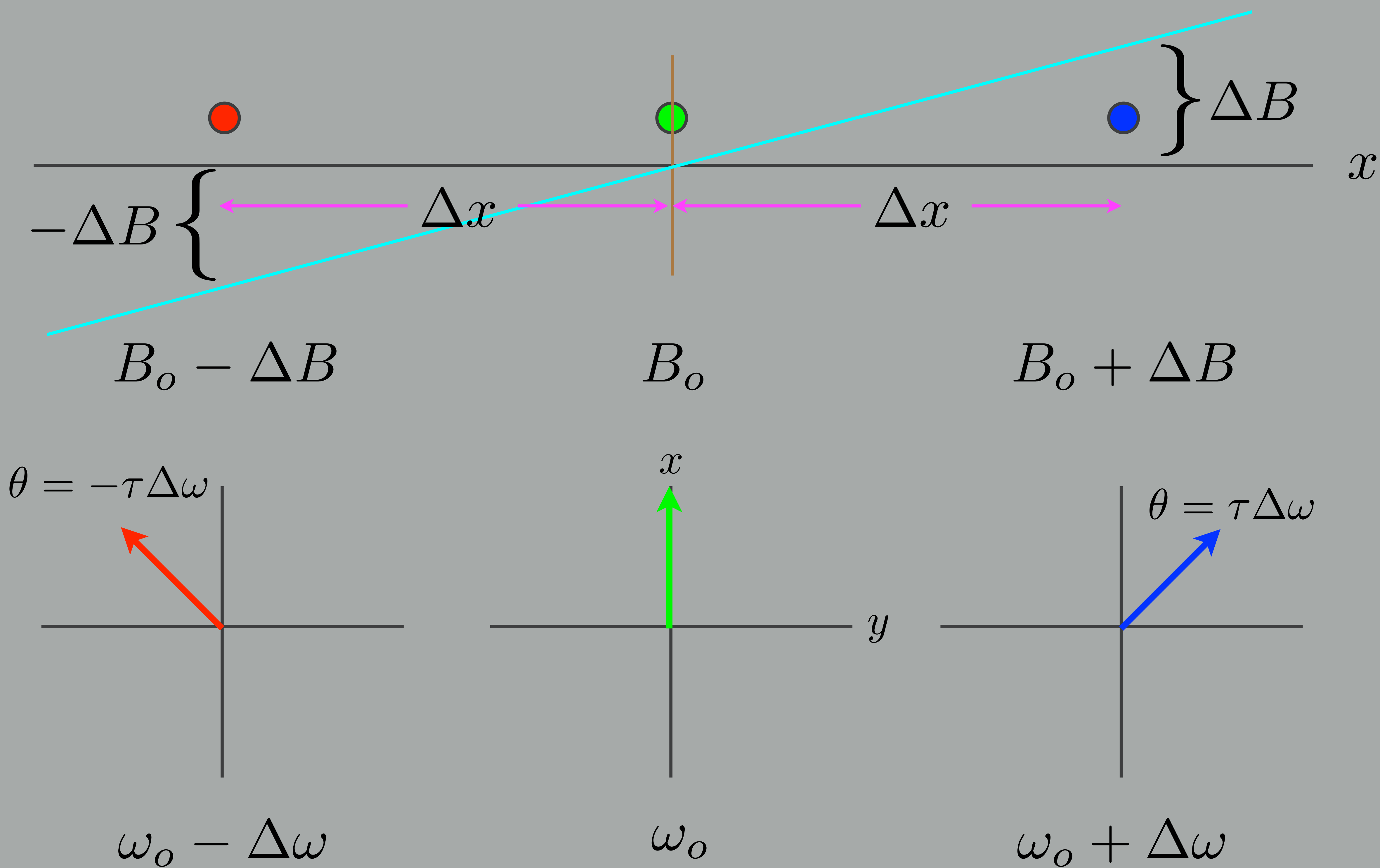
B_o B_o B_o

$\omega = \omega_o$ $\omega = \omega_o$ $\omega = \omega_o$

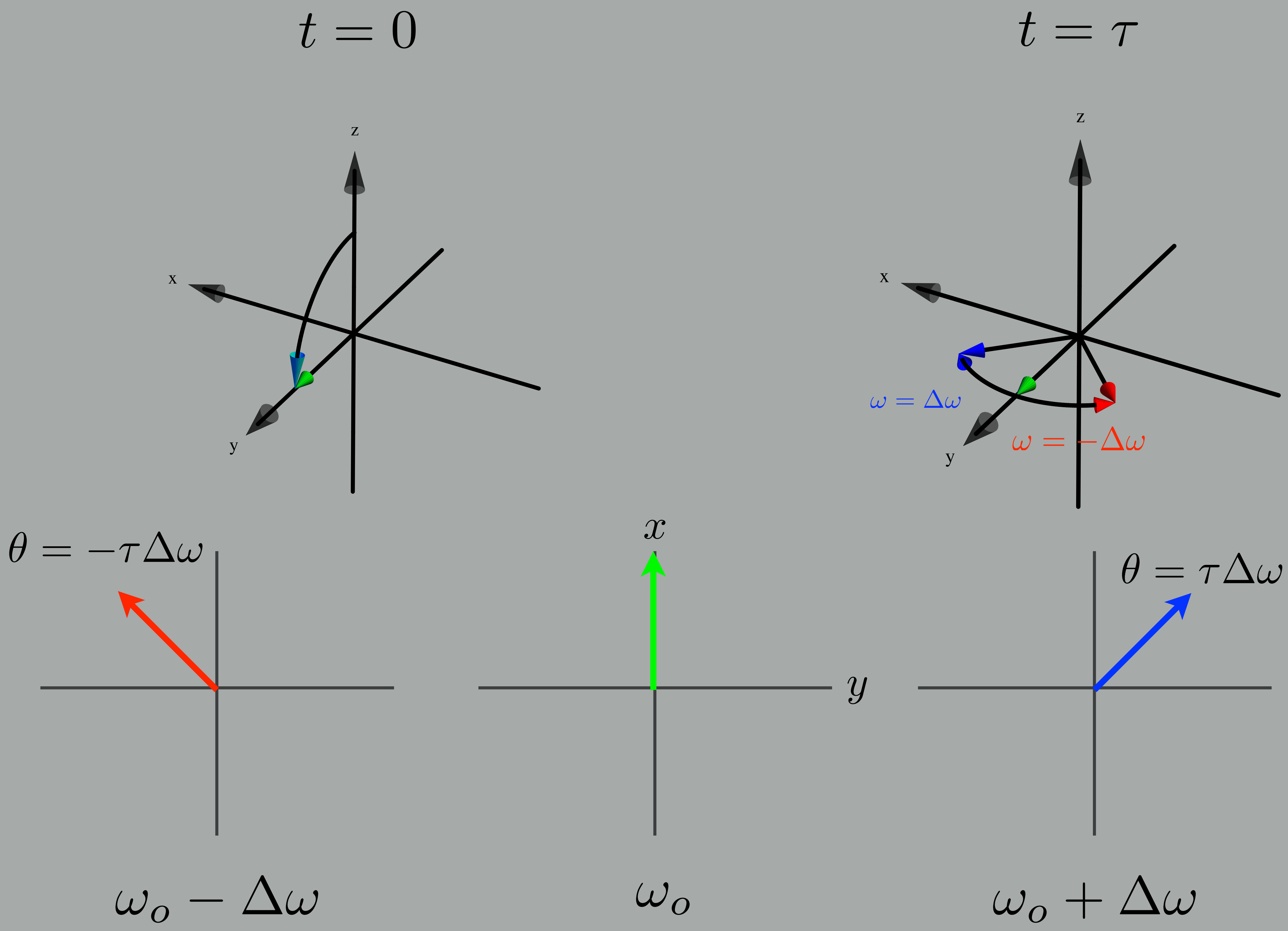
A tale of 3 isochromats



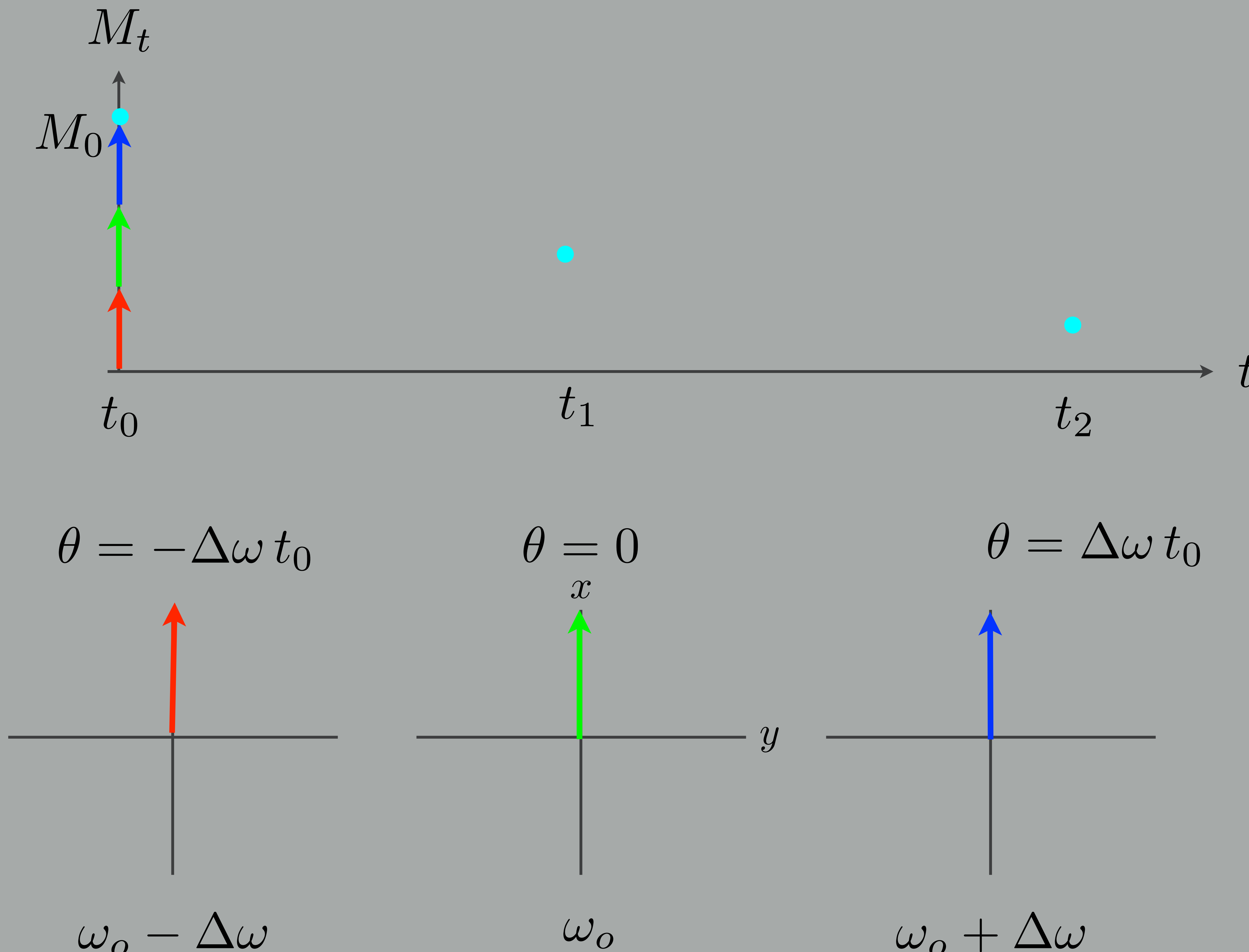
A tale of 3 isochromats



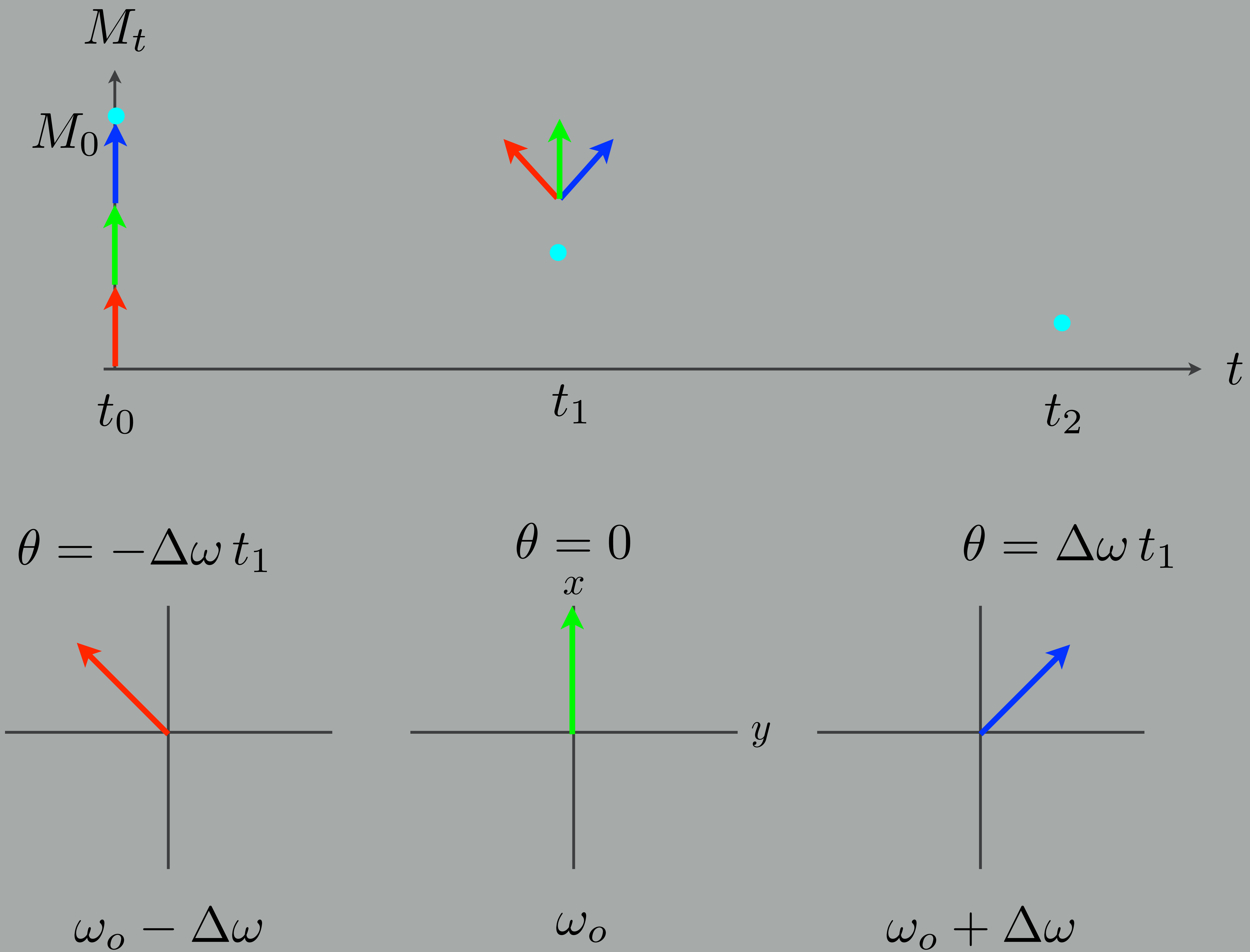
A tale of 3 isochromats



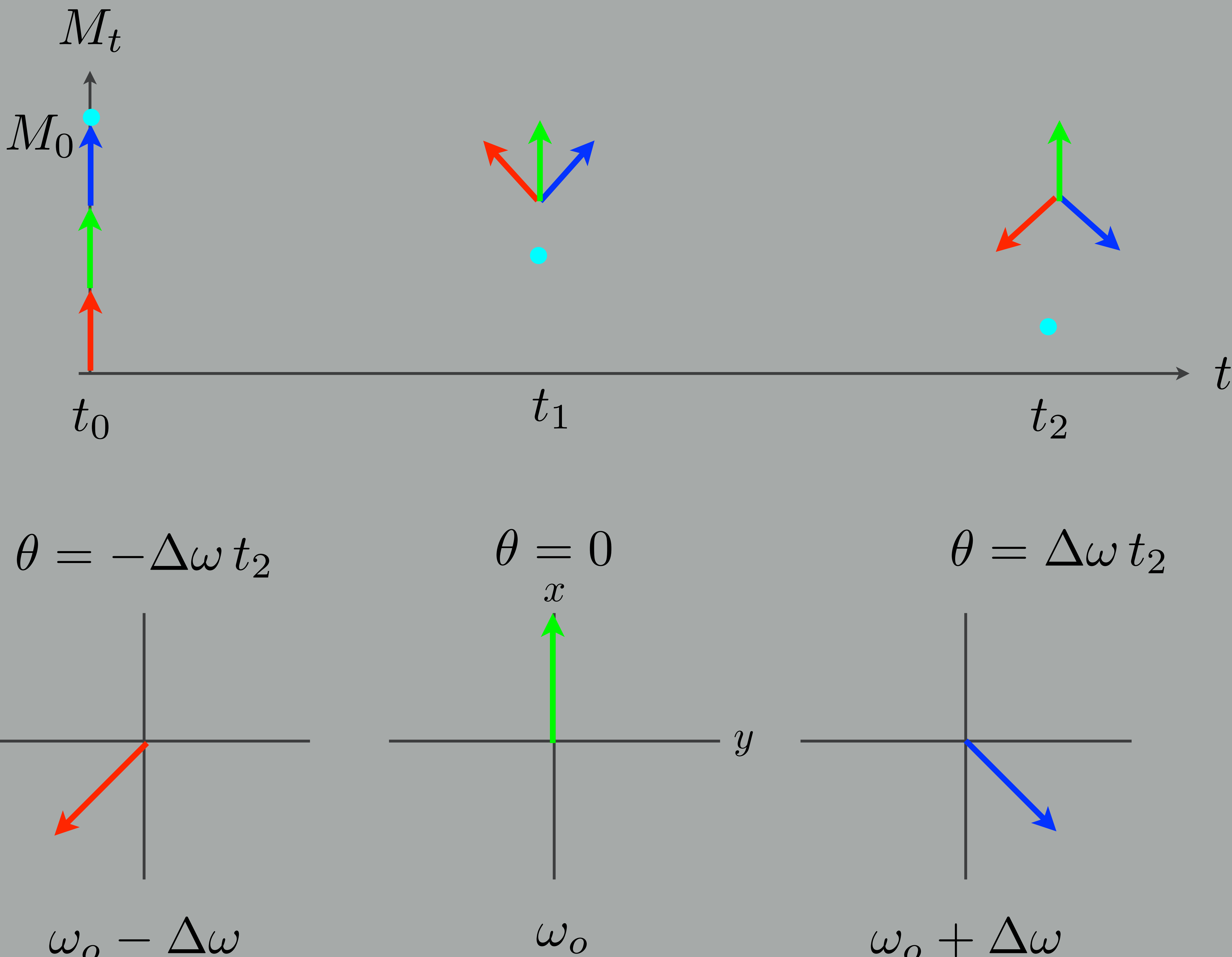
A tale of 3 isochromats



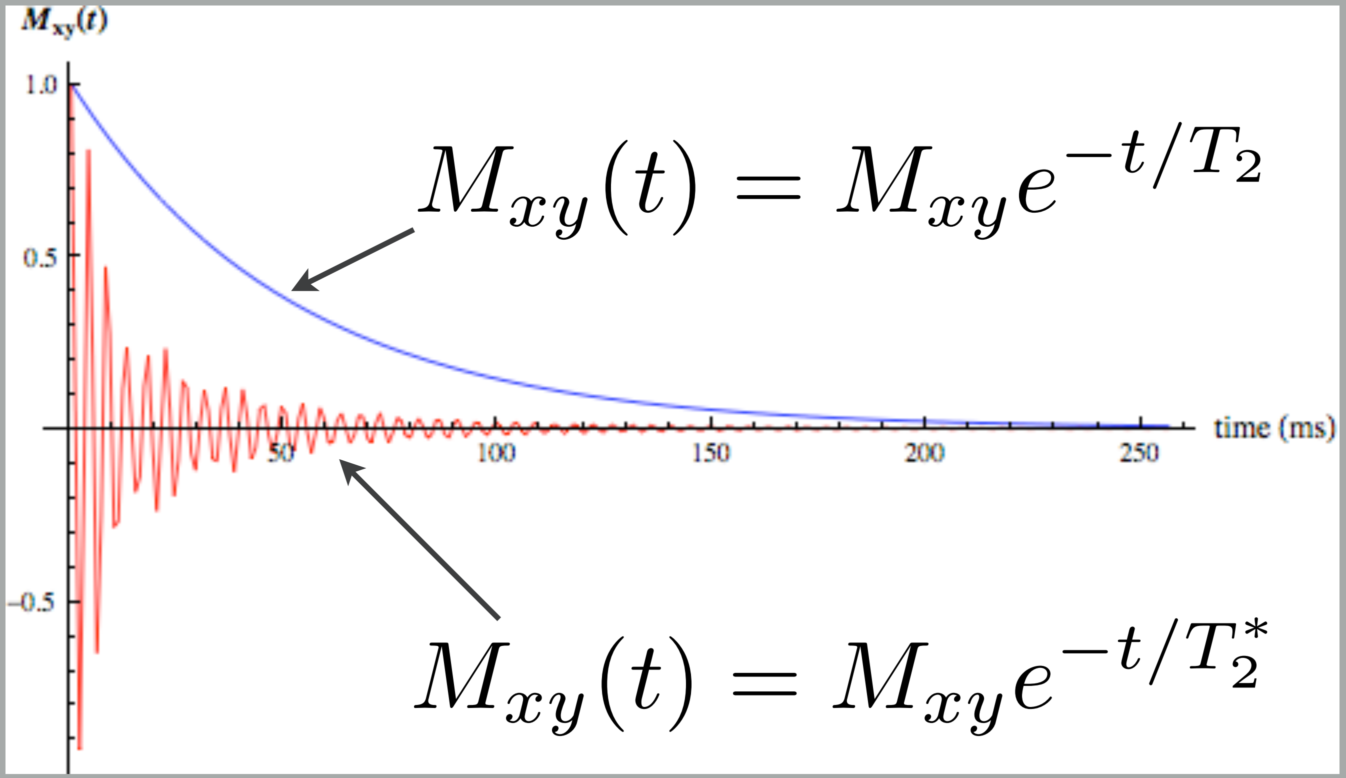
A tale of 3 isochromats



A tale of 3 isochromats



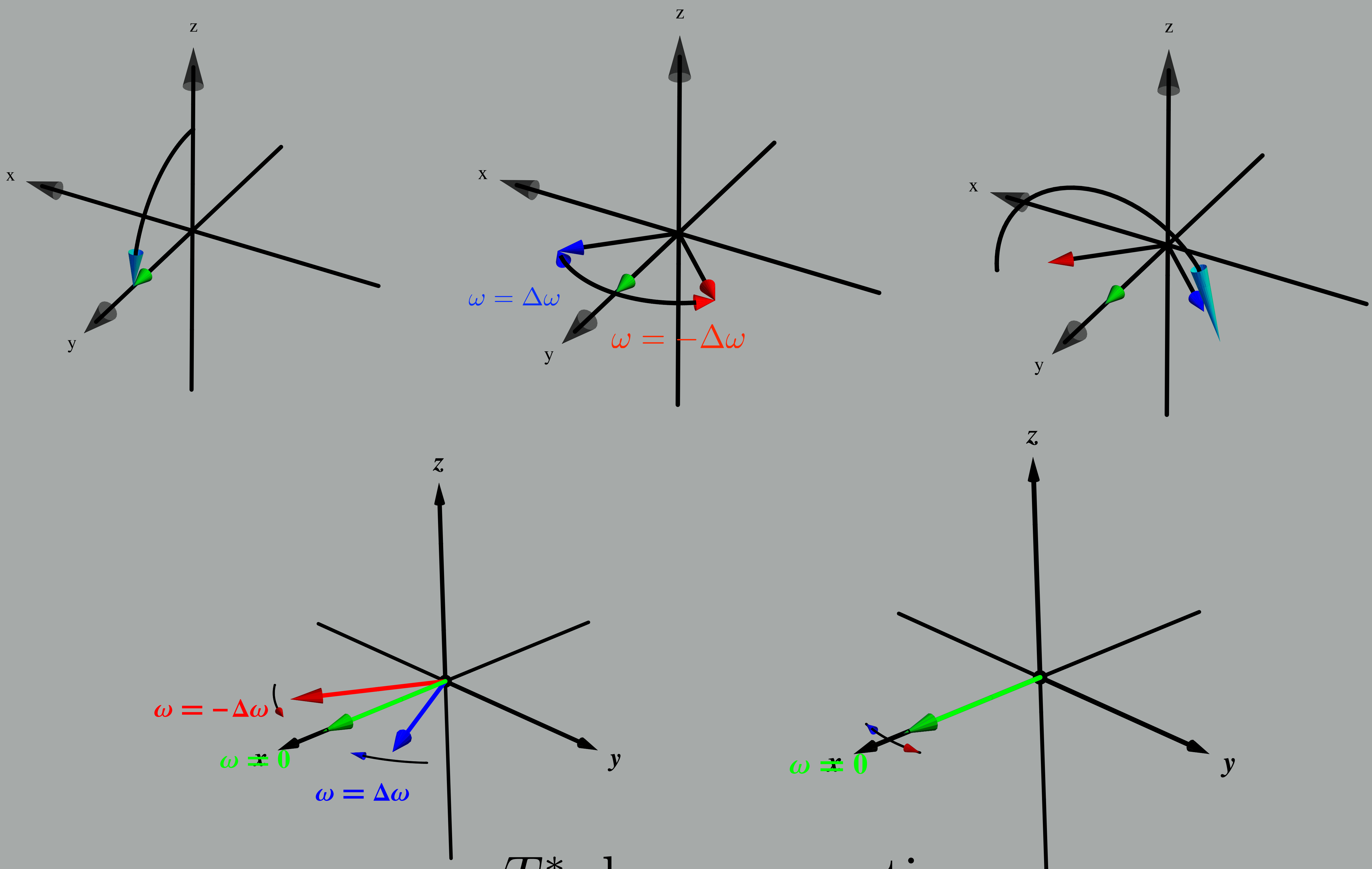
Free Induction Decay



T_2 is irreversible intrinsic transverse decay

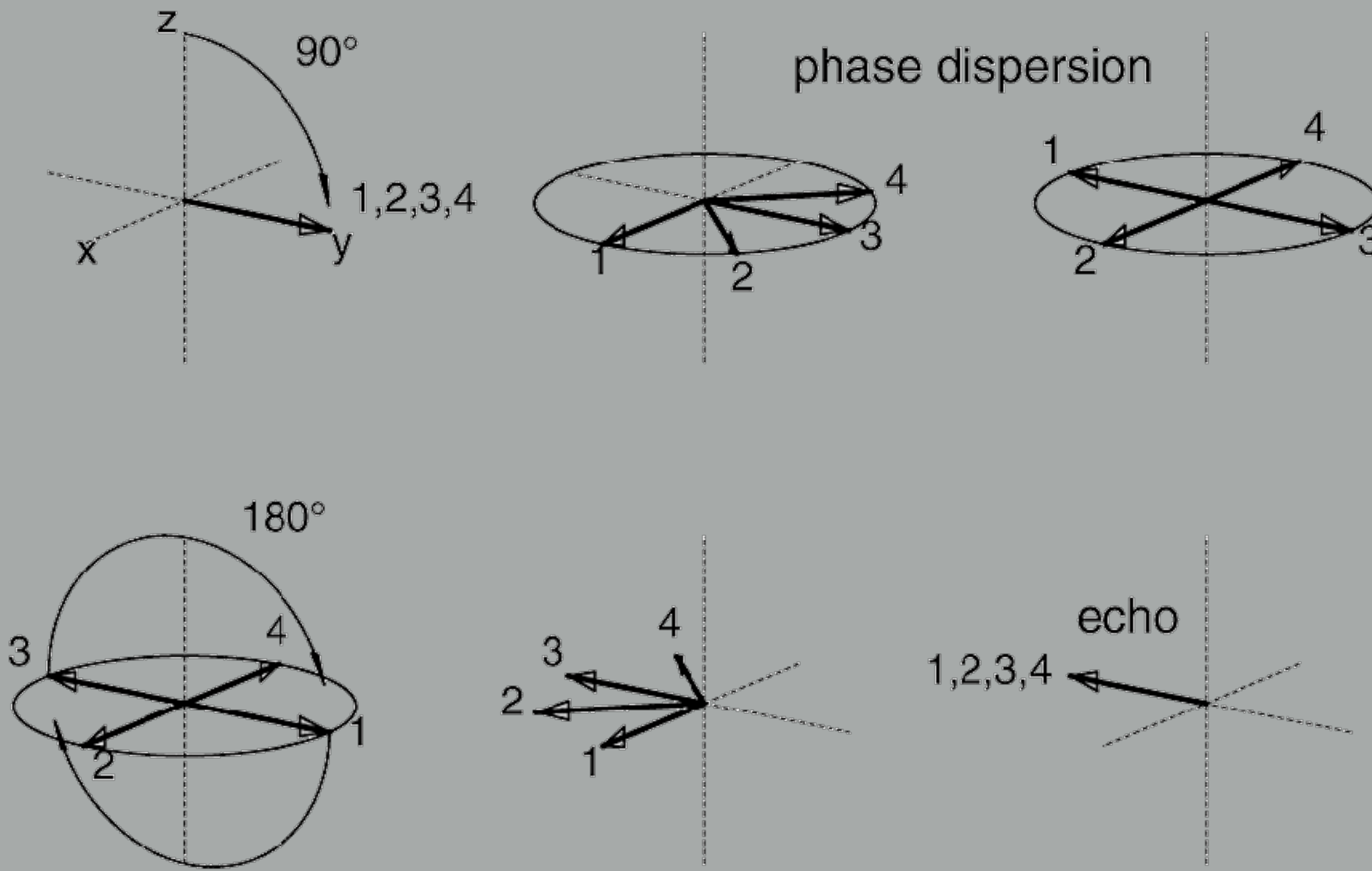
T_2^* includes reversible decay due to field inhomogeneities

Eliminating T2* effects: Spin Echo

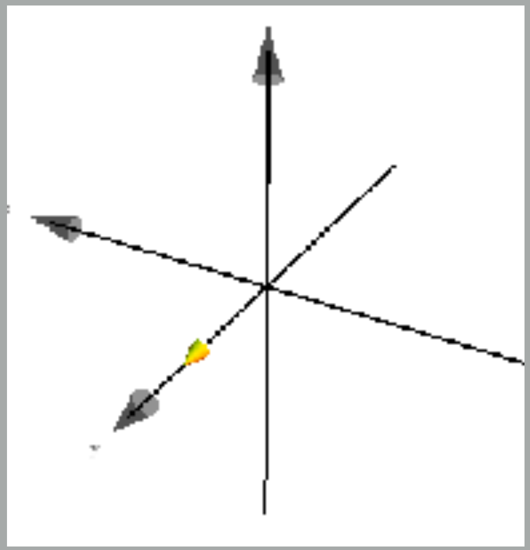


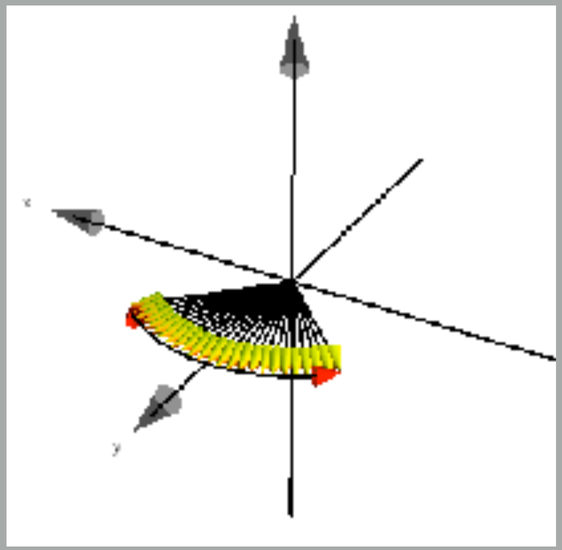
T_2^* decay correction

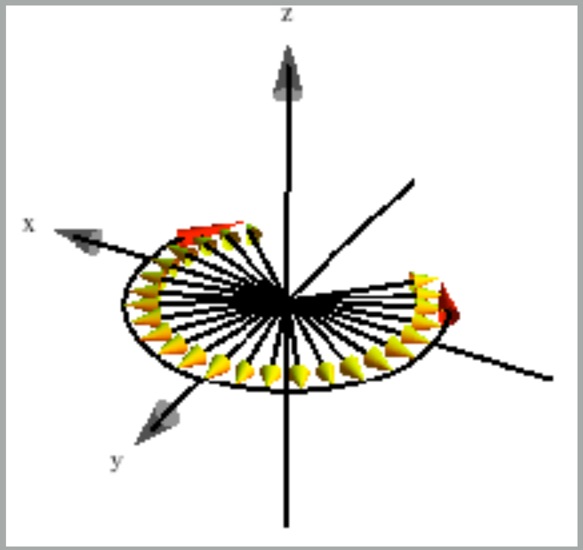
Formation of a Spin Echo

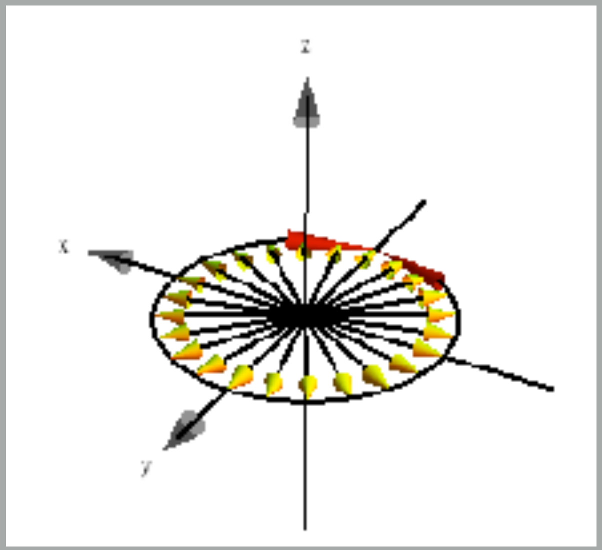


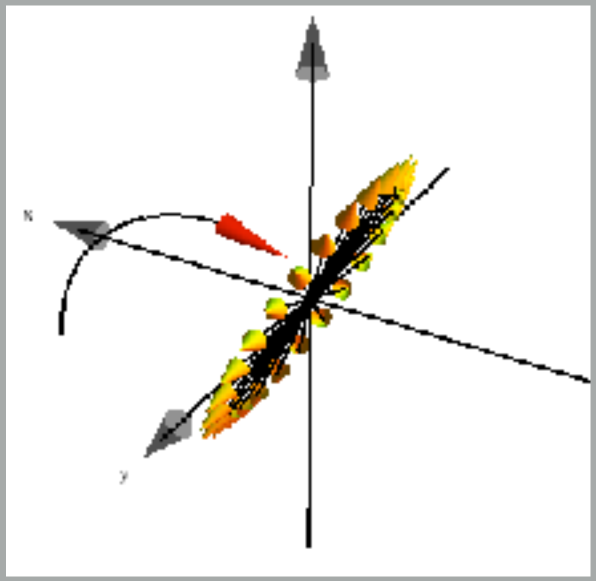
Spin echo



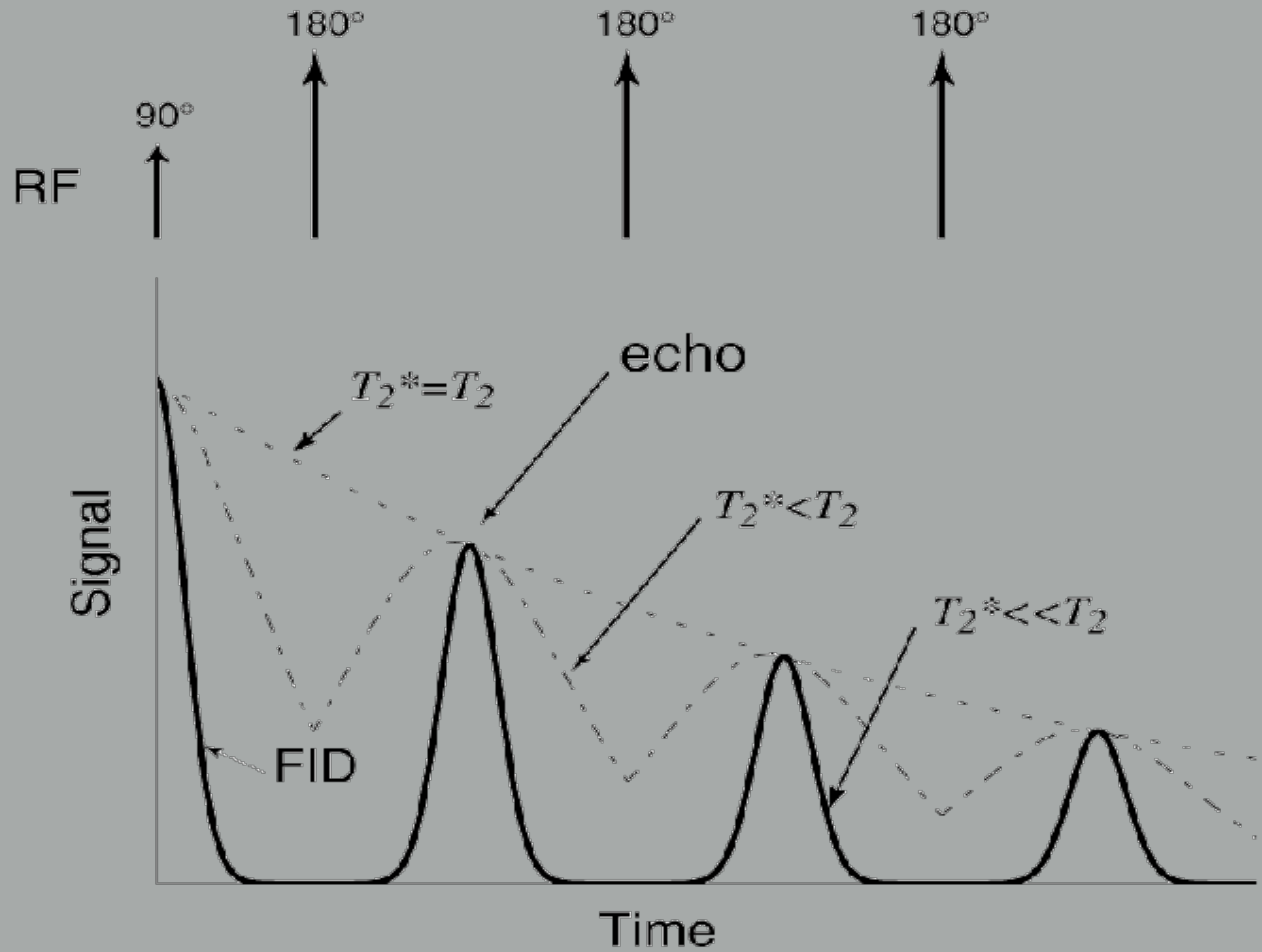




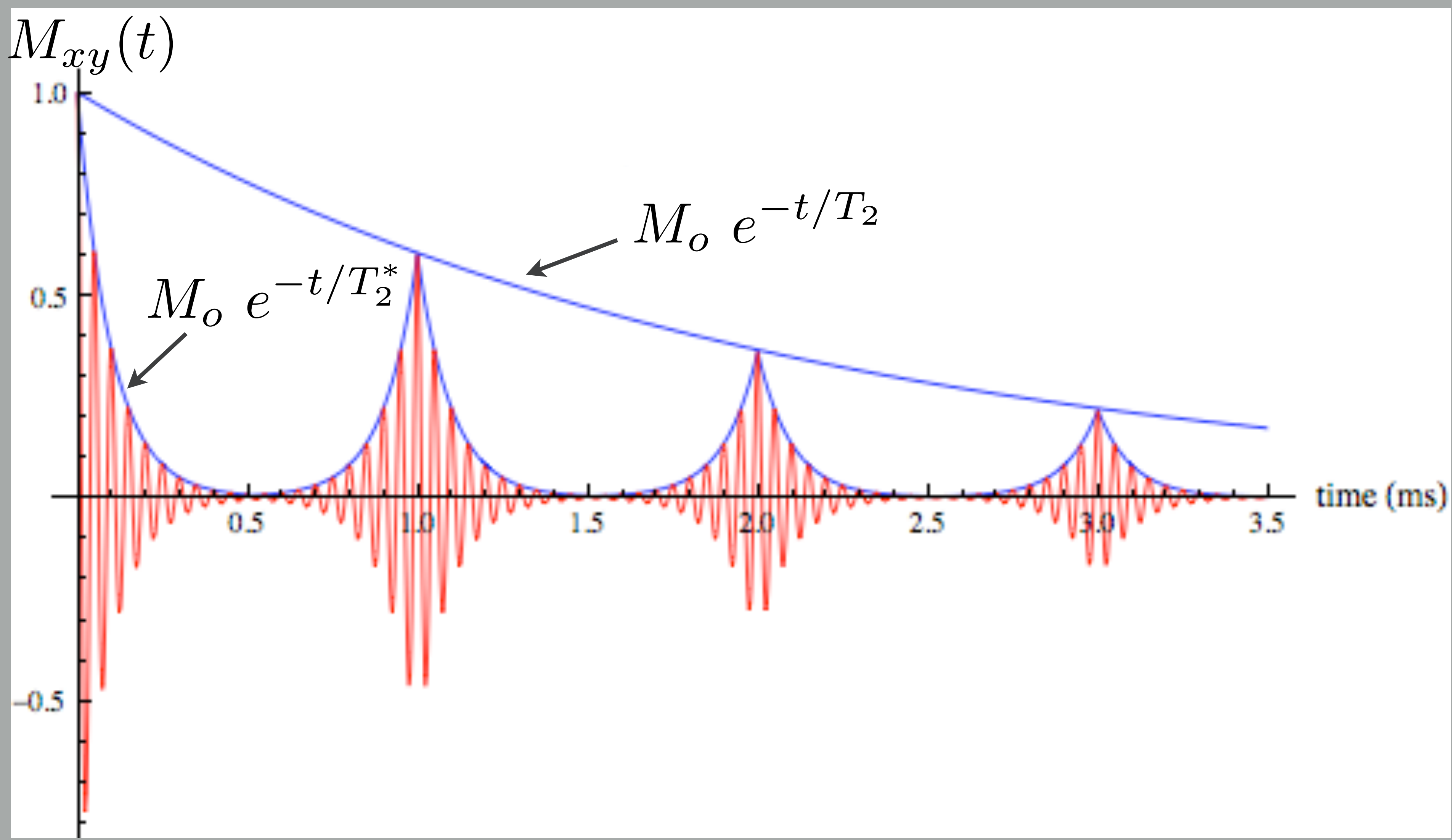




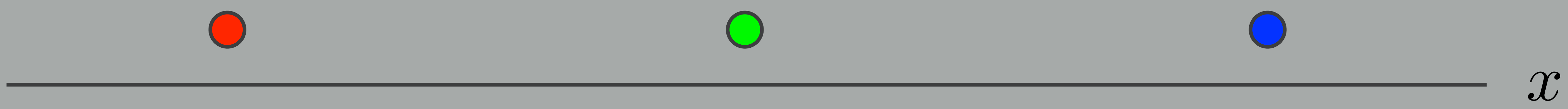
Spin Echoes



Multiple spin echos



The NMR signal



$$s(\omega) = m_\perp(x_1)e^{-i\omega_1 t} + m_\perp(x_2)e^{-i\omega_2 t} + m_\perp(x_3)e^{-i\omega_3 t}$$

$$s(\omega) = \sum_{i=1}^{3} m_\perp(x_i)e^{-i\omega_i t}$$

The NMR signal phase

$$\varphi = \gamma B_1 \Delta t + \gamma B_2 \Delta t + \gamma B_3 \Delta t + \ldots$$

$$\varphi(\tau) = \gamma \int_0^\tau B(t)\, dt$$

The NMR signal phase

$$\vec{m}_\perp(t) = |\vec{m}_\perp(t)| e^{-i\varphi(t)}$$

$$\varphi = \omega t \quad \text{where} \quad \omega = \gamma B$$

$$\varphi = \gamma B_1 \Delta t + \gamma B_2 \Delta t + \gamma B_3 \Delta t + \ldots$$

$$\varphi = \gamma \Sigma_i B_i \Delta t$$

$$\varphi(\tau) = \gamma \int_0^\tau B(t)\, dt$$

The NMR signal

$$s(\omega) = \int_\Omega m_\perp(\boldsymbol{r}, t) e^{-i\varphi(x,\tau)} \, d\boldsymbol{r}$$

where $\quad \varphi(x, \tau) = \gamma \int_0^\tau B(x, t) \, dt$

The signal is the *Fourier Transform* of the transverse magnetization

The NMR signal

$$s(\omega) = \int_{\Omega} m_{\perp}(r,t) e^{-t/T_2} e^{-i\varphi(x,\tau)} dr$$

T_2 = transverse relaxation

The signal is not exactly
the *Fourier Transform*
of the transverse magnetization

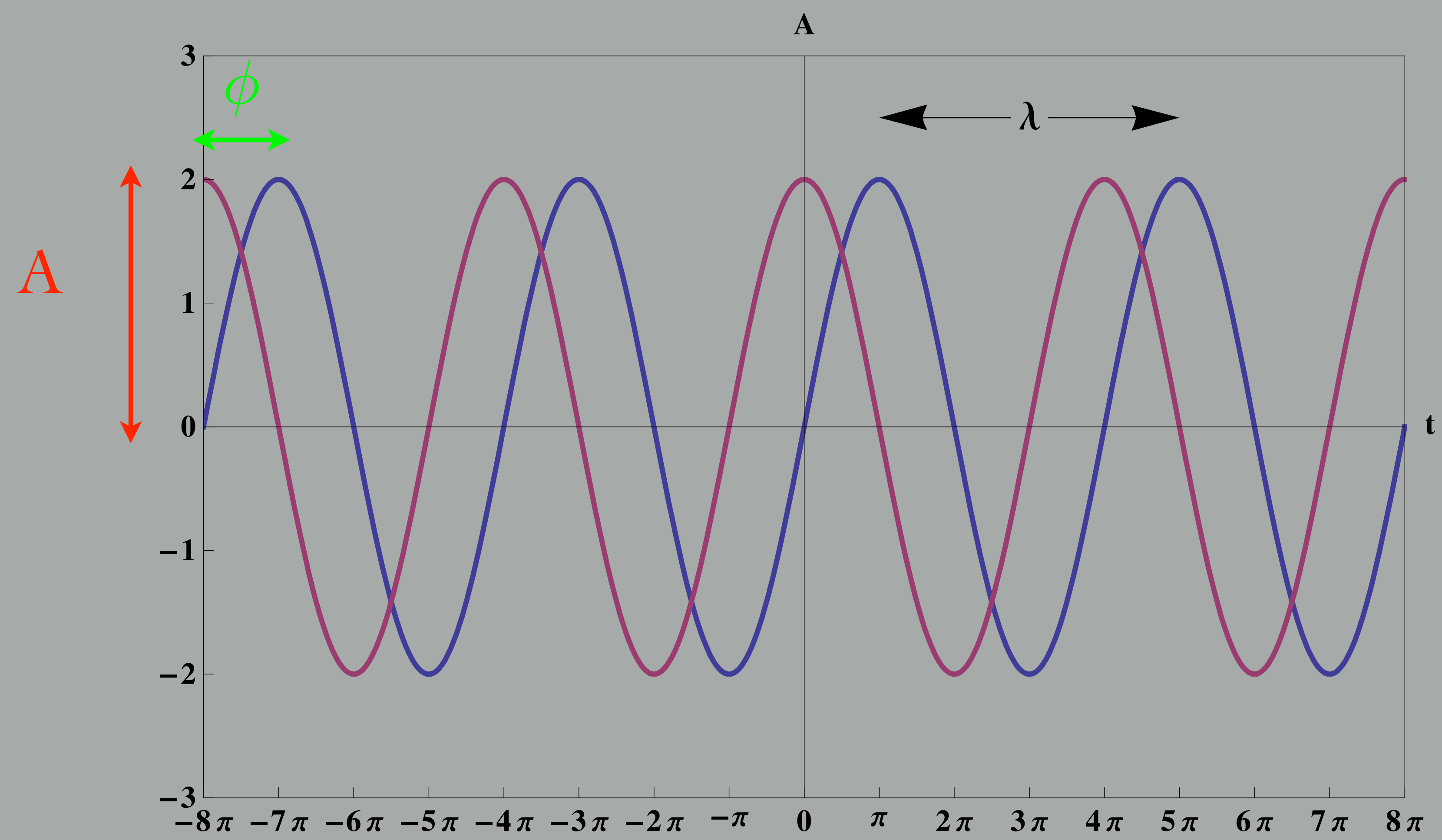
Waves in Nature

sand

water

Parameters of a wave

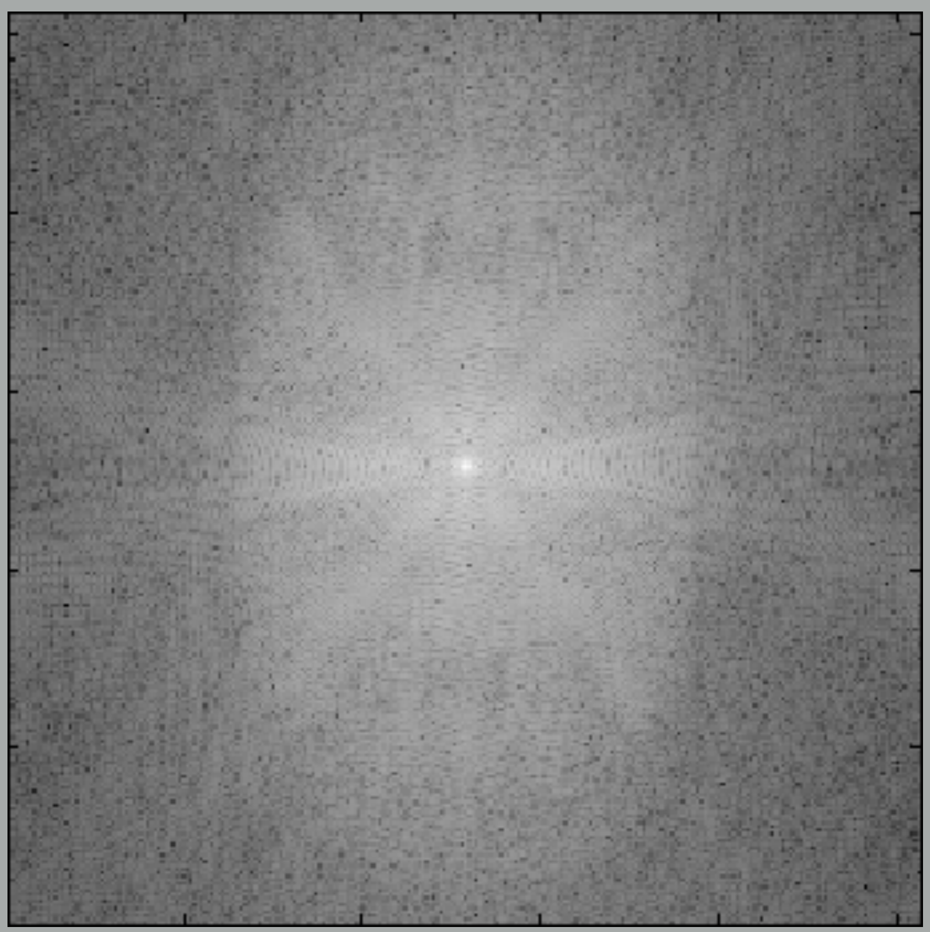
$$\omega = 2\pi/\lambda$$



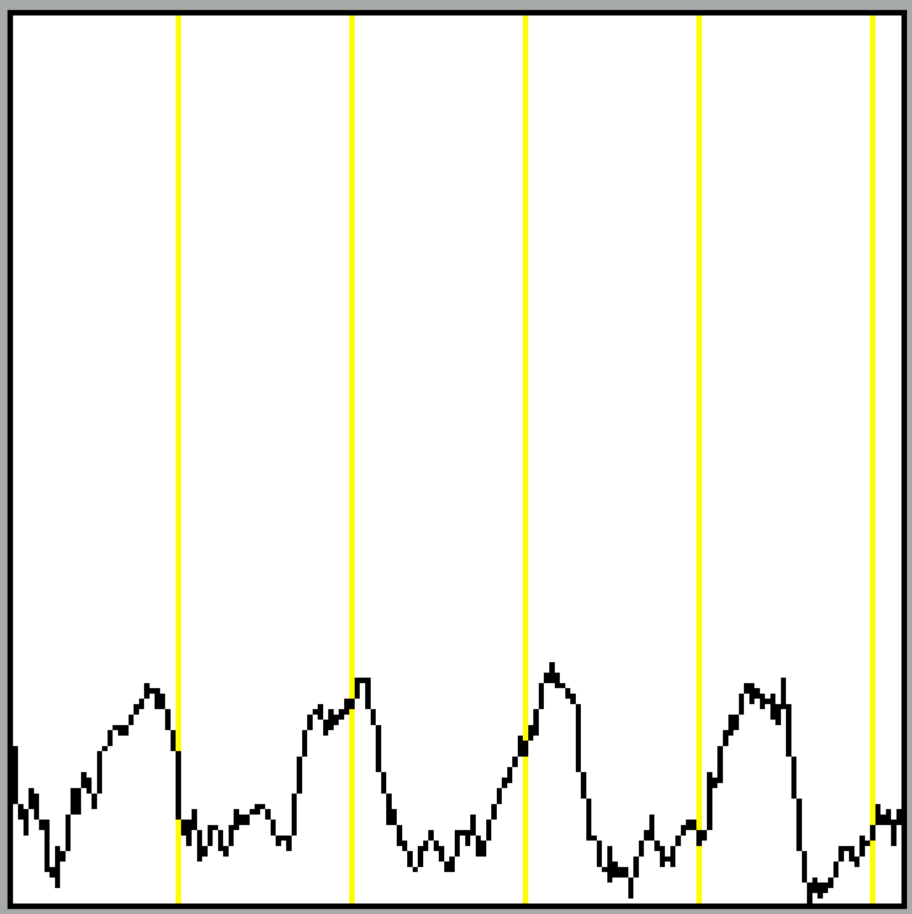
$$A\cos(\omega t + \phi)$$

$$A\cos(\omega t)$$

Waves in MRI

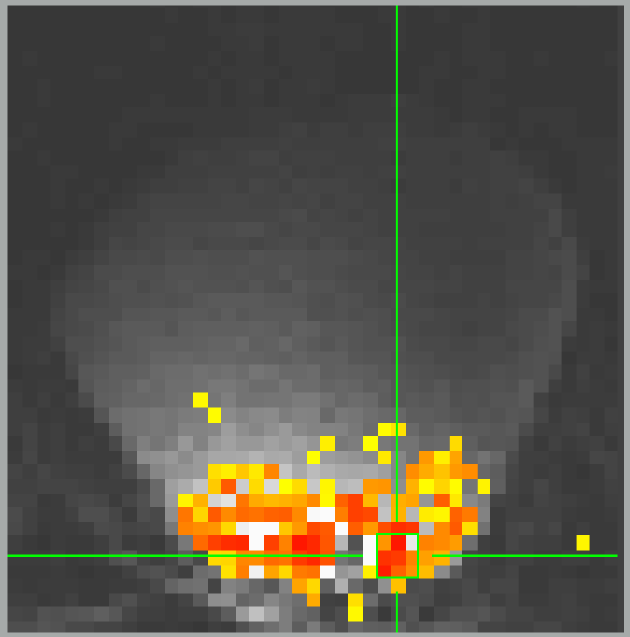


MRI

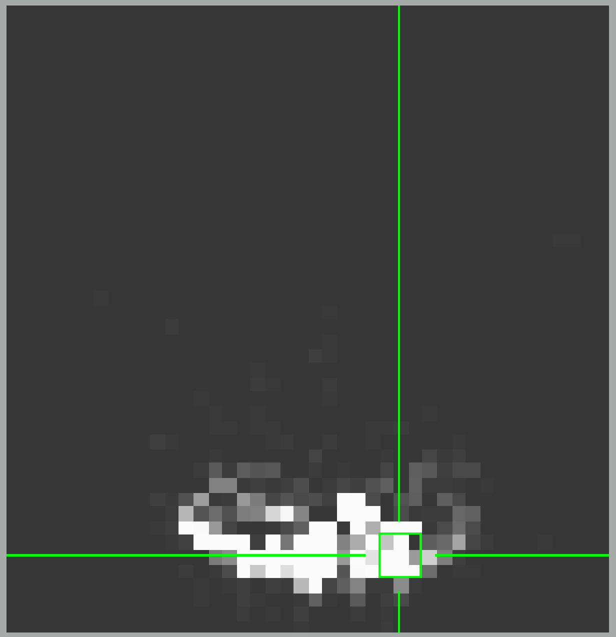


FMRI

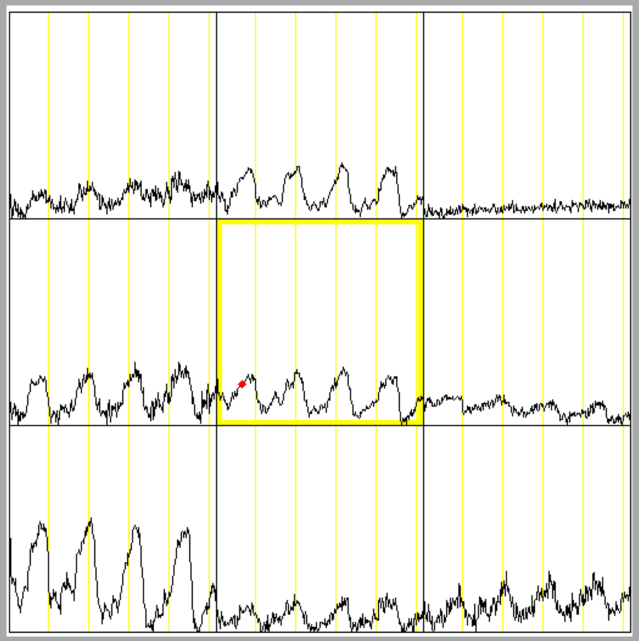
Fourier transform of the wave



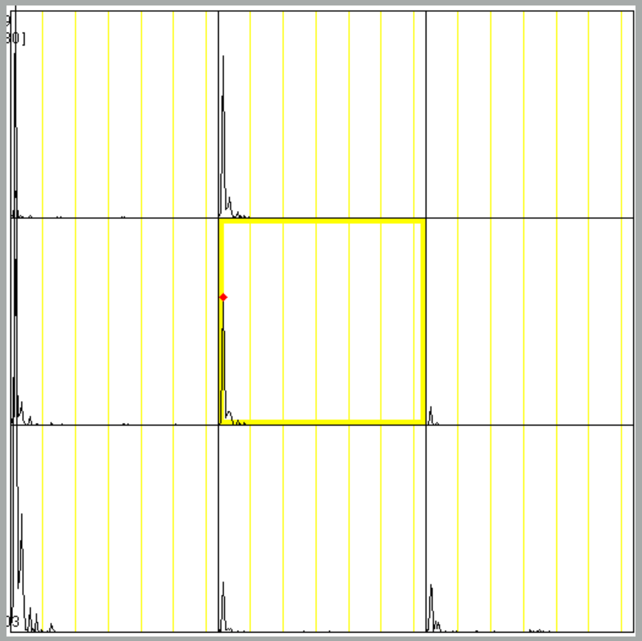
correlation coef



max |FT(time series)|



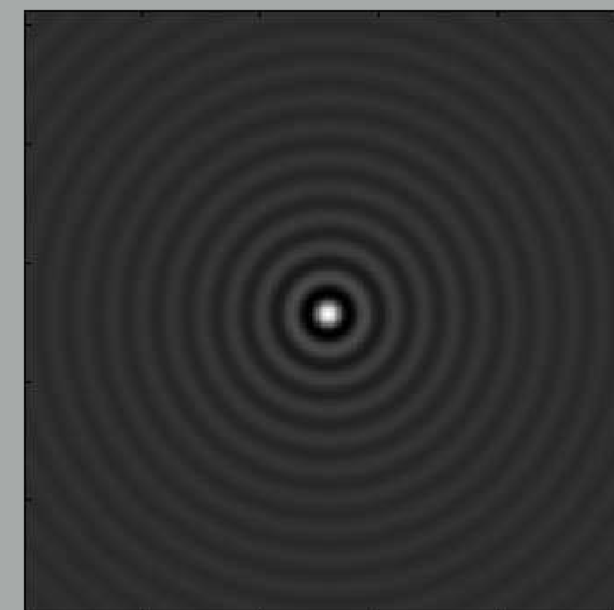
time series



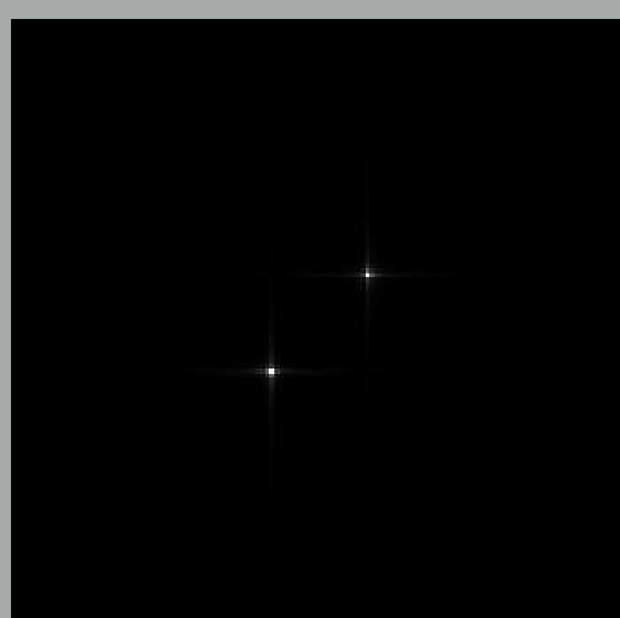
FT(time series)

Fourier transform of 2D waves

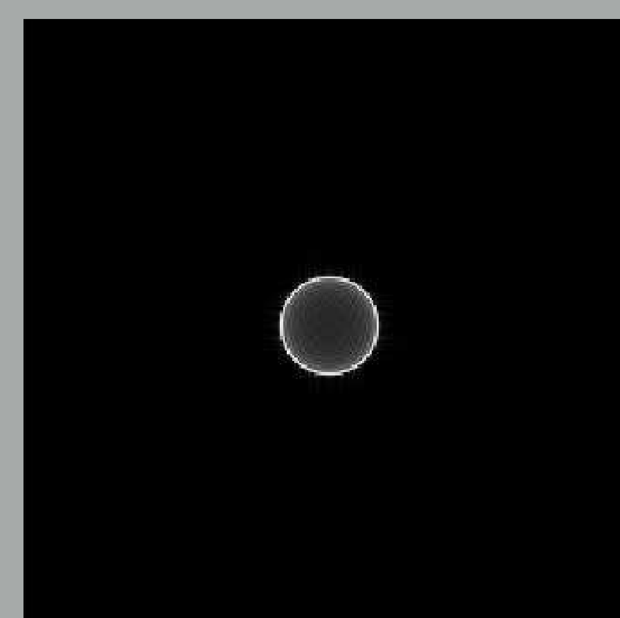
$sin(x, y)$



$sinc^2(x, y)$



$|\mathrm{FT}[sin(x, y)]|$



$|\mathrm{FT}[sinc^2(x, y)]|$